6-43. (a) For $x > 0$, $\hbar^2 k_2^2 / 2m + V_0 = E = \hbar^2 k_1^2 / 2m = 2V_0$

So, $k_2 = (2mV_0)^{1/2}/\hbar$. Because $k_1 = (4mV_0)^{1/2}/\hbar$, then $k_2 = k_1/\sqrt{2}$

(b) $R = (k_1 - k_2)^2/(k_1 + k_2)^2$ (Equation 6-68)

$= \left(1 - 1/\sqrt{2}\right)^2 / \left(1 + 1/\sqrt{2}\right)^2 = 0.0294$, or 2.94% of the incident particles are

reflected.

(c) $T = 1 - R = 1 - 0.0294 = 0.971$

(d) 97.1% of the particles, or $0.971 \times 10^6 = 9.71 \times 10^5$, continue past the step in the $+x$

direction. Classically, 100% would continue on.

6-44. (a) For $x > 0$, $\hbar^2 k_2^2 / 2m - V_0 = E = \hbar^2 k_1^2 / 2m = 2V_0$

So, $k_2 = (6mV_0)^{1/2}/\hbar$. Because $k_1 = (4mV_0)^{1/2}/\hbar$, then $k_2 = \sqrt{3/2}k_1$

(b) $R = (k_1 - k_2)^2/(k_1 + k_2)^2$

$$R = (k_1 - k_2)^2/(k_1 + k_2)^2 = \left(1 - \sqrt{3/2}\right)^2 / \left(1 + \sqrt{3/2}\right)^2 = 0.0102$$

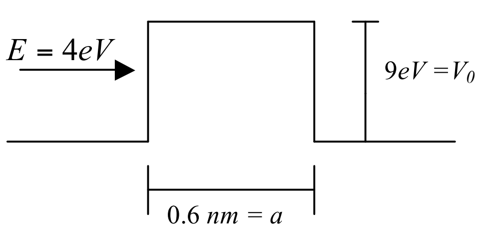
Or 1.02% are reflected at $x = 0$.

(c) $T = 1 - R = 1 - 0.0102 = 0.99$

(d) 99% of the particles, or $0.99 \times 10^6 = 9.9 \times 10^5$, continue in the $+x$ direction.

Classically, 100% would continue on.

6-45. (a)



$\alpha = \sqrt{2m(V_0 - E)}\hbar$

and $\alpha a = 0.6nm \times 11.46nm^{-1} = 6.87$

Since αa is not 1, use Equation 6-75:

The transmitted fraction

$$T = \left[1 + \frac{\sinh^2 \alpha a}{4(E/V_0)(1 - E/V_0)}\right]^{-1} = \left[1 + \left(\frac{81}{80}\right)\sinh^2(6.87)\right]^{-1}$$

Recall that $\sinh x = \left(e^x - e^{-x}\right)/2$,

$$T = \left[1 + \frac{81}{80}\left(\frac{e^{6.87} - e^{-6.87}}{2}\right)^2\right]^{-1} = 4.3 \times 10^{-6}$$ is the transmitted fraction.

(b) Noting that the size of *T* is controlled by αa through the $\sinh^2 \alpha a$ and increasing *T*

implies increasing *E*. Trying a few values, selecting $E = 4.5eV$ yields $T = 8.7 \times 10^{-6}$

or approximately twice the value in part (a).

6-48. Using Equation 6-76,

$T \approx 16\frac{E}{V_0}\left(1 - \frac{E}{V_0}\right)e^{-2\alpha a}$ where $E = 2.0eV$, $V_0 = 6.5eV$, and $a = 0.5nm$.

$T \approx 16\left(\frac{2.0}{6.5}\right)\left(1 - \frac{2.0}{6.5}\right)e^{-2(10.87)(0.5)} \approx 6.5 \times 10^{-5}$ (Equation 6-75 yields $T = 6.6 \times 10^{-5}$.)

6-49. $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$ and $T = 1 - R$ (Equations 6-68 and 6-70)

(a) For protons:

$$k_1 = \sqrt{2mc^2 E} / \hbar c = \sqrt{2(938MeV)(40MeV)} / 197.3MeV\cdot fm = 1.388$$

$$k_2 = \sqrt{2mc^2(E - V_0)} / \hbar c = \sqrt{2(938MeV)(10MeV)} / 197.3MeV\cdot fm = 0.694$$

$$R = \left(\frac{1.388 - 0.694}{1.388 + 0.694}\right)^2 = \left(\frac{0.694}{2.082}\right)^2 = 0.111 \quad \text{And } T = 1 - R = 0.889$$

(b) For electrons:

$$k_1 = 1.388\left(\frac{0.511}{938}\right)^{1/2} = 0.0324 \qquad k_2 = 0.694\left(\frac{0.511}{938}\right)^{1/2} = 0.0162$$

$$R = \left(\frac{0.0324 - 0.0162}{0.0324 + 0.0162}\right)^2 = 0.111 \quad \text{And } T = 1 - R = 0.889$$

No, the mass of the particle is not a factor. (We might have noticed that $\sqrt{m}$ could be canceled from each term.)

6-54. (a) The requirement is that $\psi^2(x) = \psi^2(-x) = \psi(-x)\psi(-x)$. This can only be true if:

$\psi(-x) = \psi(x)$ or $\psi(-x) = -\psi(x)$.

