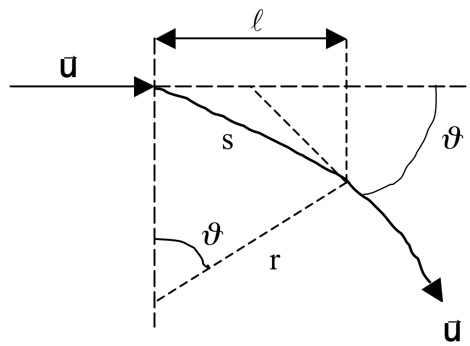
3-2.



For small values of ϑ, $s \approx \ell$; therefore, $\vartheta = \frac{s}{r} \approx \frac{\ell}{r}$

Recalling that $euB = \frac{mu^2}{r} \Rightarrow r = \frac{mu}{eB} \quad \therefore \quad \vartheta \approx \frac{\ell}{mu/eB} = \frac{eB\ell}{mu}$

3-5. (a) $R = \frac{mu}{qB} = \frac{\left[(2E_k/e)(e/m)\right]^{1/2}}{(e/m)(B)}$

$$= \frac{1}{B}\sqrt{\frac{2E_k/e}{e/m}} = \frac{1}{0.325T}\left[\frac{(2)(4.5\times10^4 eV/e)}{1.76\times10^{11}kg}\right]^{1/2} = 2.2\times10^{-3}m = 2.2mm$$

(b)

frequency $f = \frac{u}{2\pi R} = \frac{\sqrt{(2E_k/e)(e/m)}}{2\pi R}$

$$= \frac{\left[(2)(4.5\times10^4 eV/e)(1.76\times10^{11}C/kg)\right]^{1/2}}{2\pi\left(2.2\times10^{-3}m\right)} = 9.1\times10^9 Hz$$

period $T = 1/f = 1.1\times10^{-10}s$

3-6. (a) $1/2mu^2 = E_k$, so $u = \sqrt{(2E_k/e)(e/m)}$

$$\therefore \quad u = \left[(2)(2000eV/e)(1.76\times10^{11}C/kg)\right]^{1/2} = 2.65\times10^7 m/s$$

(b) $\Delta t_1 = \frac{x_1}{u} = \frac{0.05m}{2.65\times10^7 m/s} = 1.89\times10^{-9}s = 1.89ns$

(Problem 3-6 continued)

(c) $mu_y = F\Delta t_1 = e\mathbf{E}\Delta t_1$

$$\therefore \quad u_y = (e/m)\mathsf{E}\Delta t_1 = (1.76\times10^{11}C/kg)(3.33\times10^3 V/m)(1.89\times10^{-9}s) = 1.11\times10^6 m/s$$

3-12. $\lambda_m T = 2.898\times10^{-3} m\cdot K$

(a) $\lambda_m = \dfrac{2.898\times10^{-3} m\cdot K}{3K} = 9.66\times10^{-4} m = 0.966mm$

(b) $\lambda_m = \dfrac{2.898\times10^{-3} m\cdot K}{300K} = 9.66\times10^{-6} m = 9.66\mu m$

(c) $\lambda_m = \dfrac{2.898\times10^{-3} m\cdot K}{3000K} = 9.66\times10^{-7} m = 966nm$

3-13. Equation 3-4: $R = \sigma T^4$. Equation 3-6: $R = \dfrac{1}{4}cU$.

