Homework 3, more problems

1) After they are connected they need to have the same potential, Suppose they have charges Q_1 and Q_2, then

$$Q_1 + Q_2 = Q + 2Q = 3Q$$

$$\frac{Q_1}{\cancel{4\pi\varepsilon_0} \cancel{R}} = \frac{Q_2}{\cancel{4\pi\varepsilon_0} \cdot 2\cancel{R}}$$

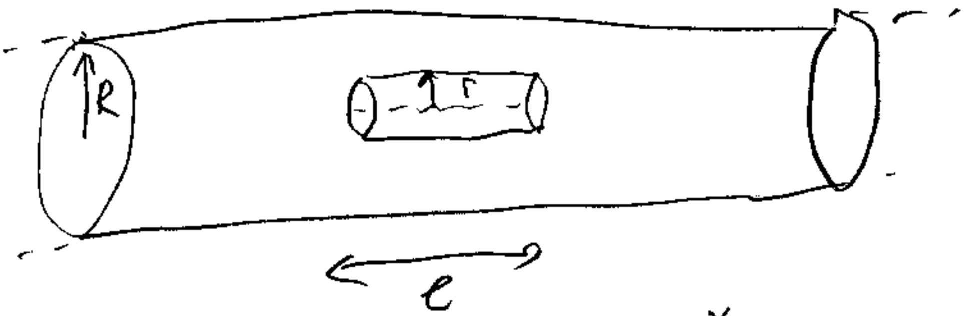
$$Q_2 = 2Q_1$$

So $Q_1 = Q$, $Q_2 = 2Q$, i.e. the charges do not change.

$$\sigma_1 = \frac{Q_1}{4\pi R^2} = \underline{\frac{Q}{4\pi R^2}}$$

$$\sigma_2 = \frac{Q_2}{4\pi (2R)^2} = \frac{2Q}{16\pi R^2} = \underline{\frac{Q}{8\pi R^2}}$$

2) Let us first find the electric field inside the cylinder using Gauss' law:



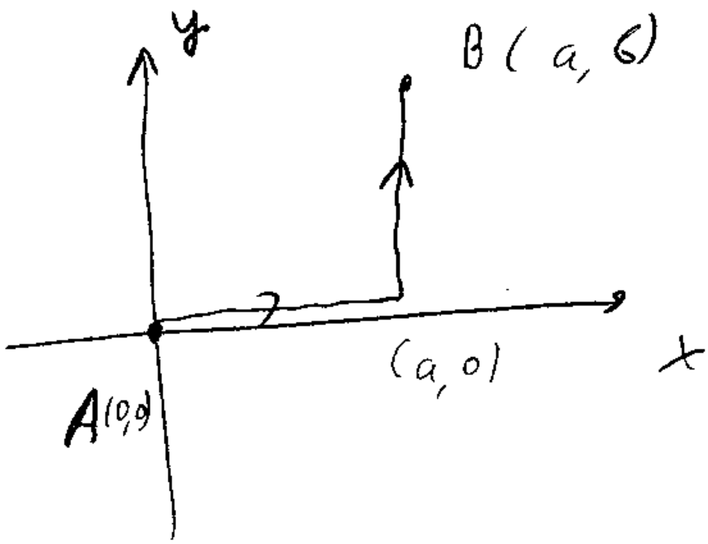
$$E \cdot 2\cancel{\pi}\cancel{r}\cancel{l} = \frac{\rho \cdot \cancel{\pi} r^{\cancel{2}} \cancel{l}}{\varepsilon_0}$$

$$E = \frac{\rho r}{2\varepsilon_0}$$

$$V(R) - V(0) = -\int_0^R \vec{E} \cdot d\vec{r} = -\int_0^R \frac{\rho r}{2\varepsilon_0}\, dr =$$

$$= -\left.\frac{\rho r^2}{4\varepsilon_0}\right|_0^R = \underline{-\frac{\rho R^2}{4\varepsilon_0}}$$

3)



$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l}$$

The integral can be taken along <u>any</u> path, we use the path shown above.

$$V_B - V_A = -\int_0^a E_x(x, 0)\, dx - \int_0^b E_y(a, y)\, dy =$$

$$= -\int_0^a (-2x)\, dx - \int_0^b (2a^2 - 4y)\, dy =$$

$$= x^2 \Big|_0^a - (2a^2 y - 2y^2)\Big|_0^b =$$

$$= \underline{a^2 - 2a^2 b + 2b^2}$$