Uncertainty, Measurement, and Models
Overview Exp #1

Lecture # 2

Physics 2BL

Winter 2011

Lab TAs

	Tuesday	Wednesday	Thursday
9:00 am	704064	704066	704069
	Mui	**Kang**	**Mui**
	Progovac	**Progovac**	**Progovac**
	A02	A04	A08
12:00 pm	704065	704067	704070
	Kang	**Lopez**	**Progovac**
	Lopez	**Kang**	**Kang**
	A03	A05	A09

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Present in labs to assist

Outline

- What uncertainty (error) analysis can for you
- Issues with measurement and observation
- What does a model do?
- General error propagation formula with example
- Overview of Experiment # 1
- Homework

What is uncertainty (error)?

- Uncertainty (or error) in a measurement is not the same as a mistake
- Uncertainty results from:
  - Limits of instruments
    - finite spacing of markings on ruler
  - Design of measurement
    - using stopwatch instead of photogate
  - Less-well defined quantities
    - composition of materials

Understanding uncertainty is important

- for comparing values
- for distinguishing between models
- for designing to specifications/planning

Measurements are less useful (often useless) without a statement of their uncertainty

An example

- Batteries
  - rated for 1.5 V potential difference across terminals
  - in reality…

Utility of uncertainty analysis

- Evaluating uncertainty in a measurement
- Propagating errors – ability to extend results through calculations or to other measurements
- Analyzing a distribution of values
- Quantifying relationships between measured values

Evaluating error in measurements

- To measure height of building, drop rock and measure time to fall: $d = \frac{1}{2} g t^2$
- Measure times

  2.6s, 2.4s, 2.5s, 2.4s, 2.3s, 2.9s
- What is the “best” value
- How certain are we of it?

Calculate “best” value of the time

- Calculate average value (2.6s, 2.4s, 2.5s, 2.4s, 2.3s, 2.9s)
  - $\bar{t} = \sum_{i=1}^{n} t_i/n$
  - $\bar{t} = 2.516666666666666666666666$ s
- Is this reasonable?

Significant figures

Uncertainty in time

- Measured values - (2.6s, 2.4s, 2.5s, 2.4s, 2.3s, 2.9s)

- By inspection can say uncertainty < 0.4 s

- Calculate **standard deviation**

$$\sigma = \sqrt{\sum (t_i - \overline{t})^2/(n-1)}$$

$\sigma = 0.2137288$ s

$\sigma = 0.2$ s (But what does this mean???)

How to quote best value

- What is uncertainty in average value?
  - Introduce **standard deviation of the mean**

    $\sigma_{\bar{t}} = \sigma/\sqrt{n} = 0.08725$ s = 0.09 s
- Now what is best quote of average value
  - $\bar{t}$ = 2.51666666666666666666666 s
  - $\bar{t}$ = 2.52 s
- Best value is
  - $\bar{t}$ = 2.52 ± 0.09 s

Propagation of error

- Same experiment, continued…
- From best estimate of time, get best estimate of distance: 31 meters
- Know uncertainty in time, what about uncertainty in distance?
- From error analysis tells us how errors propagate through mathematical functions

(2 meters)

Expected uncertainty in a calculated sum $a = b + c$

- Each value has an uncertainty
  - $b = \overline{b} \pm \delta b$
  - $c = \overline{c} \pm \delta c$
- Uncertainty for a (δa) is **at most** the sum of the uncertainties

  $\delta a = \delta b + \delta c$
- Better value for δa is

  $\delta a = \sqrt{(\delta b^2 + \delta c^2)}$
- Best value is
  - $a = \overline{a} \pm \delta a$

Expected uncertainty in a calculated product $a = b*c$

- Each value has an uncertainty
  - $b = b \pm \delta b$
  - $c = c \pm \delta c$
- Relative uncertainty for a (εa) is **at most** the sum of the RELATIVE uncertainties

  $\varepsilon a = \delta a/a = \varepsilon b + \varepsilon c$
- Better value for δa is

  $\varepsilon a = \sqrt{(\varepsilon b^2 + \varepsilon c^2)}$
- Best value is
  - $a = a \pm \varepsilon a$ (fractional uncertainty)

What about powers in a product

$a = b * c^2$

- Each value has an uncertainty
  - $b = b \pm \delta b$
  - $c = c \pm \delta c$
  - $\varepsilon a = \delta a / a$ (relative uncertainty)
- powers become a prefactor (weighting) in the error propagation
  - $\varepsilon a^2 = (\varepsilon b^2 + (2 * \varepsilon c)^2)$

How does uncertainty in t effect the calculated parameter d ?

