The Nature of Scientific Progress, More Error Analysis, Exp #2

Lecture # 3

Physics 2BL

Winter 2011

Outline

- How scientific knowledge progresses:
  - Replacing models
  - Restricting models
- What you should know about error analysis (so far) and more
- Limiting Gaussian distribution
- Exp. 2
- Reminder

Models

- Invented
- Properties correspond closely to real world
- Must be testable

How Models Fit Into Process of Doing Science

- Science is a process that studies the world by:
  - Limiting the focus to a specific topic (***making a choice***)
  - Observing (***making a measurement***)
  - Refining Intuitions (***making sense***) Creating
  - Extending (***seeking implications***) Predicting
  - Demanding consistency (***making it fit***) Refining or Replacing
  - Community evaluation and critique

- Start with simple model

How Models Change

- If models disagree with observation, we change the model
  - Refine - add to existing structure
  - Restrict - limit scope of utility
  - Replace - start over

Refining

- Original model consistent with observations, but not complete
- Extend model to account for new observations
- May include new concepts

  e.g. Model of interaction between charged objects; to include interactions between charged & uncharged add concept of induced charge

Restriction

- New model correct in situations where old isn't
- New model agrees w/ old over some range

⇨ Old still useful in limited range

e.g. General relativity vs. classical gravitational theory

Replacement

- Old model can’t be extended consistently
- Replace entire model

⇨ Earlier observations provide limits for new model

e.g. Geocentric vs heliocentric models for solar system

Random and independent?

Yes

- Estimating between marks on ruler or meter
- Releasing object from ‘rest’
- Mechanical vibration

o **Judgment**

o **Problems of definition**

No

- End of ruler screwy
- Reading meter from the side (speedometer effect)
- Scale not zeroed
  Reaction time delay

o **Calibration**

o **Zero**

Random & independent errors:

$$q = x + y - z$$

$$\delta q = \sqrt{(\delta x)^2 + (\delta y)^2 + (\delta z)^2}$$

$$q = Bx$$

$$\delta q = |B|\delta x$$

$$\frac{\delta q}{|q|} = \frac{\delta x}{|x|}$$

$$q = x \times y \div z$$

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2 + \left(\frac{\delta z}{z}\right)^2}$$

$$q = q(x, y, z)$$

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right)^2 + \left(\frac{\partial q}{\partial y}\delta y\right)^2 + \left(\frac{\partial q}{\partial z}\delta z\right)^2}$$

Propagation in formulas

Independent

Propagate error in steps

For example:

$$q = \frac{x}{y - z}$$

- First find

$$p = y - z$$

$$\delta p = \sqrt{(\delta y)^2 + (\delta z)^2}$$

- Then

$$q = \frac{x}{p}$$

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta p}{p}\right)^2}$$

An Important Simplifying Point

$$h = \frac{1}{2} g t^2$$

$$g = 2h/t^2 \,, \delta h/h = 5\%, \delta t/t = 0.1\%$$

Requires random & ind. errors!

$$\frac{\delta g}{g} = \sqrt{\left(\frac{\delta h}{h}\right)^2 + \left(2\frac{\delta t}{t}\right)^2}$$

