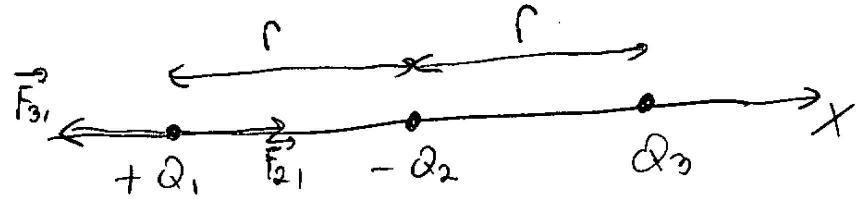
Homework 1

1.1 A



$Q_1 = Q_2$

The magnitude of force $\vec{F}_{21}$ of charge Q_2 on charge Q_1 is

$$F_{21} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1^2}{r^2}$$

Similarly

$$F_{31} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_3}{(2r)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_3}{4r^2}$$

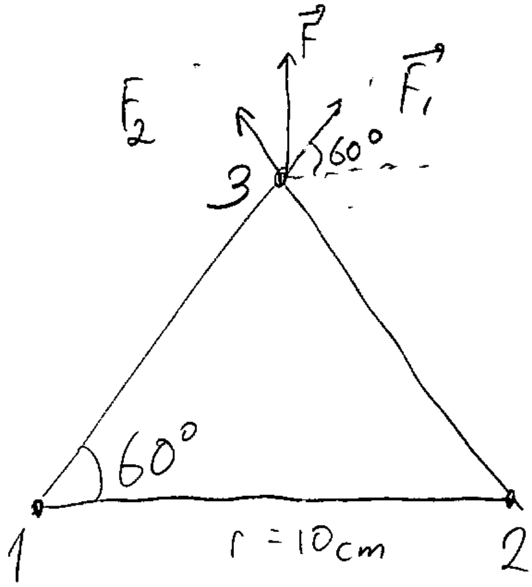
We need

$$F_{21} = F_{31}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_1^2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_3}{4r^2}$$

$$\boxed{Q_3 = 4Q_1}$$

1.1 B



Because of symmetry, the magnitude of force on each charge is the same. We will just calculate the force on charge 3. In magnitudes

$$F_1 = F_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(0.1m)^2} = 9 \cdot 10^{11} Q^2 \ N$$

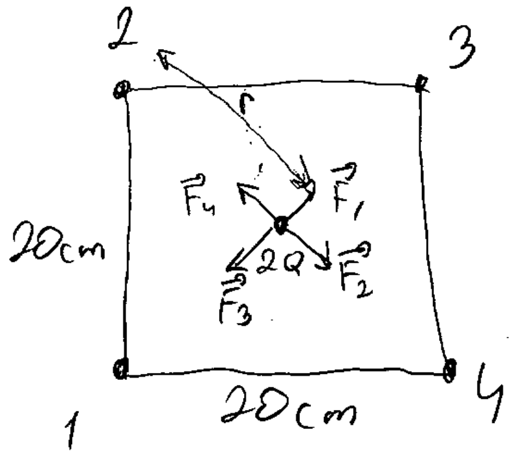
The horizontal components cancel, while the vertical components are the same and must be added together:

$$F = F_{1y} + F_{2y} = 2F_{1y} = 2F_1 \sin 60°$$

$$\boxed{F = 1.56 \cdot 10^{12} Q^2 \ N}$$

1.1 C

a.



The forces from opposite corners cancel each other, so the total force is 0.

b. Let say charge 4 is removed, $\vec{F}_1$ and $\vec{F}_3$ still cancel, the total force is equal to $\vec{F}_2$:

$$F = F_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q \cdot Q}{r^2} = 9 \cdot 10^9 \frac{2Q^2}{(0.14)^2} N$$

$$\boxed{F = 9 \cdot 10^{11} Q^2 N}$$

where we used

$$r = \frac{0.2m}{\sqrt{2}} = 0.14m$$

1.2 A

a. $\rho = \frac{Q}{V} = \frac{Q}{\frac{4\pi}{3} r^3} = \frac{Q}{\frac{4\pi}{3}(0.02\,m)^3}$

