# Physics 161: Black Holes: Lecture 13: 2 Feb 2011

Professor: Kim Griest

## 13 Extracting Energy from a Black Hole

Now let me tell you why black holes are the most efficient energy generation devices we have yet concieved. Much better than nuclear fission or even nuclear fusion. Of course, there is the small problem of not having a black hole near by and available for use.

First consider throwing some junk into a black hole from far away. We learned before that the total energy of the stuff far from the black hole is just $E = mc^2$, where $m$ is the mass of the stuff. Suppose this stuff doesn't go straight into the hole, but is slowed down in a series of orbits, finally settling into the last stable orbit we just discussed. What is the energy of the stuff now? We can use the radial geodesic equation to find out. On a circular orbit $dr/d\tau = 0$, so we have

$$\frac{E^2}{m} = \left(1 - \frac{r_S}{r}\right)\left(m + \frac{l^2}{mr^2}\right).$$

Next we plug in $l^2 = 3r_S^2 m^2$, and $r = 3r_S$,

$$\frac{E^2}{m} = (1 - \frac{1}{3})(m + \frac{3r_S^2 m^2}{m9r_S^2}) = (1 - \frac{1}{3})(1 + \frac{1}{3})m = \frac{8}{9}m.$$

Thus

$$E = \sqrt{\frac{8}{9}}mc^2.$$

Somehow during the transition from far away to the last stable orbit the energy has decreased. Note that in doing this we did *not* follow a geodesic. $E$ is conserved along geodesics, so starting at $r = \infty$ means $E = mc^2$ still at $r = 3r_S$. Thus the stuff must accelerate to get to the smaller stable circular orbit. Typically falling stuff hits other falling stuff and the collisions are what cause the acceleration. Thus, the original energy must have radiated away, and therefore be available for other use. The fractional amount of energy radiated away is $(mc^2 - E)/mc^2 = 1 - \sqrt{\frac{8}{9}} = 5.7\%$.

Let's compare this with other forms of energy generation. When burned, a gallon of gasoline produces $1.32 \times 10^8$ Joules of energy and has a mass of 2.8 kg. Thus the fractional mass energy lost when a gallon of gasoline is burned is $\Delta E/E = (1.32 \times 10^8)/((2.8)(3 \times 10^8)^2) = 5.2 \times 10^{-10}$. This is thus also the fraction of the rest mass of the gasoline that is turned into useful energy, a far cry from the 5.7% we get from the black hole.

Next, consider nuclear energy, say the complete fission of 1 kg of pure Uranium 235. This produces $8.28 \times 10^{13}$ Joules, so $\Delta E/E = (8.28 \times 10^{13})/((1)(3 \times 10^8)^2) = 0.0092\%$, still below the black hole. Finally one usually thinks of the main source of energy in the Universe as nuclear fusion of Hydrogen into Helium (as in the Sun). Actually in large stars, the Helium is then burned to Carbon, Oxygen, etc. all the way up to Iron, which is the most stable element. This whole chain of nuclear fusion releases about 0.9% of the rest mass into usable energy, still more than a factor of six below what you get from throwing your garbage into a black hole. Black holes are the best devices we know of for producing energy. In fact, using a rotating black hole we can turn even more of the rest mass of stuff into useful energy. The energy generation method discussed here is thought to be the power source of quasars, the most luminous and energetic steady objects in the Universe.