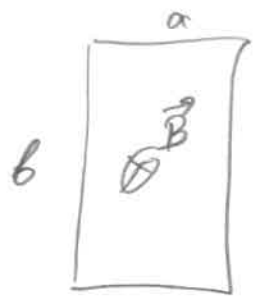
Homework 8

9.1A



$$|\mathcal{E}| = \frac{d\Phi}{dt} = \frac{d(ab\cdot B)}{dt} =$$

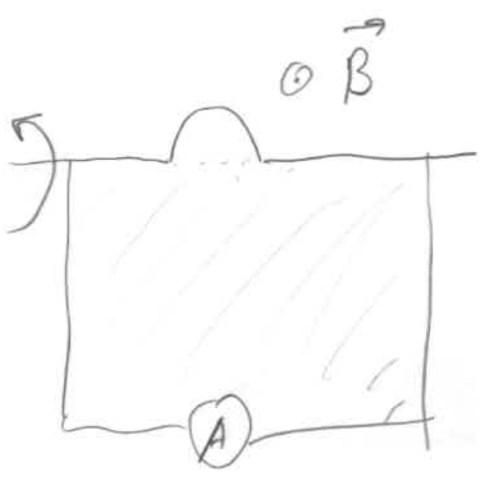
$$= ab\,\frac{dB}{dt} = \underline{abK}$$

Direction is <u>clockwise</u>.

$$|\mathcal{E}| = E_{avg}\cdot l = E_{avg}(2a+2b)$$

$$E_{avg} = \frac{|\mathcal{E}|}{2a+2b} = \underline{\frac{abK}{2a+2b}}$$

9.1C



The total area where there is a magnetic field is the shaded region plus the area enclosed by the semicircle:

$$A = A_0 + \frac{\pi R^2}{2}\cos\theta$$

where θ is the angle between the normal of the semicircle and $\vec{B}$.

$$\mathcal{E} = -\frac{d\phi}{dt} = -B\frac{dA}{dt} = +B\frac{\pi R^2}{2}\sin\theta\frac{d\theta}{dt}$$

but $\frac{d\theta}{dt} = \omega = 2\pi f$

$$\mathcal{E} = B\frac{\pi R^2}{\cancel{2}} \cdot \cancel{2}\pi f \sin(\omega t)$$

the amplitude is

$$\boxed{\mathcal{E}_0 = B\pi^2 R^2 f}$$

and the frequency is f.

$$I = \frac{\mathcal{E}}{1000\Omega}$$

with amplitude

$$I_0 = \frac{\mathcal{E}_0}{1000\Omega} = \frac{B\pi^2 R^2 f}{1000\Omega}$$

9.1 P

As view from solenoid the current in the loop will be <u>opposite</u> to that of the solenoid, i.e., <u>clockwise</u>.

<u>9.1 E</u> The volume of the wire is

$$Vol = \pi r^2 \cdot 2\pi R$$

The area enclosed:

$$A = \pi R^2$$

$$|\mathcal{E}| = \frac{d\phi}{dt} = A\frac{dB}{dt} = \pi R^2 \frac{dB}{dt}$$

$$I = \frac{|\mathcal{E}|}{\text{resistance}} = \frac{|\mathcal{E}|}{\rho l / A_1} = \frac{|\mathcal{E}| A_1}{\rho l}$$

where $l = 2\pi R$ is the length,

$A_1 = \pi r^2$ is the area of the wire.

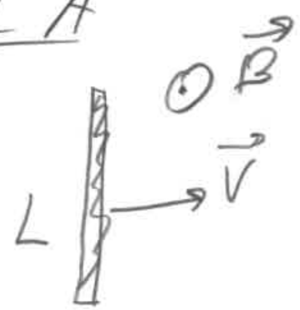
$$I = \frac{\pi R^2 \frac{dB}{dt} \cdot \pi r^2}{\rho \cdot 2\pi \cancel{R}} = \frac{\pi r^2 \cdot 2\pi R \frac{dB}{dt}}{4\pi \rho} =$$

$$= \frac{Vol \frac{dB}{dt}}{4\pi \rho}$$

but $m = Vol \cdot \delta$

so $I = \dfrac{\frac{m}{\delta} \frac{dB}{dt}}{4\pi \rho} = \dfrac{m}{4\pi \rho \delta} \dfrac{dB}{dt}$ ✓

<u>9.2 A</u>



a. $\underline{F = qvB \text{ downward}}$ on positive charges.

