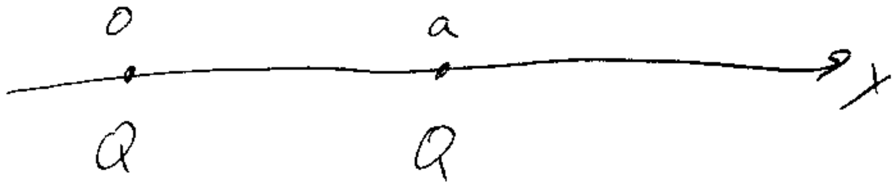
Homework 3

3.1 A a.



$$W = -Q\int_{\infty}^{a} \vec{E}\cdot d\vec{l} = -Q\int_{\infty}^{a} E_x\, dx$$

$$E_x = \frac{Q}{4\pi\varepsilon_0 x^2}$$

$$W = -Q\int_{\infty}^{a} \frac{Q}{4\pi\varepsilon_0 x^2}\, dx = \frac{Q^2}{4\pi\varepsilon_0 x}\Bigg|_{\infty}^{a} = \underline{\frac{Q^2}{4\pi\varepsilon_0 a}}$$

b. $\underline{U = \frac{Q^2}{4\pi\varepsilon_0 a}}$ in agreement with part a.

3.2 D Outside the sphere it behaves like a point charge at the center:

$$Q = \rho\,\frac{4\pi}{3}R^3$$

The electric field is

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{\rho\,\frac{\cancel{4\pi}}{3}R^3}{\cancel{4\pi}\varepsilon_0 r^2}\hat{r} = \underline{\frac{\rho R^3}{3\varepsilon_0 r^2}\hat{r}}$$

The potential is

$$V = \frac{Q}{4\pi\varepsilon_0 r} = \underline{\frac{\rho R^3}{3\varepsilon_0 r}} \qquad (*)$$

Inside the sphere the electric field is determined from Gauss' law

$$E \cdot 4\pi r^2 = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$\underline{\vec{E} = \frac{\rho r}{3\varepsilon_0}\hat{r}}$$

The potential difference between a point inside (r) and a point on the surface (R) is

$$V(r) - V(R) = -\int_R^r \vec{E}\cdot d\vec{r} = -\int_R^r \frac{\rho r}{3\varepsilon_0}\,dr =$$

$$= -\frac{\rho r^2}{6\varepsilon_0}\Big|_R^r = \frac{\rho R^2}{6\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

On the other hand, from $(*)$

$$V(R) = \frac{\rho R^2}{3\varepsilon_0}$$

$$V(r) = \frac{\rho R^2}{3\varepsilon_0} + \frac{\rho R^2}{6\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0} = \underline{\frac{\rho R^2}{2\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}}$$

<u>3.2E</u> $W = U_B - U_A = \underline{\frac{qQ}{4\pi\varepsilon_0 r_B} - \frac{qQ}{4\pi\varepsilon_0 r_A}}$

<u>3.2F</u> Outside the sphere (as well as on the surface) it behaves as a point charge at the center:

$$\underline{V(q) = \frac{q}{4\pi\varepsilon_0 R}}$$

Now suppose there is charge q and we want to add dq. The work will be

$$dW = V(q)\,dq = \frac{q\,dq}{4\pi\varepsilon_0 R}$$

To charge it up to a total charge Q we need

$$W = \int_0^Q \frac{q\,dq}{4\pi\varepsilon_0 R} = \frac{Q^2}{8\pi\varepsilon_0 R}$$

$$V = \frac{Q}{4\pi \varepsilon_0 R} \Rightarrow Q = 4\pi \varepsilon_0 R V$$

$$W = \frac{16\pi^2 \varepsilon_0^2 R^2 V^2}{8\pi \varepsilon_0 R} = \underline{2\pi \varepsilon_0 R V^2}$$

<u>3.2 G</u> a, The electric field <u>inside</u> the shell has to be 0 since it is a conductor, so on the inner surface of the shell the charge is $\underline{-Q_1}$ to cancel the charge inside. The potential on the surface is V_1 and is created by only the charge on the outer surface of the shell:

$$V_1 = \frac{Q_{out}}{4\pi \varepsilon_0 \cdot b} \qquad \underline{Q_{out} = 4\pi \varepsilon_0 b V_1}$$

b) The electric field:

$$E(r<a) = \underline{0}$$

$$E(a<r<b) = \underline{\frac{Q_1}{4\pi \varepsilon_0 r^2}}$$

$$E(r>b) = \frac{Q_{out}}{4\pi\varepsilon_0 r^2} = \frac{\cancel{4\pi\varepsilon_0} bV_1}{\cancel{4\pi\varepsilon_0} r^2} = \underline{\frac{bV_1}{r^2}}$$