From Example 3-4: $U = (8\pi^5 k^4 T^4)/(15h^3c^2)$

$$\sigma = \frac{R}{T^4} = \frac{(1/4)cU}{T^4} = \frac{1}{4}c(8\pi^5k^4T^4)/(15h^3c^2T^4)$$

$$= \frac{2\pi^5(1.38\times10^{-23}J/K)^4}{15(6.63\times10^{-34}J\cdot s)^3(3.00\times10^8 m/s)^2} = 5.67\times10^{-8}W/m^2K^4$$

3-14. Equation 3-18: $u(\lambda) = \dfrac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT}-1}$

$u(f)df = u(\lambda)d\lambda \quad \therefore \quad u(f) = u(f)\dfrac{d\lambda}{df}$ Because $c = f\lambda$, $\left|\dfrac{d\lambda}{df}\right| = c/f^2$

$$u(f) = \frac{8\pi hc(f/c)^5}{e^{hf/kT}-1}\left(\frac{c}{f^2}\right) = \frac{8\pi f^2}{c^3}\frac{hf}{e^{hf/kT}-1}$$

3-15.

(a) $\lambda_m T = 2.898\times10^{-3} m\cdot K \quad \therefore \quad \lambda_m = \dfrac{2.898\times10^{-3} m\cdot K}{2.7K} = 1.07\times10^{-3} m = 1.07mm$

(b) $c = f\lambda \quad \therefore \quad f = \dfrac{c}{\lambda_m} = \dfrac{3.00\times10^{8}\,m/s}{1.07\times10^{-3}\,m} = 2.80\times10^{11}\,Hz$

(c) Equation 3-6:

$$R = \frac{1}{4}cU = \frac{c}{4}\left(8\pi^5 k^4 T^4/15h^3c^3\right)$$
$$= \frac{\left(3.00\times10^{8}\,m/s\right)\left(8\pi^5\right)\left(1.38\times10^{-23}\,J/K\right)^4\left(2.7\right)^4}{\left(4\right)\left(15\right)\left(6.63\times10^{-34}\,J\cdot s\right)^3\left(3.00\times10^{8}\,m/s\right)^3} = 3.01\times10^{-6}\,W/m^2$$

Area of Earth: $A = 4\pi r_E^2 = 4\pi\left(6.38\times10^{6}\,m\right)^2$

Total power = $RA = \left(3.01\times10^{-6}\,W/m^2\right)\left(4\pi\right)\left(6.38\times10^{6}\,m\right)^2 = 1.54\times10^{9}\,W$

3-16. $\lambda_m T = 2.898\times10^{-3}\,m\cdot K$

(a) $T = \dfrac{2.898\times10^{-3}\,m\cdot K}{700\times10^{-9}\,m} = 4140K$

(b) $T = \dfrac{2.898\times10^{-3}\,m\cdot K}{3\times10^{-2}\,m} = 9.66\times10^{-2}\,K$

(c) $T = \dfrac{2.898\times10^{-3}\,m\cdot K}{3m} = 9.66\times10^{-4}\,K$

3-17. Equation 3-4: $R_1 = \sigma T_1^4$ $\qquad R_2 = \sigma T_2^4 = \sigma\left(2T_1\right)^4 = 16\sigma T_1^4 = 16R_1$

3-18. (a) Equation 3-17: $\bar{E} = \dfrac{hc/\lambda}{e^{hc/\lambda kT}-1} = \dfrac{hc/\left(10hc/kT\right)}{e^{\left(hc/kT\right)/\left(10hc/kT\right)}-1} = \dfrac{0.1kT}{e^{0.1}-1} = 0.951kT$

(b) $\bar{E} = \dfrac{hc/\lambda}{e^{hc/\lambda kT}-1} = \dfrac{hc/\left(0.1hc/kT\right)}{e^{\left(hc/kT\right)/\left(0.1hc/kT\right)}-1} = \dfrac{10kT}{e^{10}-1} = 4.59\times10^{-4}\,kT$

Equipartition theorem predicts $\overline{E} = kT$. The long wavelength value is very close to *kT*, but the short wavelength value is much smaller than the classical prediction.