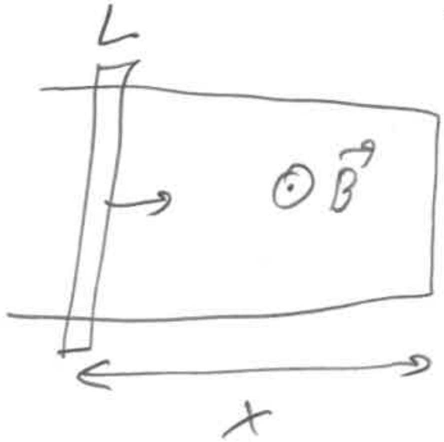
b. Since there is no current the magnetic and electric forces cancel so

$E = \frac{F}{q} = \underline{vB \quad \text{upward.}}$

c. $\Delta V = EL = \underline{vBL}$

d. $I = \frac{\Delta V}{R} = \underline{\frac{vBL}{R}}$

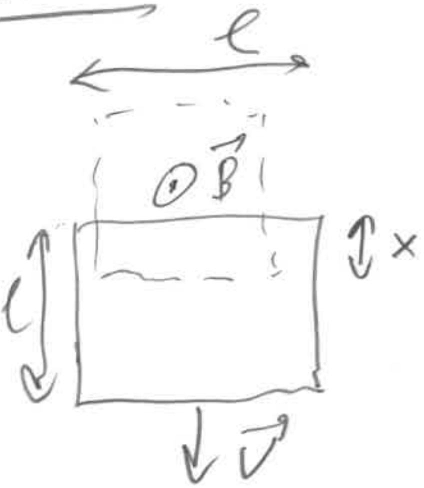
e.



$|\mathcal{E}| = \frac{d\phi}{dt} = \frac{d(AB)}{dt} = \frac{d(BLx)}{dt} =$

$= BL\frac{dx}{dt} = \underline{BLv}$

9.2C



$$|\mathcal{E}| = \frac{d\phi}{dt} = \frac{d(x\ell \cdot B)}{dt} =$$

$$= \ell B \frac{dx}{dt} = \ell B v$$

$$I = \frac{|\mathcal{E}|}{R} = \frac{\ell B v}{R}$$

Magnetic force on the top side

$$F = B\ell I = B\ell \frac{\ell B v}{R} = \frac{B^2 \ell^2 v}{R}$$

At terminal velocity the magnetic force exactly cancels the weight

$$F = mg$$

$$\frac{B^2 \ell^2 v}{R} = mg$$

$$\boxed{v = \frac{Rmg}{B^2 \ell^2}}$$

If the cross section of the wire is doubled then

$m \to 2m$
$R \to R/2$

v does not change.

9.3 A

$$|\mathcal{E}| = L \frac{dI}{dt}$$

$$50V = 2H \frac{dI}{dt}$$

$$\frac{dI}{dt} = 25 \frac{A}{s}$$

Need to change the current at the rate $25 \frac{A}{s}$.