$$\delta g/g = \sqrt{5\%^2 + (2\times 0.1\%)^2}$$

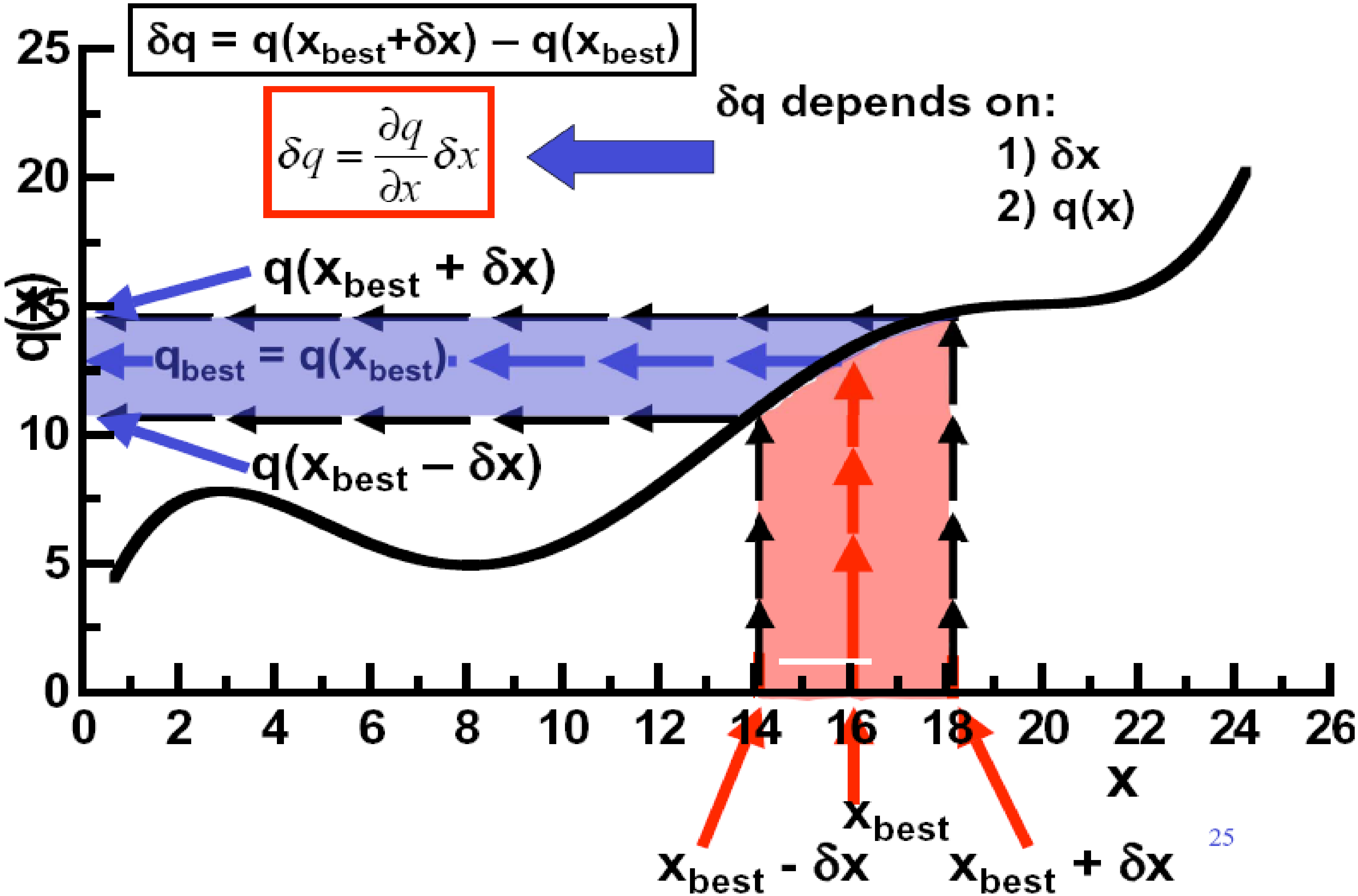
$$\delta g/g = 0.050039984 = 5\%$$

- Often the error is dominated by error in least accurate measurement

⇨Simplifies calc.