3-19. (a) $\lambda_m T = 2.898 \times 10^{-3} m \cdot K \quad \therefore \quad T_1 = \dfrac{2.898 \times 10^{-3} m \cdot K}{27.0 \times 10^{-6} m} = 107K$

$$R_1 = \sigma T_1^4 \quad \text{and} \quad R_2 = \sigma T_2^4 = 2R_1 = 2\sigma T_1^4$$

$$\therefore \quad T_2^4 = 2T_1^4 \text{ or } T_2 = 2^{1/4} T_1 = \left(2^{1/4}\right)\left(107K\right) = 128K$$

(b) $\lambda_m = \dfrac{2.898 \times 10^{-3} m \cdot K}{128K} = 23 \times 10^{-6} m$

3-20. (a) $\lambda_m T = 2.898 \times 10^{-3} m \cdot K$ (Equation 3-5)

$$\lambda_m = \frac{2.898 \times 10^{-3} m \cdot K}{2 \times 10^4 K} = 1.45 \times 10^{-7} m = 145 nm$$

(b) λ_m is in the ultraviolet region of the electromagnetic spectrum.

3-21. Equation 3-4: $R = \sigma T^4$

$P_{abs} = \left(1.36 \times 10^3 W/m^2\right)\left(\pi R_E^2 m^2\right)$ where R_E = radius of Earth

$$P_{emit} = \left(RW/m^2\right)\left(4\pi R_E^2\right) = \left(1.36 \times 10^3 W/m^2\right)\left(\pi R_E^2 m^2\right)$$

$$R = \left(1.36 \times 10^3 W/m^2\right)\left(\frac{\pi R_E^2}{4\pi R_E^2}\right) = \frac{1.36 \times 10^3}{4}\frac{W}{m^2} = \sigma T^4$$

$$T^4 = \frac{1.36\times10^3 W/m^2}{4\left(5.67\times10^{-8} W/m^2\cdot K^4\right)} \quad \therefore \quad T = 278.3K = 5.3°C$$

3-22. (a) $\lambda_m T = 2.898\times10^{-3} m\cdot K \quad \therefore \quad \lambda_m = \dfrac{2.898\times10^{-3} m\cdot K}{3300K} = 8.78\times10^{-7} m = 878nm$

$$f_m = c/\lambda_m = \frac{3.00\times10^8 m/s}{8.78\times10^{-7} m} = 3.42\times10^{14} Hz$$

(b) Each photon has average energy $E = hf$ and $NE = 40J/s$.

$$N = \frac{40J/s}{hf_m} = \frac{40J/s}{\left(6.63\times10^{-34} J\cdot s\right)\left(3.42\times10^{14} Hz\right)} = 1.77\times10^{20}\, photons/s$$

(c) At $5m$ from the lamp N photons are distributed uniformly over an area $A = 4\pi r^2 = 100\pi m^2$. The density of photons on that sphere is $\left(N/A\right)/s\cdot m^2$.

The area of the pupil of the eye is $\pi\left(2.5\times10^{-3} m\right)^2$, so the number of photons entering the eye per second is:

$$n = \left(N/A\right)\left(\pi\right)\left(2.5\times10^{-3} m\right)^2 = \frac{\left(1.77\times10^{20}/s\right)\left(\pi\right)\left(2.5\times10^{-3} m\right)^2}{100\pi m^2}$$

$$= \left(1.77\times10^{20}/s\right)\left(\pi\right)\left(2.5\times10^{-3} m\right)^2 = 1.10\times10^{13}\, photons/s$$

3-23. Equation 3-18:

$$u(\lambda)=\frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT}-1} \quad \text{Letting } A=\pi hc, \quad B=hc/kT, \text{and } U(\lambda)=\frac{A\lambda^{-5}}{e^{B/\lambda}-1}$$

$$\frac{du}{d\lambda}=\frac{d}{d\lambda}\left[\frac{A\lambda^{-5}}{e^{B/\lambda}-1}\right]=A\left[\frac{\lambda^{-5}(-1)e^{B/\lambda}\left(-B\lambda^{-2}\right)}{\left(e^{B/\lambda}-1\right)^2}-\frac{5\lambda^{-6}}{e^{B/\lambda}-1}\right]$$

$$=\frac{A\lambda^{-6}}{\left(e^{B/\lambda}-1\right)^2}\left[\frac{B}{\lambda}e^{B/\lambda}-5\left(e^{B/\lambda}-1\right)\right]=\frac{A\lambda^{-6}e^{B/\lambda}}{\left(e^{B/\lambda}-1\right)^2}\left[\frac{B}{\lambda}-5\left(1-e^{-B/\lambda}\right)\right]=0$$