9.3 B The total flux is

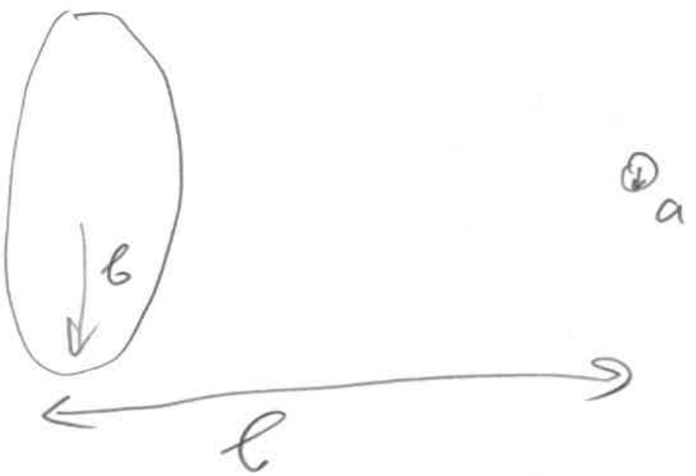
$$\Phi_{total} = LI = 10 \cdot 10^{-3} H \cdot 0.5 \cdot 10^{-3} A =$$

$$= 5 \cdot 10^{-6} Wb$$

which is through 100 turns. The flux through the coil is then

$$\phi_{coil} = \frac{\phi_{total}}{100} = 5 \cdot 10^{-8}\, Wb$$

9.4B



$b >> a$

If $b >> a$ then the magnetic field throug the small loop can be approximated by the value at the center. If there is current I through the big loop then

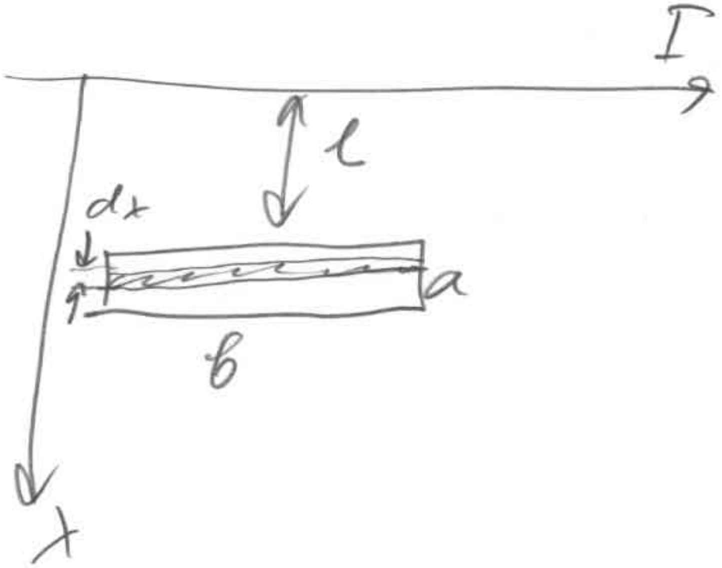
$$B = \frac{\mu_0 I}{2} \frac{b^2}{(b^2 + \ell^2)^{3/2}} \quad (eq.\ 8.8)$$

$$\phi = BA = B\pi a^2 = \frac{\mu_0 I}{2} \frac{b^2}{(b^2+\ell^2)^{3/2}} \pi a^2$$

$$M = \frac{\Phi}{I} = \frac{\mu_0}{2} \frac{b^2}{(b^2+c^2)^{3/2}} \pi a^2$$

9.4C

We first calculate the mutual inductance at distance l.



The flux through the shaded region is

$$d\Phi = B \cdot dA = B \cdot b\,dx$$

$$B = \frac{\mu_0 I}{2\pi x}$$

$$d\Phi = \frac{\mu_0 I}{2\pi x} b\,dx$$

$$\phi = \int_{l}^{l+a} \frac{\mu_0 I}{2\pi x} b\,dx = \frac{\mu_0 I b}{2\pi} \ln x \Big|_{l}^{l+a}$$

$$\phi = \frac{\mu_0 I b}{2\pi} \ln \frac{l+a}{l}$$

$$M = \frac{\phi}{I} = \underline{\frac{\mu_0 b}{2\pi} \ln \frac{l+a}{l}}$$

Now if it moves with velocity v then

$$l(t) = l + vt$$

(since at $t=0$ $l(t) = l$)