$\rho = 2.98 \cdot 10^4 \, Q \, \frac{C}{m^3}$

b. In the inner shell

$Q_{in} = \rho \cdot V_{in} = \rho \, \frac{4\pi}{3} (0.01m)^3 = 4.19 \cdot 10^{-6} m^3 \cdot \rho$

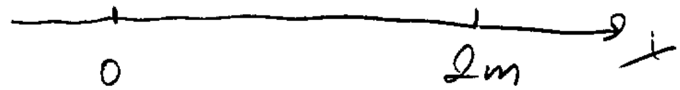
$Q_{in} = 0.125\, Q$

In the outer shell

$Q_{out} = Q - Q_{in}$

$\boxed{Q_{out} = 0.875\, Q}$

1.2 B

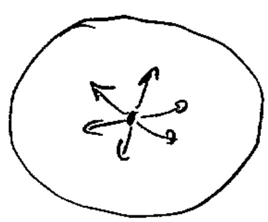


$$\mu = \mu_0 + 2x \, \frac{C}{m}$$

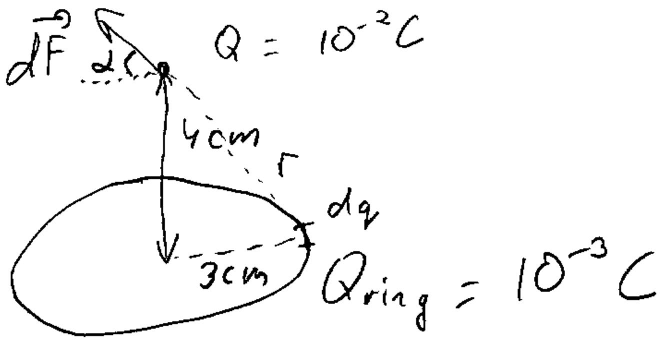
$$Q = \int_0^{2m} \mu(x)\, dx = \int_0^{2m} (\mu_0 + 2x) \frac{C}{m} \, dx =$$

$$= (\mu_0 x + x^2) \frac{C}{m} \Big|_0^{2m} = \underline{(4 + 2\mu_0)\, C}$$

1.3 A



If a point charge is placed at the center of a ring with uniform charge distribution, the forces from opposite parts of the ring will cancel each other and <u>the total force will be 0.</u>



Let us consider the force from a small part of the ring with charge dq.

$$r^2 = (4cm)^2 + (3cm)^2$$

$$r = 5\ cm = 0{,}05\ m$$

$$dF = \frac{1}{4\pi\varepsilon_0} \frac{Q \cdot dq}{r^2}$$

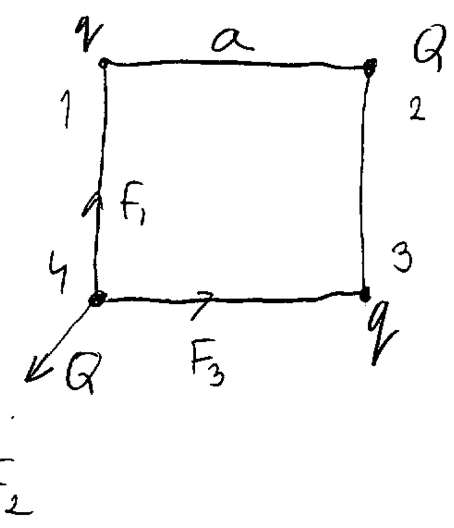
If we add the forces from all parts of the ring, the horizontal components will cancel each other from opposite parts, the vertical components will add,

$$dF_y = dF \sin\alpha = dF \cdot \frac{4}{5} = \frac{4}{5} \frac{1}{4\pi\varepsilon_0} \frac{Q\, dq}{r^2}$$

$$F_y = \int_{ring} dF_y = \frac{4}{5} \frac{1}{4\pi\varepsilon_0} \frac{Q \int_{ring} dq}{r^2} = \frac{4}{5} \cdot \frac{1}{4\pi\varepsilon_0} \frac{Q \cdot Q_{ring}}{r^2}$$

$$\boxed{F_y = 2.88 \cdot 10^7\ N}$$

1.3 B



Consider the total force on charge q. It is clear that cancellation can be achived only if charges q have opposite sign to Q.