⇨Suggests improvements in experiment

Error Propagation



From Yagil

General Formula for error propagation

For independent, random errors

$$\delta q = \left| \frac{dq}{dx} \right| \delta x$$

$$\boxed{\delta q = \sqrt{\left(\frac{\partial q}{\partial x} \delta x \right)^2 + \left(\frac{\partial q}{\partial y} \delta y \right)^2}}$$

From Yagil

Analyzing Multiple Measurements

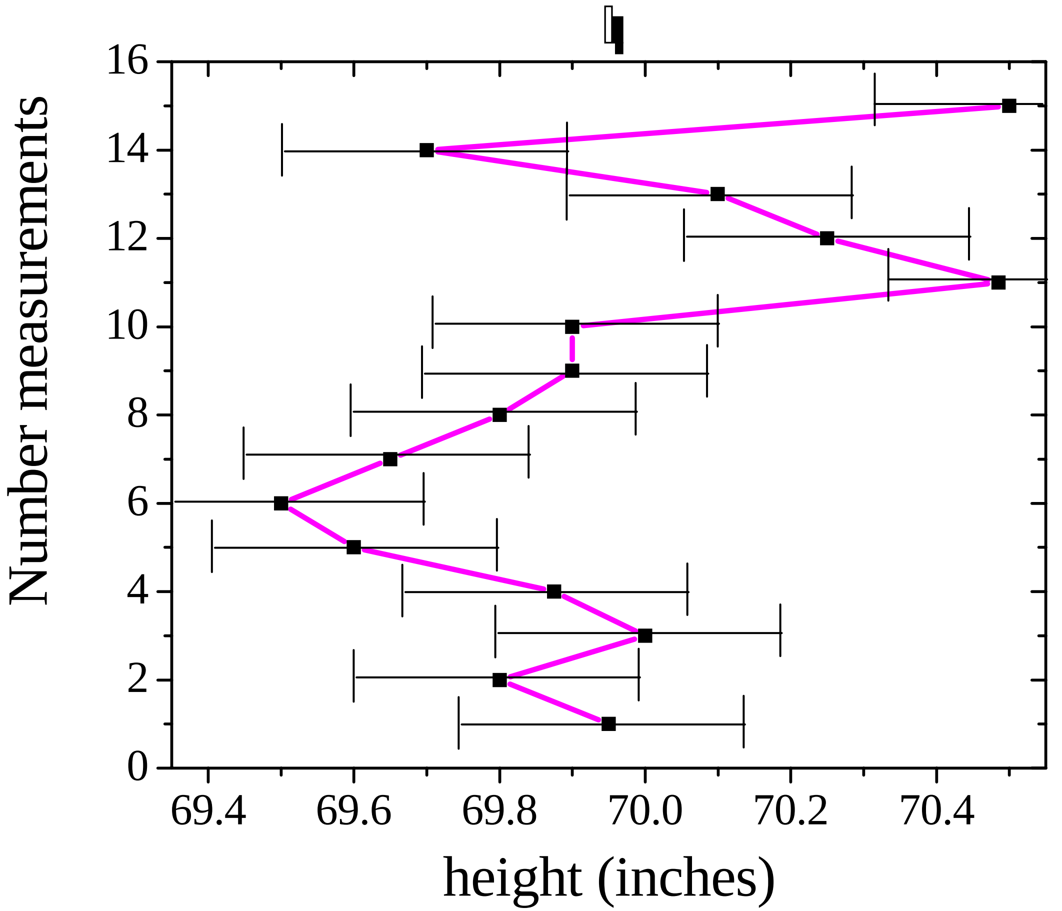
- Repeat measurement of *x* many times
- Best estimate of *x* is average (mean)