The maximum corresponds to the vanishing of the quantity in brackets. Thus, $5\lambda\left(1-e^{-B/\lambda}\right)=B$. This equation is most efficiently solved by iteration; i.e., guess at a value for *B/λ* in the expression $5\lambda\left(1-e^{-B/\lambda}\right)$, solve for a better value of *B/λ*; substitute the new value to get an even better value, and so on. Repeat the process until the calculated value no longer changes. One succession of values is: 5, 4.966310, 4.965156, 4.965116, 4.965114, 4.965114. Further iterations repeat the same value (to seven digits), so we have:

$$\frac{B}{\lambda_m}=4.965114=\frac{hc}{\lambda_m kT} \quad \therefore \quad \lambda_m T=\frac{hc}{(4.965114)k}=\frac{\left(6.63\times10^{-34}J\cdot s\right)\left(3.00\times10^{8}m/s\right)}{(4.965114)\left(1.38\times10^{-23}J/K\right)}$$

$$\lambda_m T=2.898\times10^{-3}m\cdot K \quad \text{(Equation 3-5)}$$

3-26. (a) $\lambda_t=\frac{hc}{\phi}=\frac{1240eV\cdot nm}{1.9ev}=653nm, \quad f_t=\frac{\phi}{h}=\frac{1.9eV}{4.136\times10^{-15}eV\cdot s}=4.59\times10^{4}Hz$

(b) $V_0=\frac{1}{e}\left(\frac{hc}{\lambda}-\phi\right)=\frac{1}{e}\left(\frac{1240eV\cdot nm}{300nm}-1.9eV\right)=2.23V$

(c) $V_0=\frac{1}{e}\left(\frac{hc}{\lambda}-\phi\right)=\frac{1}{e}\left(\frac{1240eV\cdot nm}{400nm}-1.9eV\right)=1.20V$

3-27. (a) Choose λ =550nm for visible light. $nhf = E \quad \rightarrow \quad \dfrac{dn}{dt}hf = \dfrac{dE}{dt} = P$

$$\frac{dn}{dt} = \frac{P}{hf} = \frac{P\lambda}{hc} = \frac{(0.05\times100W)(550\times10^{-9}m)}{(6.63\times10^{-34}J\cdot s)(3.00\times10\ m/s)} = 1.38\times10^{19}/s$$

(b) $$flux = \frac{number\ \ radiated\ /\ unit\ time}{area\ of\ the\ sphere} = \frac{1.38\times10^{19}/s}{4\pi(2m)^2} = 2.75\times10^{17}/m^2\cdot s$$

3-28. (a) $hf = \phi \quad \therefore \quad f_t = \dfrac{\phi}{h} = \dfrac{4.22eV}{4.14\times10^{-15}eV\cdot s} = 1.02\times10^{15}Hz$

(b) $f = c/\lambda = \dfrac{3.00\times10^{8}m/s}{560\times10^{-9}m} = 5.36\times10^{14}Hz$ No.

Available energy/photon $hf = (4.14\times10^{-15}eV\cdot s)(5.36\times10^{14}Hz) = 2.22eV$.

This is less than ϕ.

3-32. (a) $\phi = \dfrac{hc}{\lambda} = \dfrac{1240eV\cdot nm}{653nm} = 1.90eV$

(b) $E_k = \dfrac{hc}{\lambda} - \phi = \dfrac{1240eV\cdot nm}{300nm} - 1.90eV = 2.23eV$