– $d = \frac{1}{2} g t^2$

$\varepsilon d = \sqrt{(2 * \varepsilon t)^2} = 2 * \varepsilon t$

$\varepsilon d = 2 * (.09/2.52) = 0.071$

$\delta d = .071 * 31 \text{ m} = 2.2 \text{ m} = 2 \text{ m}$

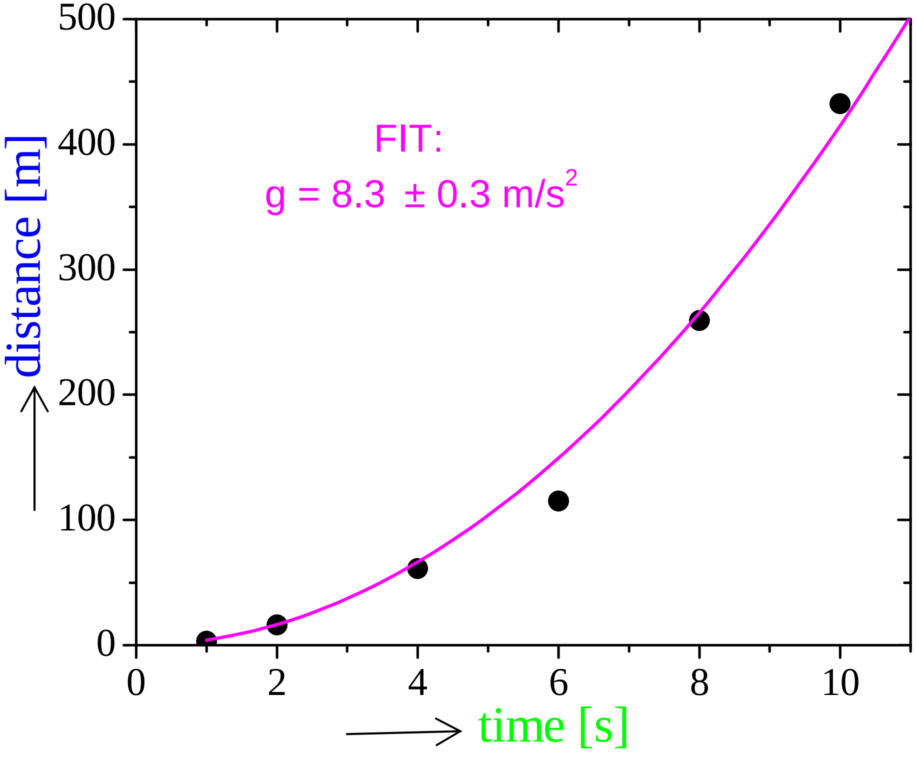
Statistical error

Relationships

- Know there is a functional relation between d and t $d = \frac{1}{2} g t^2$
- d is directly proportional to t^2
- Related through a constant ½ g
- Can measure time of drop (t) at different heights (d)
- plot d versus t to obtain constant