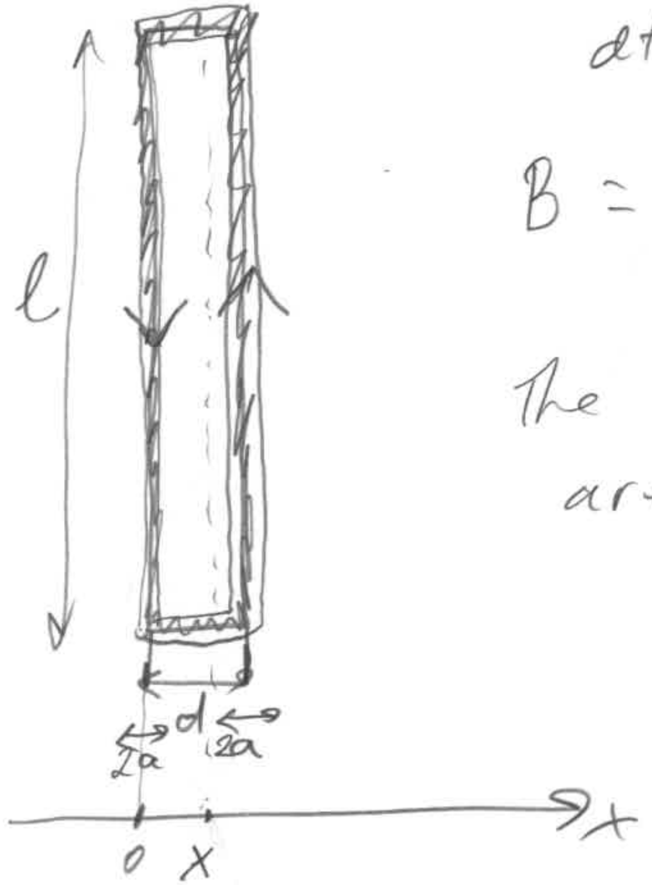
$$\underline{M(t) = \frac{\mu_0 b}{2\pi} \ln \frac{l+vt+a}{l+vt}}$$

$$\phi(t) = \frac{\mu_0 b I}{2\pi} \ln \frac{l+vt+a}{l+vt}$$

$$|\varepsilon| = \left|\frac{d\phi}{dt}\right| = \left|\frac{\mu_0 b I}{2\pi} \cdot \left(\frac{v}{l+vt+a} - \frac{v}{l+vt}\right)\right| =$$

$$= \frac{\mu_0 a b I v}{2\pi (l+vt)(l+vt+a)}$$

9.40



The magnetic field at a point x is

$$B = \frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi (d-x)}$$

The flux through an area of width dx is

$$d\phi = B \cdot l \, dx$$

$$d\phi = \frac{\mu_0 I l}{2\pi} \left(\frac{1}{x} + \frac{1}{d-x} \right) dx$$

The total flux (neglecting the flux within the wires)

$$\phi = \int_a^{d-a} \frac{\mu_0 I l}{2\pi} \left(\frac{1}{x} + \frac{1}{d-x} \right) dx$$

$$\phi = \frac{\mu_0 I l}{2\pi}\left(\ln x - \ln(d-x)\right)\Big|_a^{d-a} =$$

$$= \frac{\mu_0 I l}{2\pi}\left(\ln(d-a) - \ln a - \ln a + \ln(d-a)\right) =$$

$$= \frac{\mu_0 I l}{\pi} \ln \frac{d-a}{a}$$

$$L = \frac{\phi}{I} = \frac{\mu_0 l}{\pi} \ln \frac{d-a}{a}$$

9.5A $\quad L = 5H$

$I = 10A$

$$U = \frac{1}{2} L I^2 = \underline{250 J}$$

$I' = 1A \qquad \Delta t = \frac{1}{20} s$

$$\Delta I = I - I' = 9A$$

$$|\mathcal{E}| = L \frac{\Delta I}{\Delta t} = \underline{900 V}$$

95B $I = 10A$

$\phi = 10W$

$N = 200$

$\phi_{coil} = \phi N = 2000W$

$L = \frac{\phi_{coil}}{I} = 200H$

$U = \frac{1}{2} L I^2 = \underline{10000J}$