$$x_1, x_2, \cdots, x_N$$

$$x_{best} = \bar{x} = \frac{x_1 + x_2 + \cdots + x_N}{N}$$

$$\bar{x} = \frac{\sum x_i}{N}$$

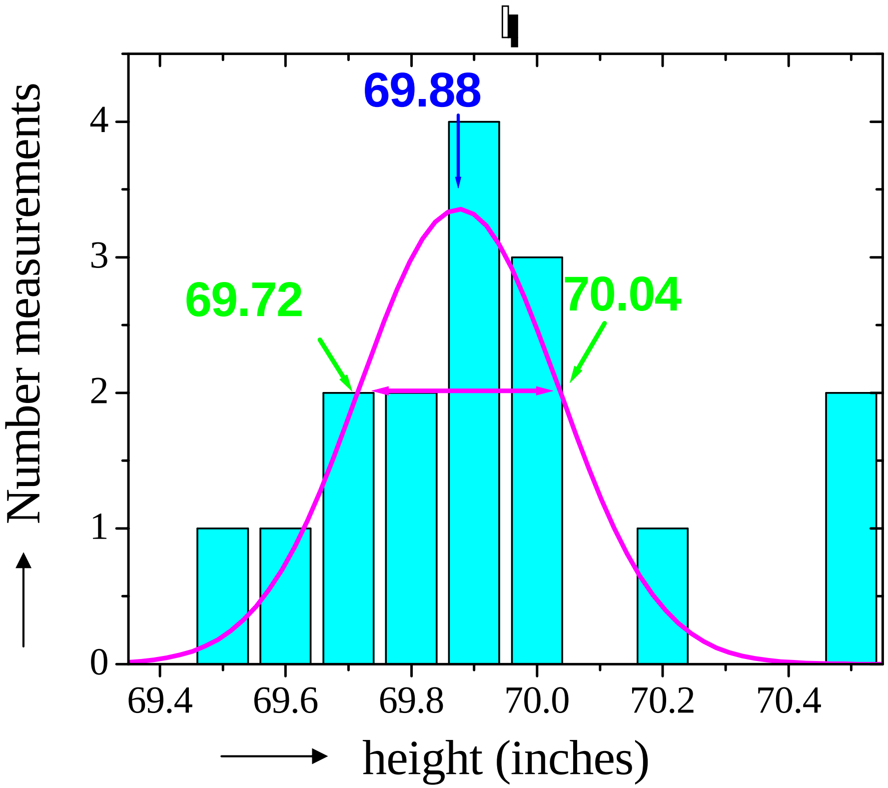
Repeated Measurements



How are Measured Values Distributed?

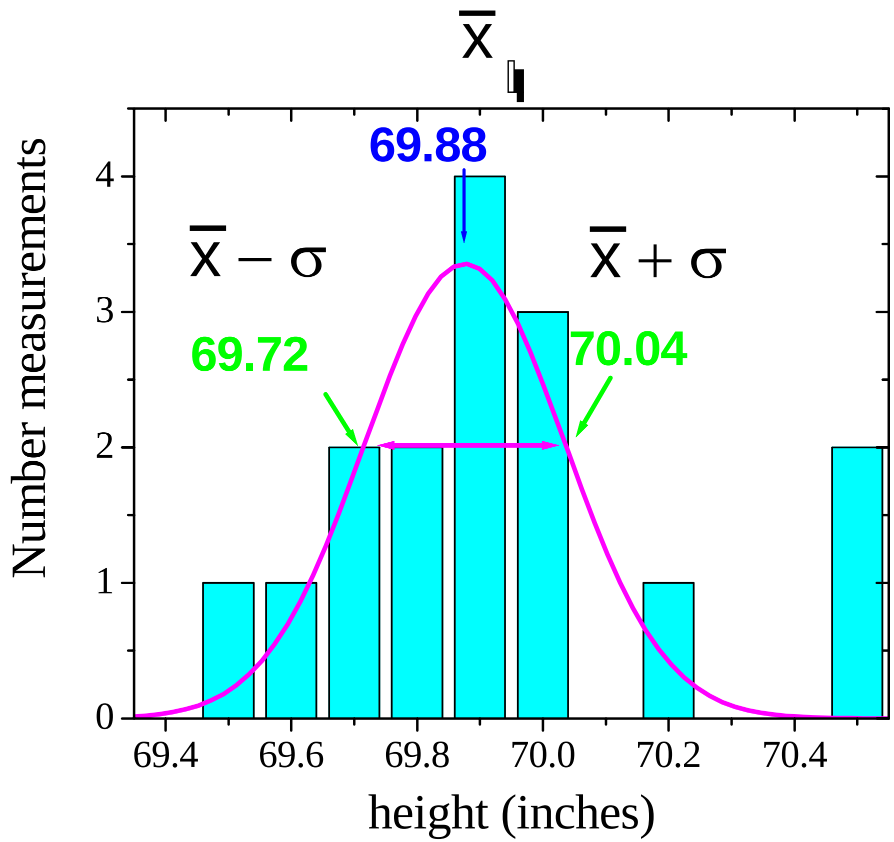
- If errors are random and independent:
  - Expect most values near true value
  - Expect few values far from true value