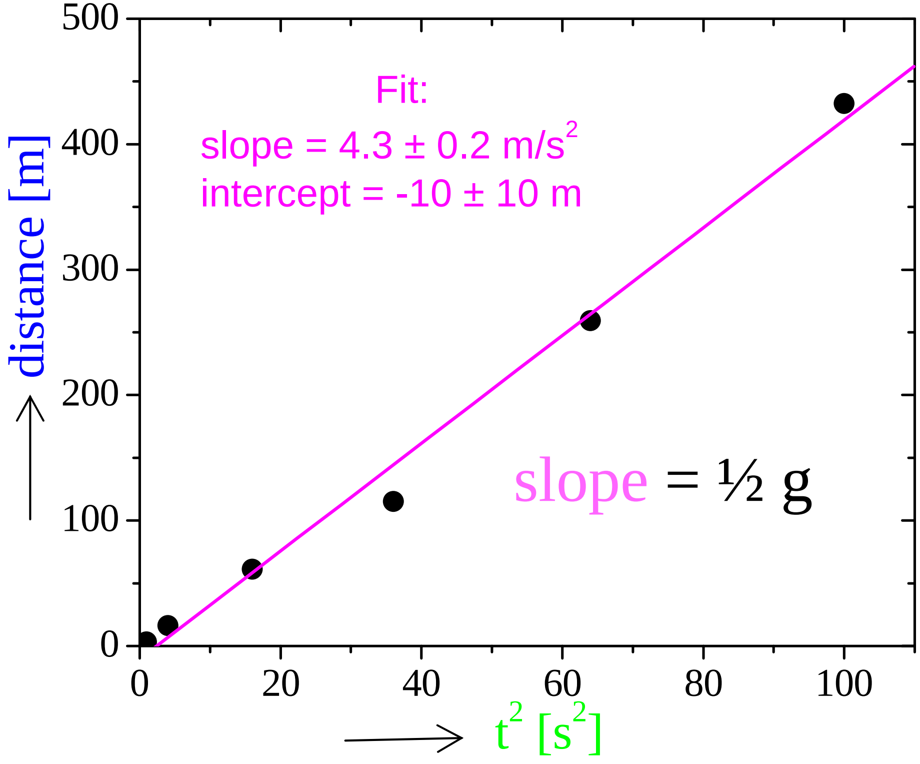
Quantifying relationships

$d = \frac{1}{2} g t^2$



$g = 8.3 \pm 0.3\ m/s^2$

$d = \frac{1}{2} g(t^2)$



$g = 8.6 \pm 0.4\ m/s^2$

General Formula for error propagation

For independent, random errors

$$\delta q = \left| \frac{dq}{dx} \right| \delta x$$

$$\boxed{\delta q = \sqrt{\left(\frac{\partial q}{\partial x} \delta x \right)^2 + \left(\frac{\partial q}{\partial y} \delta y \right)^2}}$$

From Yagil

Measurement and Observation

- Measurement: deciding the amount of a given property by observation
- Empirical
- Not logical deduction
- Not all measurements are created equal…