The potential

$$V(r>b) = \frac{Q_{out}}{4\pi\varepsilon_0 r} = \frac{4\pi\varepsilon_0 bV_1}{4\pi\varepsilon_0 r} = \underline{\frac{bV_1}{r}}$$

Now consider a point r s.t. $a<r<b$

$$V(r) - V(b) = -\int_b^r \vec{E}\cdot d\vec{r} = -\int_b^r \frac{Q_1\, dr}{4\pi\varepsilon_0 r^2} =$$

$$= \frac{Q_1}{4\pi\varepsilon_0 r}\Big|_b^r = \frac{Q_1}{4\pi\varepsilon_0 r} - \frac{Q_1}{4\pi\varepsilon_0 b}$$

$$V(r) = V(b) + \frac{Q_1}{4\pi\varepsilon_0 r} - \frac{Q_1}{4\pi\varepsilon_0 b} = \underline{V_1 + \frac{Q_1}{4\pi\varepsilon_0 r} - \frac{Q_1}{4\pi\varepsilon_0 b}}$$

At point a

$$V(a) = V_1 + \frac{Q_1}{4\pi\varepsilon_0 a} - \frac{Q_1}{4\pi\varepsilon_0 b}$$

For $r<a$ the electric field is 0, so the potential is constant:

$$V(r<a) = V(a) = \underline{V_1 + \frac{Q_1}{4\pi\varepsilon_0 a} - \frac{Q_1}{4\pi\varepsilon_0 b}}$$

3.2 H The electric field is determined by Gauss' law

$$E \cdot 2\pi r \cancel{\ell} = \frac{Q\cancel{\ell}}{\varepsilon_0}$$

$$E = \frac{Q}{2\pi\varepsilon_0 r}$$

$$V(r_2) - V(r_1) = -\int_{r_1}^{r_2} E\,dr = -\int_{r_1}^{r_2} \frac{Q}{2\pi\varepsilon_0 r}\,dr =$$

$$= -\frac{Q}{2\pi\varepsilon_0} \ln r \Big|_{r_1}^{r_2} = \underline{\frac{Q}{2\pi\varepsilon_0} \ln \frac{r_1}{r_2}}$$

3.2 I Suppose they have radius R_1, charge Q_1:

$$V_1 = \frac{Q_1}{4\pi\varepsilon_0 R_1} \Rightarrow Q_1 = 4\pi\varepsilon_0 R_1 V_1$$

After they coalesce: $Q_2 = 2Q_1 = 8\pi\varepsilon_0 R_1 V_1$

The total volume is double the volume of one:

$$\cancel{\frac{4}{3}\pi} R_2^3 = 2 \cdot \cancel{\frac{4}{3}\pi} R_1^3$$

$R_2 = 2^{1/3} R_1$

The potential

$$V_2 = \frac{Q_2}{4\pi\varepsilon_0 R_2} = \frac{\cancel{8\pi}^{\,2}\varepsilon_0 R_1 V_1}{\cancel{4\pi\varepsilon_0}\, 2^{1/3} \cancel{R_1}} = \underline{2^{2/3} V_1}$$

<u>3.3 A</u> a. $\underline{V = \frac{-Q}{4\pi\varepsilon_0 r}}$

b. $E_r = -\frac{\partial V}{\partial r} = \frac{-Q}{4\pi\varepsilon_0 r^2} \Rightarrow \underline{\vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}}$

c. $\underline{\vec{E}_{inside} = 0}$

<u>3.3 B</u> $V = \frac{ax}{(x^2+y^2)^{3/2}} + \frac{b}{(x^2+y^2)^{1/2}}$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{a\left((x^2+y^2)^{3/2} - x\frac{3}{\cancel{2}}(x^2+y^2)^{1/2}\cancel{2}x\right)}{(x^2+y^2)^3} -$$