⇨ Assume values are distributed *normally*



$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-(x-\overline{x})^2 / 2\sigma^2)$$

Normal Distribution



$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-(x-\bar{x})^2 / 2\sigma^2)$$

Error of an Individual Measurement

- How precise are measurements of *x*?
- Start with each value's deviations from mean
- Deviations average to zero, so square, then average, then take square root
- ~68% of time, x_i will be w/in σ_x of true value

$$d_i \equiv x_i - \bar{x}$$

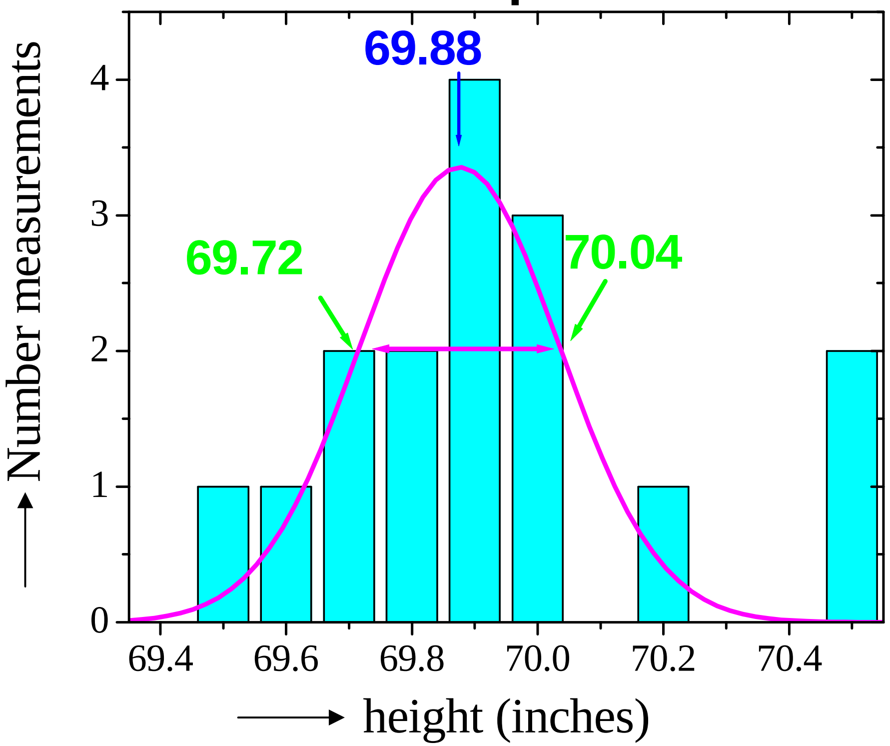
$$\bar{d} = 0$$

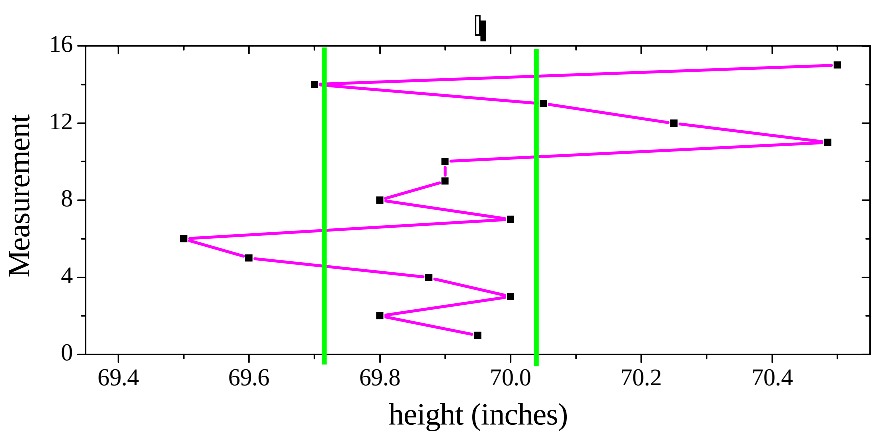
$$\sigma_x \equiv \sqrt{\overline{(d_i)^2}}$$