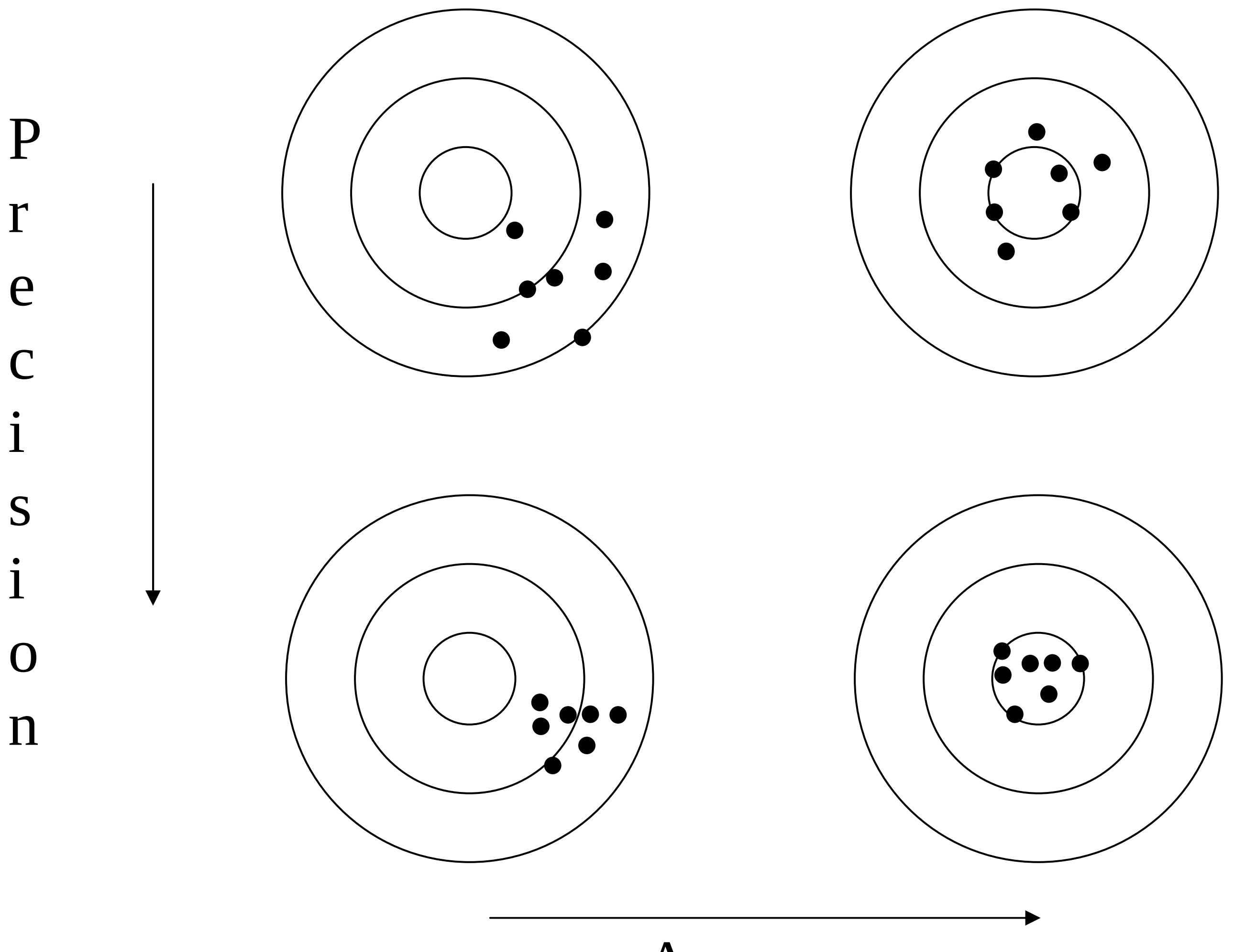
Reproducibility

- Same results under similar circumstances
  - Reliable/precise
- ‘Similar’ - a slippery thing
  - Measure resistance of metal
    - need same sample purity for repeatable measurement
    - need same people in room?
    - same potential difference?
  - Measure outcome of treatment on patients
    - Can’t repeat on same patient
    - Patients not the same

Precision and Accuracy

- Precise - reproducible
- Accurate - close to true value
- Example - temperature measurement
  - thermometer with
    - fine divisions
    - or with coarse divisions
  - and that reads
    - 0 C in ice water
    - or 5 C in ice water

Accuracy vs. Precision



Random and Systematic Errors

- Accuracy and precision are related to types of errors
  - random (thermometer with coarse scale)
    - can be reduced with repeated measurements, careful design
  - systematic (calibration error)
    - difficult to detect with error analysis
    - compare to independent measurement

Observations in Practice

- Does a measurement measure what you think it does? Validity
- Are scope of observations appropriate?
  - Incidental circumstances
  - Sample selection bias
- Depends on model

Models

- Model is a construction that represents a subject or imitates a system
- Used to predict other behaviors (extrapolation)
- Provides context for measurements and design of experiments
  - guide to features of significance during observation

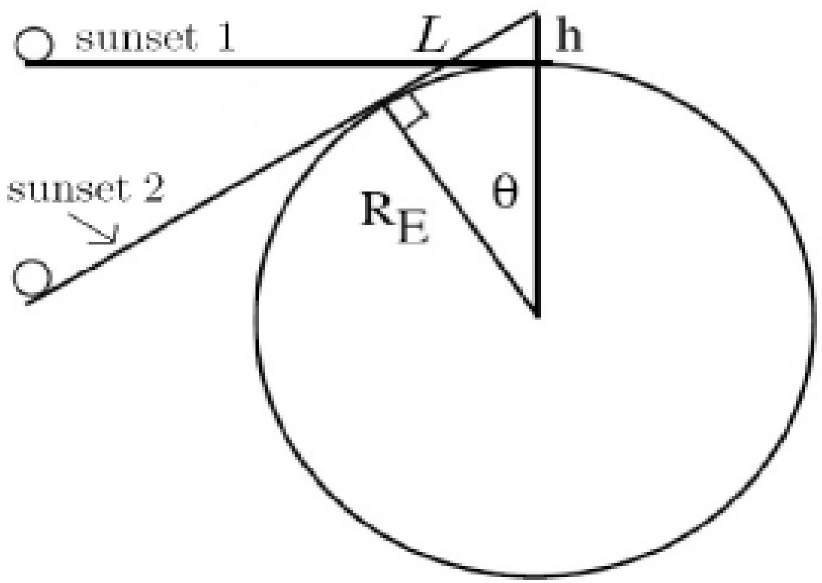
Testing model

- Models must be consistent with data
- Decide between competing models
  - elaboration: extend model to region of disagreement
  - precision: prefer model that is more precise
  - simplicity: Ockham’s razor