$$-b\left(-\frac{1}{\cancel{2}}\right)(x^2+y^2)^{-3/2}\cancel{2}x =$$

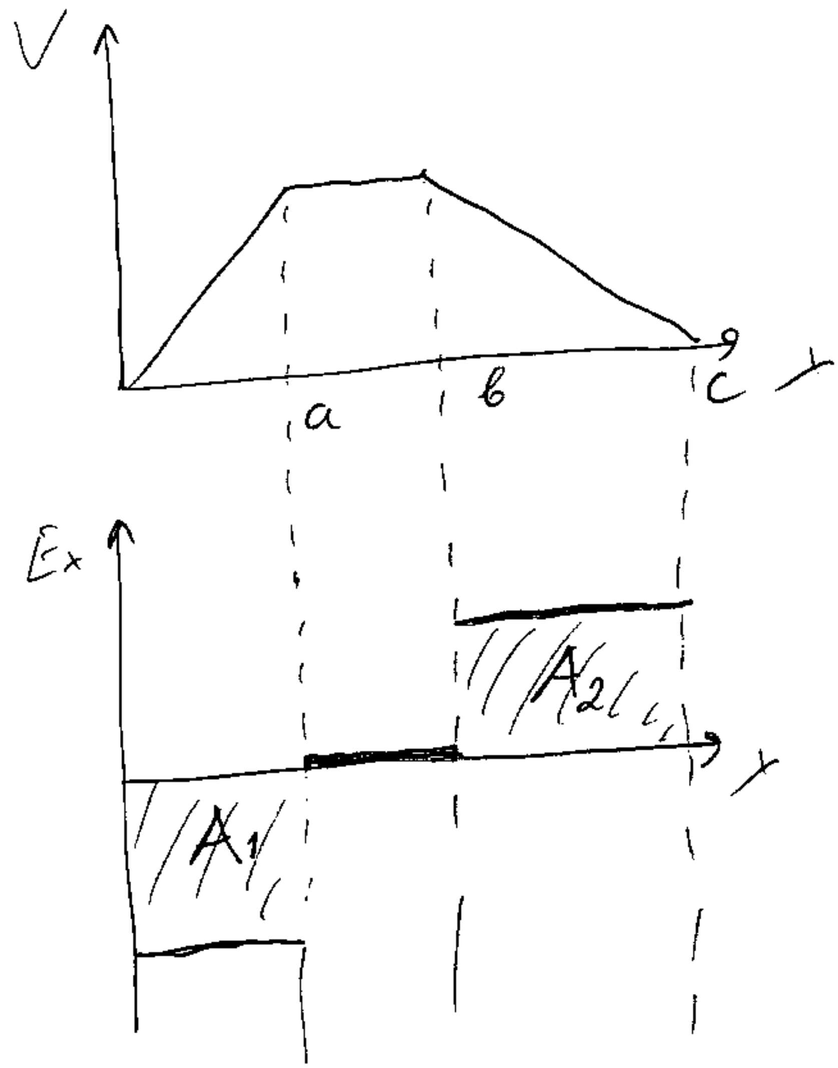
$$= \underline{-\frac{a(y^2-2x^2)}{(x^2+y^2)^{5/2}} + \frac{bx}{(x^2+y^2)^{3/2}}}$$

$$E_y = -\frac{\partial V}{\partial y} = -ax\left(-\frac{3}{2}\right)(x^2+y^2)^{-5/2}\cdot(2y) -$$

$$-b\left(-\frac{1}{2}\right)(x^2+y^2)^{-3/2}\cdot(2y) =$$

$$= \frac{3axy}{(x^2+y^2)^{5/2}} + \frac{by}{(x^2+y^2)^{3/2}}$$

<u>3.3 D</u> $E_x = -\frac{dV}{dx}$



$\Delta V = -\int E_x \, dx = -$ (area under E_x curve)

since $\Delta V = 0$ from 0 to c it means the total area under E_x is 0, i.e,

$$-A_1 + A_2 = 0$$

$$A_1 = A_2 \quad \checkmark$$

3.3E For a sphere the electric field is maximal near the surface:

$$E = \frac{Q}{4\pi\varepsilon_0 R^2} \qquad V = \frac{Q}{4\pi\varepsilon_0 R}$$

$$\Downarrow$$

$$E = \frac{V}{R}$$

$$V_{max} = E_{max} \cdot R = 10^4 \, V/cm \cdot 10\,cm$$

$$\boxed{V_{max} = 10^5 \, V}$$

3,4 A

$$E = \Delta V \cdot q = 100V \cdot e = \underline{100eV}$$

$$E = 100V \cdot 1.6 \cdot 10^{-19} C = \underline{1.6 \cdot 10^{-17} J}$$

$$E = 1.6 \cdot 10^{-17} \cdot 10^{7} erg = \underline{1.6 \cdot 10^{-10} erg}$$

$$\frac{m_e v^2}{2} = E$$

$$v = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2 \cdot 1.6 \cdot 10^{-17} J}{9.1 \cdot 10^{-31} kg}}$$

$$\underline{v = 5.9 \cdot 10^{6}\ m/s}$$