$$= \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

Take σ_x as error in individual measurement - called standard deviation

Standard Deviation





Drawing a Histogram

1. Determine the range of your data by subtracting the smallest number from the largest one.

2. The number of bins should be approximately $\sqrt{N}$ and the width of a bin should be the range divided by $\sqrt{N}$.

3. Make a list of the boundaries of each bin and determine which bin each of your data points should fall into.

4. Draw the histogram. The y axis should be the number of values that fall into each bin.

5. Sometimes this procedure will not produce a good histogram. If you make too many bins the histogram will be flat and too few bins will not show the curve on either side of the maximum. You might need to play around with the number of bins to produce a better histogram.

Error of the Mean

- Expect error of mean to be lower than error of the measurements it's calculated from
- Divide SD by square root of number of measurements
- Decreases slowly with more measurements

Standard Deviation of the Mean (SDOM)

or

Standard Error

or

Standard Error of the Mean

$$\sigma_{\bar{x}} = \sigma_x \big/ \sqrt{N}$$

Summary

- Average

$$\bar{x} = \frac{\sum x_i}{N}$$

- Standard deviation

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

- Standard deviation of the mean

$$\sigma_{\bar{x}} = \sigma_x / \sqrt{N}$$

The Four Experiments

- **Determine the average density of the earth**

Weigh the Earth, Measure its volume

– Measure simple things like lengths and times

– Learn to estimate and propagate errors

- **Non-Destructive measurements of densities, inner structure of objects**

– Absolute measurements *vs.* Measurements of variability

– Measure moments of inertia

– Use repeated measurements to reduce random errors

- **Construct and tune a shock absorber**

– Adjust performance of a mechanical system

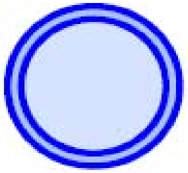
– Demonstrate critical damping of your shock absorber

- **Measure coulomb force and calibrate a voltmeter.**

– Reduce systematic errors in a precise measurement.

Experiment 2

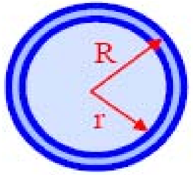
- Devise a simple, fast, and non-destructive method to measure the variation in thickness of the shell of a large number of racquet balls to determine if the variation in thickness is much less than 10%.



Racquet Balls

We should check if the variation in *d* is much less than 10%.

Moments of Inertia



$$I = \frac{2}{5} M \frac{R^5 - r^5}{R^3 - r^3}$$

We want R-r to much less than 10%.

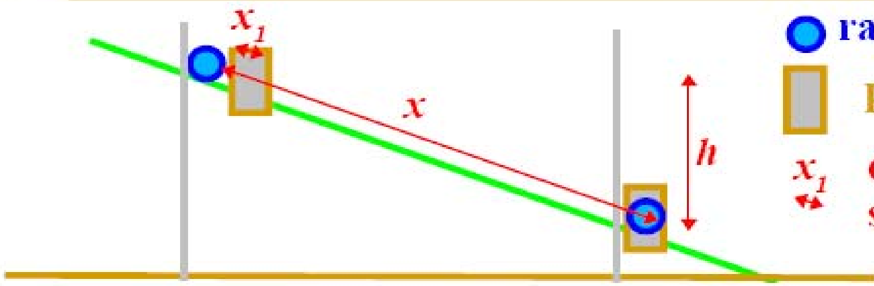
Problem can be solved by

Measuring the mass and

moment of inertia of the balls

- For the balls, we need to measure the variation in thickness.