Experiment 1 Overview: Density of Earth

density
$$\boxed{\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3} = \frac{3g}{4\pi G R_E}}$$

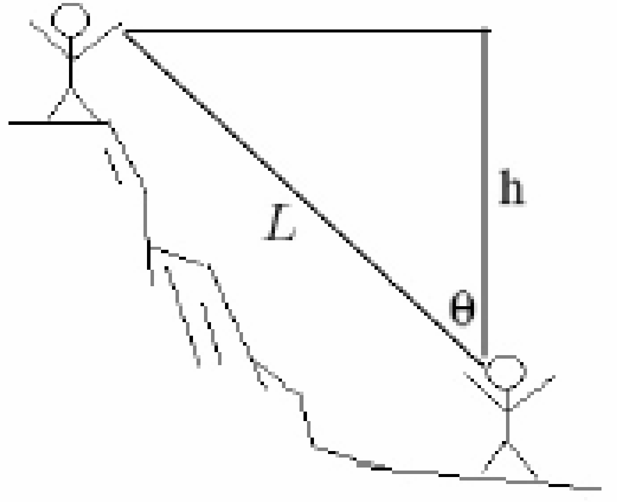
$$\frac{G M_E m}{R_E^2} = \text{mg}$$

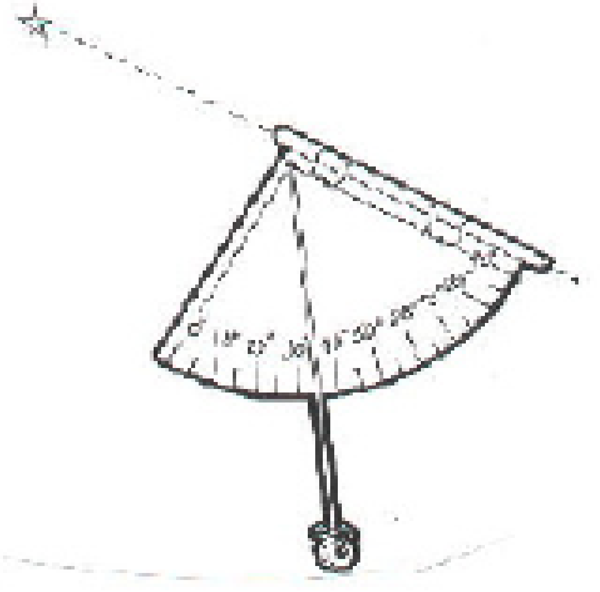


$$\boxed{R_E = \frac{2h}{\omega^2 (\Delta t)^2}}$$

measure Δt between sunset on cliff and at sea level

Experiment 1: Height of Cliff





rangefinder to get *L*

Sextant to get θ

Wear comfortable shoes

Make sure you use θ and not (90 – θ)

Measure Earth's Radius using Δt Sunset

Now, is this time delay measurable?

