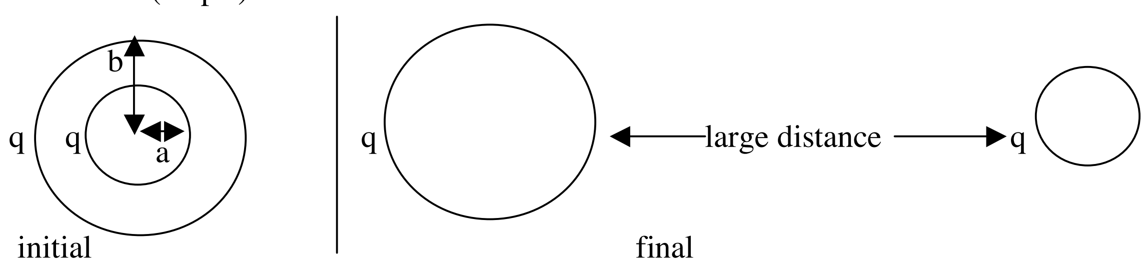
PHYSICS 4C FINAL EXAM (open book) Fall QUARTER 2011
PROF. HIRSCH December 6

Problem 1 (10 pts)
The electric field in a certain region of space is given by
$\vec{E}(x,y,z) = E_x\hat{x} + E_y\hat{y}$
with $E_x = \frac{E_0}{\ell}x, \quad E_y = \frac{E_0}{\ell}y$, and E_0=3V/m, ℓ=2m. It is time-independent.
(a) Find the charge density $\rho(x,y)$ that gives rise to this electric field, in C/m^3.
(b) Find how much work you have to do to bring a negative charge Q = -1.5C from the origin (x,y)=(0,0) to a point at distance 10ℓ from the origin. Give your answer in J, and state whether the work you do is positive or negative.
(c) Find the electric potential V(x,y) that gives rise to this electric field.
(d) Consider an electric dipole $\vec{p}$ of magnitude 3Cm located at position (x,y)=(5ℓ,0) and pointing in the $+\hat{x}$ direction. Find how much work you have to do (in J) to turn this dipole so that it points in the $+\hat{y}$ direction.
$\varepsilon_0=8.85\text{x}10^{-12}\text{C}^2/(\text{Nm}^2)$

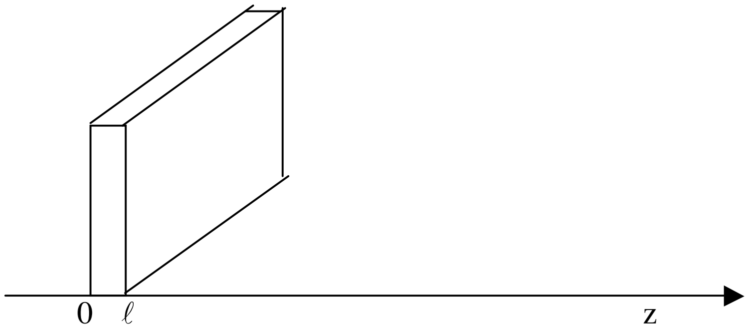
Problem 2 (10 pts)
The electric field in the region of space
$x^2 + y^2 \leq R^2, \qquad 0 \leq z \leq z_0$ is given by
$\vec{E}(x,y,z) = -\alpha\, y\,\hat{x} + \alpha\, x\,\hat{y}$ with $\alpha = 3V/m^2$. It is time-independent.
(a) Show that the charge density is zero everywhere in this region.
(b) Find $\vec{\nabla}\times\vec{E}$
(c) Assume at t=0 there is no magnetic field in this region. Find the magnitude and direction of the magnetic field in this region when t=2s.
(d) Draw schematically the electric field lines for this electric field in this region, and put arrows on them to indicate the direction.

Problem 3 (10 pts)



Consider two concentric spherical conducting shells of radius a and b respectively, each with the same charge q (of the same sign).
(a) Give expressions for the electric field in the regions (i) r<a, (ii) a<r<b, (iii) r>b.
(b) Find the electrostatic energy in the region r<b (use that the electrostatic energy density is $u = (1/2)\varepsilon_0 E^2$)
(c) Find the electrostatic energy in the region r>b.
(d) Find for what value of the ratio b/a the energies calculated in (b) and (c) are equal.
(e) For the value of b/a found in (d): assume you separate the shells a large distance without changing their charges. Calculate the work done by you in the process. Is it positive or negative? (answer this even if you didn't calculate the work, and justify why).