Measuring I by Rolling Objects



racketball

photogate timer

x_1 distance before starting timer

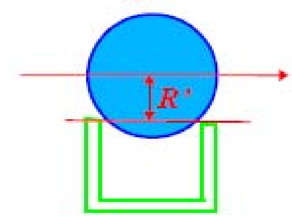
1. Measure M and R
2. Using photo gate timer measure the time, t, to travel distance x

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$ energy conservation

$$v = R'\omega$$ rolling radius

$$v = \frac{2x}{t}$$ for uniform acceleration

rolling radius R'



$$Mgh = \frac{1}{2}v^2\left(M + \frac{I}{R'^2}\right)$$

$$gh = \frac{2x^2}{t^2}\left(1 + \frac{I}{MR'^2}\right)$$

$$\frac{I}{MR'^2} = \left(\frac{ght^2}{2x^2} - 1\right)$$

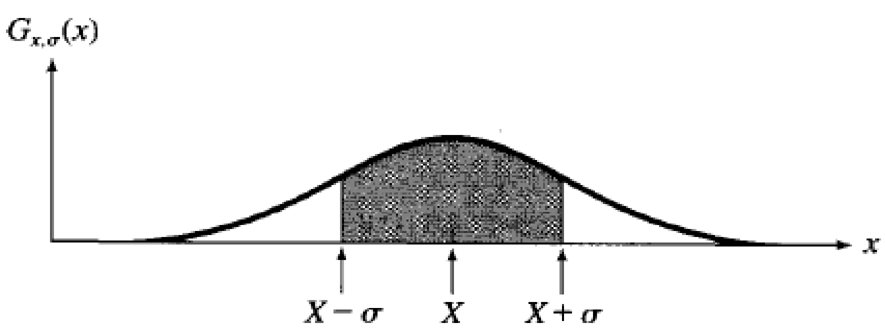
$$\tilde{I} \equiv \frac{I}{MR^2} = \frac{R'^2}{R^2}\left(\frac{ght^2}{2x^2} - 1\right)$$

The Gauss, or Normal Distribution

normalize $\quad e^{-(x-X)^2/2\sigma^2} \longrightarrow \int_{-\infty}^{+\infty} f(x)dx = 1$

$$\boxed{G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-X)^2/2\sigma^2}}$$

standard deviation σ_x = width parameter of the Gauss function σ
the mean value of x = true value X

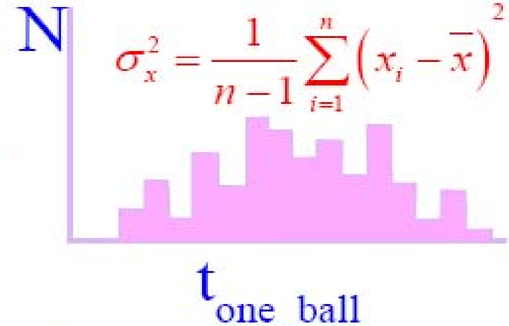


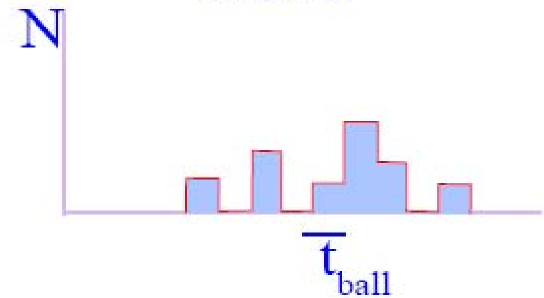
Measuring the Variation in Thickness of the Shell