h - height above the sea level

L - distance to the horizon line

$$t = \frac{L}{2\pi R_e} T = \frac{T}{2\pi}\sqrt{\frac{2h}{R_e}}$$

$$T = 24\,\text{hr} = 24 \cdot 60 \cdot 60\ \text{s}$$

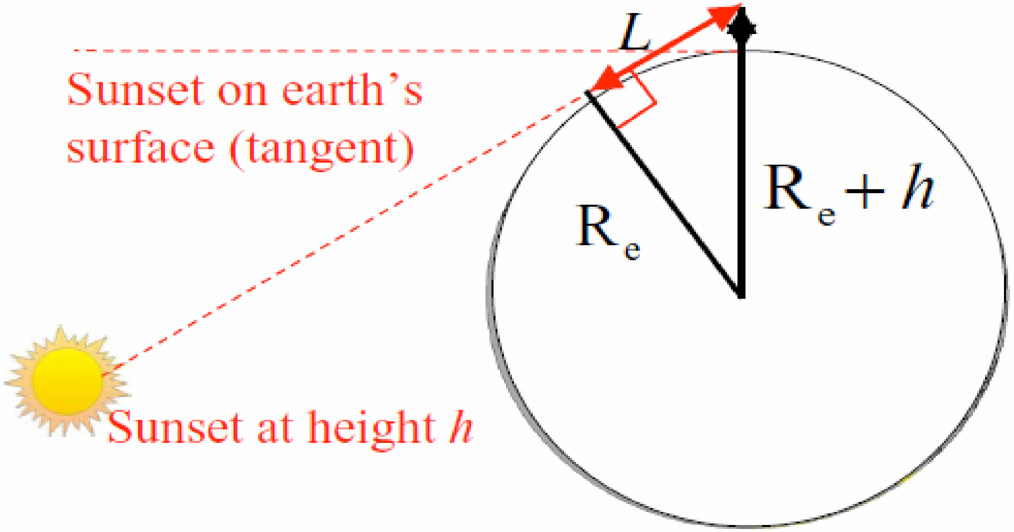
$$= 86400\ \text{s}$$

$$R_e = 6{,}000{,}000\ \text{m}$$

$h \sim 100$ m - our cliff

$$t = \frac{86400\ \text{s}}{2\pi}\sqrt{\frac{200}{6 \times 10^6}} \approx 80\ \text{s}$$

Looks doable!



Have we forgotten something?

From Yagil

"The Equation" for Experiment 1a

$$t = \frac{T}{2\pi}\sqrt{\frac{2Ch}{R_e}} = \frac{1}{\omega}\sqrt{\frac{2Ch}{R_e}} \qquad \omega = \frac{2\pi}{24 \text{ hr}}$$

from previous page.

Which are the variables that contribute to the error significantly?

$$\Delta t = t_1 - t_2 = \frac{1}{\omega}\sqrt{\frac{2\overline{C}}{\mathrm{R_e}}}\left(\sqrt{h_1} - \sqrt{h_2}\right)$$

Time difference between the two sunset observers.

$$C \equiv \frac{1}{\cos^2(\lambda)\cos^2(\lambda_s) - \sin^2(\lambda)\sin^2(\lambda_s)}$$

Season dependant factor slightly greater than 1.

What other methods could we use to measure the radius of the earth?

The formula for your error analysis.

$$\mathrm{R_e} = \frac{2C}{\omega^2}\left(\frac{\sqrt{h_1} - \sqrt{h_2}}{\Delta t}\right)^2$$

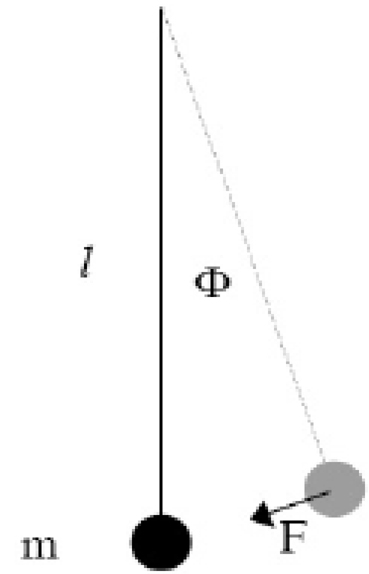
Eratosthenes

angular deviation = angle subtended

From Yagil

Experiment 1: Determine g

pendulum



$F = -mg\sin(\phi) = -mg\phi$

$F = m\alpha = ml\ddot{\phi}$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}}$$

period

Experiment 1: Pendulum

- For release angle θ_i, you should have a set of time data $(t_1^p, t_2^p, t_3^p, ..., t_N^p)$.

- Calculate the average, $\bar{t}^p$, and the the standard deviation, σ_{t^p}, of this data.

- Divide $\bar{t}^p$ and σ_{t^p} by p to get average time of a *single* period, $\bar{T}$ and standard deviation of a single period σ_T.

- Calculate SDOM, $\sigma_{\bar{T}} = \frac{\sigma_T}{\sqrt{N}}$.

- Now you should have $\bar{T} \pm \sigma_{\bar{T}}$ for you data at θ_i.

- Repeat these calculations for data at each release angle.

Grading rubric uploaded on website

Error Propagation - example

Ws saw earlier how to determine the acceleration of gravity, g.