(b) Writing the Schrödinger equation in the form $\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi$, the general solutions of this 2nd order differential equation are:

$\psi(x) = A\sin kx$ and $\psi(\mathrm{x}) = A\cos kx$

where $k = \sqrt{2mE}/\hbar$. Because the boundaries of the box are at $x = \pm L/2$, both solutions are allowed (unlike the treatment in the text where one boundary was at $x = 0$). Still, the solutions are all zero at $x = \pm L/2$ provided that an integral number of half wavelengths fit between $x = -L/2$ and $x = +L/2$. This will occur for:

$\psi_n(x) = (2/L)^{1/2}\cos n\pi x/L$ when $n = 1,3,5,\cdots$. And for

$\psi_n(x) = (2/L)^{1/2}\sin n\pi x/L$ when $n = 2,4,6,\cdots$.

The solutions are alternately even and odd.

(c) The allowed energies are: $E = \hbar^2 k^2 / 2m = \hbar^2 \left(n\pi L^2\right) / 2m = n^2 h^2 / 8mL^2$.

6-55. $\psi_0 = Ae^{-x^2/2L^2}$

(a) $\dfrac{d\psi_0}{dx} = \left(-x/L^2\right)Ae^{-x^2/2L^2}$ and $\psi_1 = L\dfrac{d\psi_0}{dx} = L\left(-x/L^2\right)Ae^{-x^2/2L^2} = \left(-x/L\right)\psi_0$

So, $\dfrac{d\psi_1}{dx} = -\left(1/L\right)\psi_0 - \left(x/L\right)d\psi_0/dx$

And $\dfrac{d^2\psi_1}{dx^2} = -\left(1/L\right)d\psi_0/dx - \left(1/L\right)d\psi_0/dx - \left(x/L\right)d^2\psi_0/dx^2$

$= \left(2x/L^3\right)\psi_0 + \left(x/L^3\right)\psi_0 + \left(x^3/L^5\right)\psi_0$

Recalling from Problem 6-3 that $V\left(x\right) = \hbar^2 x^2 / 2mL^4$, the Schrödinger

equation

becomes $\left(-\hbar^2/2m\right)\left(3m/L^3 + x^3/L^5\right)\psi_0 + \left(\hbar^2 x^3/2mL^5\right)\psi_0 = E\left(-x/L\right)\psi_0$ or,

simplifying: $\left(-3\hbar^2 x/2mL^3\right)\psi_0 = E\left(-x/L\right)\psi_0$. Thus, choosing E

appropriately

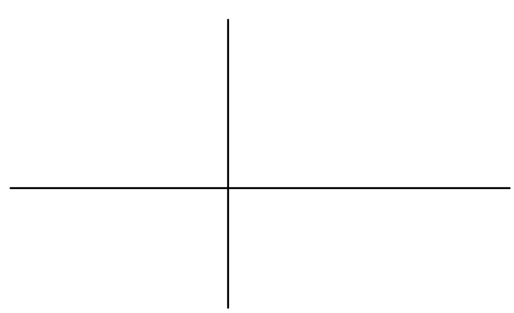
will make ψ_1 a solution.

(b) We see from (a) that $E = 3\hbar^2/2mL^2$, or three times the ground state energy.

(c) ψ_1 plotted looks as below. The single node indicates that ψ_1 is the first

excited state.

(The energy value in [b] would also tell us that.)



6-56. $\langle x^2 \rangle = \int_0^L \frac{2}{L} x^2 \sin^2 \frac{n\pi x}{L} dx$ Letting $u = n\pi x / L,\ du = (n\pi / L)dx$

$$\langle x^2 \rangle = \frac{2}{L}\left(\frac{L}{n\pi}\right)^2 \left(\frac{L}{n\pi}\right)\int_0^{n\pi} u^2 \sin^2 u du$$

$$= \frac{2}{L}\left(\frac{L}{n\pi}\right)^3 \left[\frac{u^3}{6} - \left(\frac{u^2}{4} - \frac{1}{8}\right)\sin 2u - \frac{u\cos 2u}{4}\right]\Bigg|_0^{n\pi}$$

$$= \frac{2}{L}\left(\frac{L}{n\pi}\right)^3 \left[\frac{(n\pi)^3}{6} - 0 - \frac{n\pi}{4} - 0\right] = \frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}$$

6-58. (a) For $\Psi(x,t) = A\sin(kx - \omega t)$

$\frac{d^2\Psi}{dx^2} = -k^2\Psi$ and $\frac{\partial \Psi}{\partial t} = -\omega A\cos(kx - \omega t)$ so the Schrödinger equation becomes:

$$-\frac{\hbar^2 k^2}{2m} A\sin(kx - \omega t) + V(x) A\sin(kx - \omega t) = -i\hbar\omega\cos(kx - \omega t)$$

Because the *sin* and *cos* are not proportional, this Ψ cannot be a solution. Similarly,

for $\Psi(x,t) = A\cos(kx - \omega t)$, there are no solutions.

(b) For $\Psi(x,t) = A[\cos(kx - \omega t) + i\sin(kx - \omega t)] = Ae^{i(kx - \omega t)}$, we have that

$\frac{d^2\Psi}{dx^2} = -k^2\Psi$ and $\frac{\partial \Psi}{\partial t} = -i\omega\Psi$. And the Schrödinger equation becomes:

$$-\frac{\hbar^2 k^2}{2m}\Psi + V(x)\Psi = -\hbar\omega\Psi \text{ for } \hbar\omega = \hbar^2 k^2 / 2m + V.$$