- 1. Measure rolling time of one ball many times to determine the measurement error in *t*, $\sigma_{measurement}$
- 2. Measure rolling time of many balls to determine the total spread in *t*, σ_{total}
- 3. Calculate the spread in time due to ball manufacture, $\sigma_{manufacture}$, by subtracting the measurement error
- 4. Propagate error on *t* into error on *I* and then into error on thickness *d*

$$\sigma_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

statistical analysis





$$\sigma_{total} = \sigma_{manufacture} \oplus \sigma_{measruement}$$

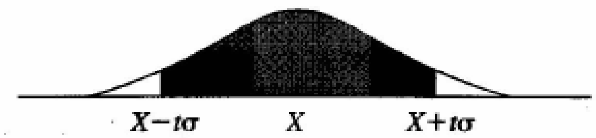
$$\sigma_t \longrightarrow \sigma_I \longrightarrow \sigma_d$$

variation in *t* → variation in *I* → variation in *d*

t score

$$t = \frac{x - \bar{x}}{\sigma_x}$$

Table A. The percentage probability, $Prob(\text{within } t\sigma) = \int_{X-t\sigma}^{X+t\sigma} G_{X,\sigma}(x)\,dx$, as a function of t.



t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00	0.80	1.60	2.39	3.19	3.99	4.78	5.58	6.38	7.17
0.1	7.97	8.76	9.55	10.34	11.13	11.92	12.71	13.50	14.28	15.07
0.2	15.85	16.63	17.41	18.19	18.97	19.74	20.51	21.28	22.05	22.82
0.3	23.58	24.34	25.10	25.86	26.61	27.37	28.12	28.86	29.61	30.35
0.4	31.08	31.82	32.55	33.28	34.01	34.73	35.45	36.16	36.88	37.59
0.5	38.29	38.99	39.69	40.39	41.08	41.77	42.45	43.13	43.81	44.48
0.6	45.15	45.81	46.47	47.13	47.78	48.43	49.07	49.71	50.35	50.98
0.7	51.61	52.23	52.85	53.46	54.07	54.67	55.27	55.87	56.46	57.05
0.8	57.63	58.21	58.78	59.35	59.91	60.47	61.02	61.57	62.11	62.65
0.9	63.19	63.72	64.24	64.76	65.28	65.79	66.29	66.80	67.29	67.78
1.0	68.27	68.75	69.23	69.70	70.17	70.63	71.09	71.54	71.99	72.43
1.1	72.87	73.30	73.73	74.15	74.57	74.99	75.40	75.80	76.20	76.60
1.2	76.99	77.37	77.75	78.13	78.50	78.87	79.23	79.59	79.95	80.29
1.3	80.64	80.98	81.32	81.65	81.98	82.30	82.62	82.93	83.24	83.55
1.4	83.85	84.15	84.44	84.73	85.01	85.29	85.57	85.84	86.11	86.38
1.5	86.64	86.90	87.15	87.40	87.64	87.89	88.12	88.36	88.59	88.82
1.6	89.04	89.26	89.48	89.69	89.90	90.11	90.31	90.51	90.70	90.90
1.7	91.09	91.27	91.46	91.64	91.81	91.99	92.16	92.33	92.49	92.65
1.8	92.81	92.97	93.12	93.28	93.42	93.57	93.71	93.85	93.99	94.12
1.9	94.26	94.39	94.51	94.64	94.76	94.88	95.00	95.12	95.23	95.34
2.0	95.45	95.56	95.66	95.76	95.86	95.96	96.06	96.15	96.25	96.34
2.1	96.43	96.51	96.60	96.68	96.76	96.84	96.92	97.00	97.07	97.15
2.2	97.22	97.29	97.36	97.43	97.49	97.56	97.62	97.68	97.74	97.80

$t = 1$

p. 287 Taylor

$t = 1.47$

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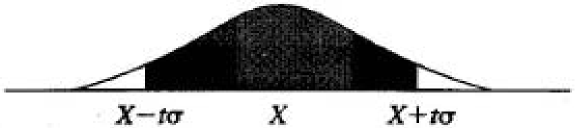

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2.3	97.86	97.91	97.97	98.02	98.07	98.12	98.17	98.22	98.27	98.32

Remember

- Lab #2
- Read lab description, prepare
- Read Taylor through Chapter 6 & 7
- Problems #6.4, #7.2