Using a simple pendulum, measuring its length and period:

-Length l : $l = l_{best} \pm \delta l$

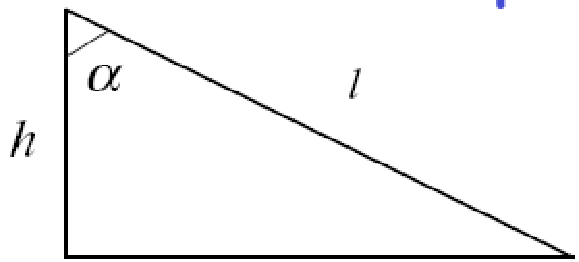
-Period T : $T = T_{best} \pm \delta T$

Determine g by solving: $\boxed{g = l \cdot (2\pi / T)^2}$

The question is what is the resulting uncertainty on g, **δg** ??

From Yagil

Example



Given that: $l = 10 \pm 0.1\ \text{m}$

$\alpha = 20 \pm 3^0$

==> Find *h*.

$$h = l \cdot \cos\alpha = 10 \cdot \cos 20^0 = 10 \cdot 0.94 = 9.4\ \text{m}$$

$$\delta h = \sqrt{\left(\frac{\partial h}{\partial l}\delta l\right)^2 + \left(\frac{\partial h}{\partial \alpha}\delta\alpha\right)^2}$$

$$\frac{\partial h}{\partial l} = \cos\alpha$$

$$\frac{\partial h}{\partial \alpha} = l \cdot (-\sin\alpha)$$

always use radians when calculating the errors on trig functions

$$\delta\alpha = 3^0 = \frac{2\pi\ \text{rad}}{360^0} \cdot 3^0 = 0.05\ \text{rad}$$

$$\delta h = \sqrt{(\cos\alpha \cdot \delta l)^2 + (l \cdot (-\sin\alpha) \cdot \delta\alpha)^2} = \sqrt{(0.94 \cdot 0.1)^2 + (10 \cdot [-0.34] \cdot 0.05)^2} = 0.2\ \text{m}$$

$$h = 9.4 \pm 0.2\ \text{m}$$

From Yagil

Propagating Errors for Experiment 1

$$\rho = \frac{3}{4\pi}\frac{g}{GR_e}$$ **Formula for density.**

$$\sigma_\rho = \frac{3}{4\pi}\frac{1}{GR_e}\sigma_g \oplus \frac{-3}{4\pi}\frac{g}{GR_e^2}\sigma_{R_e}$$ **Take partial derivatives and add errors in quadrature**

Or, in terms of relative uncertainties: $$\frac{\sigma_\rho}{\rho} = \frac{\sigma_g}{g} \oplus \frac{\sigma_{R_e}}{R_e}$$

shorthand notation for quadratic sum: $$\sqrt{a^2 + b^2} = a \oplus b$$

From Yagil

Propagating Errors for R_e

$$R_e = \frac{2C}{\omega^2}\left(\frac{\sqrt{h_1}-\sqrt{h_2}}{\Delta t}\right)^2$$

basic formula

$$\sigma_{R_e} = \frac{\partial R_e}{\partial \Delta t}\sigma_{\Delta t} \oplus \frac{\partial R_e}{\partial h_1}\sigma_{h_1} \oplus \frac{\partial R_e}{\partial h_2}\sigma_{h_2}$$

Propagate errors (use shorthand for addition in quadrature)

$$\sigma_{R_e} = \frac{2R_e}{\Delta t}\sigma_{\Delta t} \oplus \frac{R_e}{\sqrt{h_1}\left(\sqrt{h_1}-\sqrt{h_2}\right)}\sigma_{h_1} \oplus \frac{R_e}{\sqrt{h_2}\left(\sqrt{h_1}-\sqrt{h_2}\right)}\sigma_{h_2}$$

Note that the error blows up at $h_1=h_2$ and at $h_2=0$.

From Yagil

Reminder

- Prepare for lab
- Read Taylor chapter 4
- Homework due next week (Jan. 18-20) - Taylor 4.6, 4.14, 4.18, 4.26 (separate sheet)
- No lecture next Monday which is Martin Luther King Day
- Next lecture (Jan. 24) on Gaussian Distributions, lab #2, confidnece in data
- Homework for lab #2 starting the following week (Jan. 25-27) - read Taylor through Chapter 5 and do problems 5.2, 5.36.