Physics 4E **Quiz 4 Solutions** Spring 2011

Problem 1

$$E_{n_1 n_2 n_3} = \frac{\hbar^2 \pi^2}{2 m_e L^2}\left(n_1^2 + n_2^2 + \frac{n_3^2 \cdot L^2}{L_3^2}\right) = E_0\left(n_1^2 + n_2^2 + \frac{n_3^2}{(L_3/L)^2}\right)$$

(a) For $L_3 = \sqrt{2} L \Rightarrow (L_3/L)^2 = 2 \Rightarrow$

$$E_{n_1, n_2, n_3} = E_0\left(n_1^2 + n_2^2 + \frac{n_3^2}{2}\right)$$

n_1, n_2, n_3	$E_{n_1,n_2,n_3}/E_0$
1 1 1	2.5
1 1 2	4
1 2 1	5.5
2 1 1	5.5
1 1 3	6.5
1 2 2	7
2 1 2	7
1 1 4	10

Energy level diagram

$n_1 n_2 n_3$		Energy
212, 122	(2)	$7 E_0$
113	(1)	$6.5 E_0$
121, 211	(2)	$5.5 E_0$
112	(1)	$4 E_0$
111	(1) (degeneracy)	$2.5 E_0$

(b) To get a triply degenerate state: need to have $E_{211} = E_{113} \Longrightarrow$

$$4 + 1 + \frac{1}{(L_3/L)^2} = 1 + 1 + \frac{9}{(L_3/L)^2} \Longrightarrow$$

$$\Rightarrow \frac{8}{(L_3/L)^2} = 3 \Rightarrow \left(\frac{L_3}{L}\right)^2 = \frac{8}{3} \Rightarrow \boxed{\frac{L_3}{L} = \sqrt{\frac{8}{3}} = 1.633}$$

$$\boxed{E_{n_1 n_2 n_3} = E_0\left(n_1^2 + n_2^2 + \frac{3}{8} n_3^2\right)}$$

n_1 n_2 n_3	$E_{n_1 n_2 n_3}/E_0$
1 1 1	2.375
1 1 2	3.5
1 2 1, 2 1 1	5.375
1 1 3	5.375
1 2 2, 2 1 2	6.5

$n_1 n_2 n_3$		Energy
212, 122	(2)	$6.5 E_0$
113, 211, 121	(3)	$5.375 E_0$
112	(1)	$3.5 E_0$
111	(1)	$2.375 E_0$

Problem 2

$$\psi(r,\theta,\phi) = \frac{1}{(\pi a_0^3)^{1/2}} e^{-r/a_0} = R(r)\, Y(\theta,\phi)$$

Clearly, Y is constant for this state. Since

$$\int d\Omega\, |Y|^2 = 1 = |Y|^2 \cdot \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta = 4\pi \Rightarrow \boxed{Y = \frac{1}{2\sqrt{\pi}}}$$

Therefore, $\boxed{R(r) = \frac{2}{a_0^{3/2}} e^{-r/a_0}}$

(b) Probability is $\boxed{P(r) = r^2 R(r)^2 = \frac{4r^2}{a_0^3} e^{-2r/a_0}}$

So
$$\frac{P(r = \alpha a_0)}{P(r = a_0)} = \alpha^2 e^{-2(\alpha-1)}$$

$$\Rightarrow \boxed{\frac{P(r=0.9a_0)}{P(r=a_0)} = 0.9^2 e^{+0.2} = 0.989}; \boxed{\frac{P(1.1a_0)}{P(a_0)} = 1.1^2 e^{-0.2} = 0.991}$$

(c) $$\langle r \rangle = \int_0^\infty dr\, r P(r) = \frac{4}{a_0^3} \int_0^\infty dr\, r^3 e^{-2r/a_0} = \frac{4}{a_0^3} \cdot \frac{3!}{2^4} a_0^4 = \frac{3}{2} a_0$$

$$\langle r^2 \rangle = \int_0^\infty dr\, r^2 P(r) = \frac{4}{a_0^3} \int_0^\infty dr\, r^4 e^{-2r/a_0} = \frac{4}{a_0^3} \cdot \frac{4!}{2^5} a_0^5 = 3a_0^2$$

$$\Rightarrow \langle r \rangle = \frac{3}{2} a_0, \quad \langle r^2 \rangle = 3a_0^2$$

$$\Rightarrow \Delta r = \sqrt{9a_0^2 - \frac{9}{4} a_0^2} = \frac{\sqrt{27}}{2} a_0 \Rightarrow \boxed{\Delta r = 2.6 a_0}$$

Problem 3

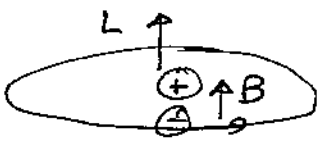
(a) The magnetic moment due to spin is

$$\mu_z = -\frac{e\hbar}{2m_e} \cdot g \cdot m_s = \mp \mu_B \cdot 2 \cdot \frac{1}{2} \quad ; \; \mu_B = 5.79\times 10^{-5}\, eV/T$$

Interaction energy with B: $U = -\mu_z B = \pm \mu_B B$

For $B = 2.5T$, $\boxed{U = \pm 1.4475 \times 10^{-4}\, eV}$

(b) The magnetic field seen by the electron is in same direction as the electron orbital angular momentum:



electron magnetic moment is in opposite direction to its spin.

Here, $m_\ell = -3$. For $m_s = -\frac{1}{2}$, spin // to $\vec{L} \Rightarrow \vec{\mu}_s$ antiparallel to $\vec{L}$

$\Rightarrow \vec{\mu}_s$ antiparallel to $\vec{B} \Rightarrow$ $\boxed{\text{energy high}(+)}$ $\boxed{\text{For } m_s = +\frac{1}{2}, \text{ energy low}(-)}$

(c) If the nuclear charge increases, spin orbit interaction energy increases.

$$B = \frac{\mu_0 I}{2\pi r} \quad I = \frac{Ze}{T} \quad ; \quad v = \frac{2\pi r}{T} \Rightarrow \frac{1}{T} = \frac{v}{2\pi r} \Rightarrow I = \frac{Zev}{2\pi r} \Rightarrow$$

$$\Rightarrow B = \frac{\mu_0 Zev}{4\pi^2 r^2} = \frac{\mu_0 Ze(m_e v r)}{4\pi^2 r^3 m_e} = \frac{\mu_0 Ze \cdot L}{4\pi^2 r^3 m_e}$$

$L = m_e v r$ = angular momentum doesn't change.

$r = a_0/Z$ changes. So we have

$$\boxed{B = \frac{\mu_0 Ze\, L}{4\pi^2 (a_0/Z)^3} \propto Z^4} \Rightarrow \boxed{\text{energy shift increases by } 2^4 = 16}$$