Problem 4 (10 pts)

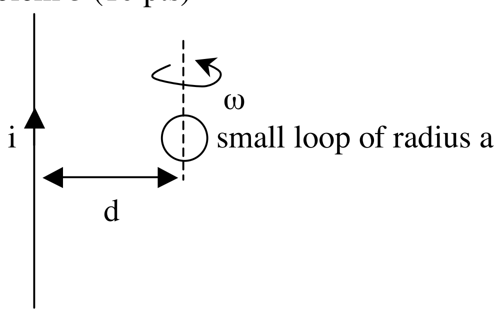


Consider an infinite slab of charge with normal in the z direction. The left end of the slab is at z=0 and the thickness of the slab is ℓ. The charge density in the slab is uniform in the x and y directions and non-uniform in the z direction, given by

$$\rho(z) = \rho_0 \frac{z}{\ell}$$

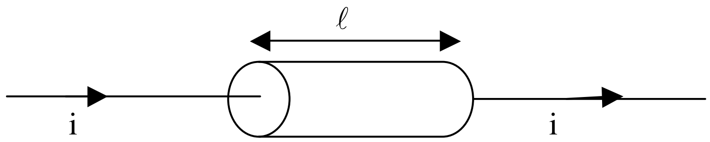
i.e. it is zero on the left edge and increases linearly to ρ_0 on the right edge.
(a) Find an expression for the electric field at position z, E(z), for $0<z<\ell$ in terms of ρ_0, ℓ, and E(z=0). Use $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$.
(b) Find the values of E(z=0) and E(z=ℓ). Hint: use the fact that for an infinite sheet of charge, E outside the sheet is independent of the distance to the sheet.
(c) Find for which value(s) of z the electric field is zero.
(d) Make a schematic plot of E(z) versus z extending from $z<-\ell$ to $z>\ell$.

Problem 5 (10 pts)



The long straight wire carries a constant current i. A small loop of wire of radius a is at distance d from the long wire, with d>>a. The loop rotates around an axis (dashed line) parallel to the long wire, with angular velocity ω, and has resistance R.
(a) Find an expression for the flux of magnetic field through the loop as function of the rotation angle of the loop, θ. Use the convention that θ=0 or π when the loop is on the plane of the paper. Use that d>>a.
(b) Find an expression for the induced current through the loop, $i_{ind}(t)$, as function of time, assuming the self-inductance of the loop can be ignored.
(c) For which orientation of the loop will the current be maximum?
(d) Assume now the loop has a small self-inductance L. Will the maximum current occur (i) at <u>the same</u> orientation as found in (c), or (ii) at an orientation attained slightly <u>earlier</u> or (iii) slightly <u>later</u>? Justify your answer.

Problem 6 (10 pts)



Consider a homogeneous cylinder of radius a, length ℓ and resistivity ρ, through which a constant current i flows fed by wires.
(a) Give an expression for the electric field in the interior of the cylinder as function of ρ, a, i and ℓ (there is no net charge density in the cylinder, only current).
(b) Give an expression for the magnetic field at the lateral surface of the cylinder.
(c) Calculate the magnitude and direction of the Poynting vector at the lateral surface of the cylinder.
(d) Calculate the flux of the Poynting vector through the lateral surface of the cylinder $\int \vec{S}\cdot d\vec{A}$ in terms of ρ, a, ℓ and i.
(e) Explain the relation between the answer in (d) and the conservation law

$$\frac{\partial U}{\partial t} = -\oint \vec{S}\cdot d\vec{A}$$

where U is the energy.

Problem 7 (10 pts)
Consider a particle of mass m and charge q moving in a circular orbit of radius r. Assume a magnetic field in direction perpendicular to the orbit is turned on and increases from initial value 0 to final value B_0 according to some continuous function B(t). Assume the charge stays in the same circular orbit of radius r during this process.
(a) Give an expression for the induced emf $\varepsilon(t)$ at radius r in terms of B(t) and r.
(b) Give an expression for the induced electric field E(t) at radius r in terms of B(t) and r.
(c) Give an expression for the force exerted on the charge q during this process.
(d) Calculate the change Δv in the speed of the charge q in this process using Newton's law, between the time when B=0 and when it is B_0. Show that it is independent of the particular form of B(t).
(e) Is the final speed of the particle always smaller than the initial speed, or is it always larger, or can it be either? Does it depend on the sign of q? Explain clearly all possibilities.

Problem 8 (10 pts)
The magnetic field in an electromagnetic wave is

$$\vec{B}(x,y,z) = B_0[\cos(kz + \omega t) + \sin(kz + \omega t)]\,\hat{y}$$

(a) Find an expression for the electric field in this wave, with the amplitude given in terms of B_0 and the speed of light only.
(b) Find an expression for the Poynting vector and state in which direction it is poynting (sorry I meant pointing).

Justify all your answers to all problems