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(\sqrt{2}\,a)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2a^2}$$

$$F_1 = F_3 = \frac{1}{4\pi\varepsilon_0}\frac{|Q||q|}{a^2}$$

Cancellation will be achived if the horizontal component of $\vec{F_2}$ cancels $\vec{F_3}$ and the vertical component of $\vec{F_2}$ cancels $\vec{F_1}$.

$$F_{2x} = -F_2\cos 45^\circ = -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2a^2}\cdot\frac{\sqrt{2}}{2}$$

$$F_{2x} + F_3 = 0$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2a^2}\frac{\sqrt{2}}{2} = \frac{1}{4\pi\varepsilon_0}\frac{|Q||q|}{a^2}$$

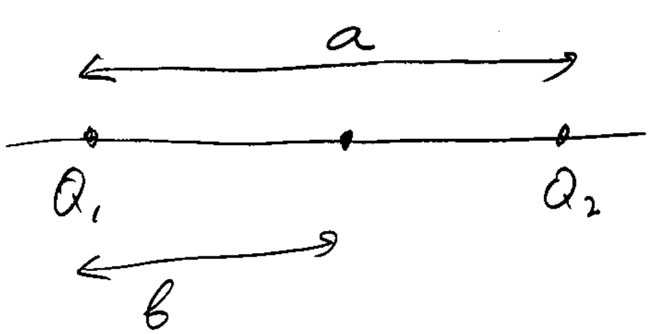
$$|q| = \frac{\sqrt{2}}{4} |Q|$$

$$\boxed{q = -\frac{\sqrt{2}}{4} Q}$$

It is easy to check that in this case the vertical forces also cancel (there is symmetry).

We see that in order to cancel the forces on charges Q we need <u>smaller</u> charges q at the other corners. This means that the charges Q will not ~~be~~ able to cancel the forces on charges q since $|Q| > |q|$. It is <u>not possible</u> to cancel the forces on all the charges.

1.4 A



At a distance x the force on Q_2 is

$$F_x = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{x^2}$$

so at distance a

$$\boxed{F_x = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{a^2}}$$

To move it to distance b we need to apply equal and opposite force

$$F_{appl\,x} = -F_x = -\frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{x^2}$$

The work is

$$W = \int_a^b F_{appl\,x}\,dx = -\int_a^b \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{x^2}\,dx =$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{x}\Big|_a^b = \underline{\frac{1}{4\pi\varepsilon_0} Q_1 Q_2\left(\frac{1}{b} - \frac{1}{a}\right)}$$

1.5 A

$$F_{el} = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{r^2}$$

$$F_{grav} = G \frac{m^2}{r^2}$$

$$\frac{F_{el}}{F_{grav}} = \frac{\frac{1}{4\pi\varepsilon_0} q^2}{G \ m^2} = \frac{9\cdot 10^9 \cdot (1.6\cdot 10^{-19})^2}{6.67\cdot 10^{-11} \cdot (9\cdot 10^{-31})^2}$$

$$\boxed{\frac{F_{el}}{F_{grav}} = 4.26\cdot 10^{42}}$$

1.6 A

$r = 0.528\cdot 10^{-10}\ m$

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2} = 8.26\cdot 10^{-8}\ N$$

The centripetal acceleration is

$$a = \frac{F}{m_{el}} = 9.18\cdot 10^{22}\ m/s^2 = \frac{v^2}{r}$$

$$v = \sqrt{ar}$$

The frequency is

$$f = \frac{v}{2\pi r} = \frac{\sqrt{ar}}{2\pi r} = \frac{1}{2\pi}\sqrt{\frac{a}{r}}$$

$$\boxed{f = 6{,}64 \cdot 10^{15}\ s^{-1}}$$

$$L = m_{el} v r = m_{el} \sqrt{ar} \cdot r$$

$$\boxed{L = 1{,}05 \cdot 10^{-34}\ kg \frac{m^2}{s}}$$

(this is the reduced Planck's constant you will learn about in 4E)

Now suppose the force was gravitational

$$F_{grav} = G \frac{M_{prot} \cdot M_{el}}{r^2}$$

$$a = \frac{F_{grav}}{M_{el}} = \frac{G\, M_{prot}}{r^2}$$

$$L = M_{el} \sqrt{ar} \cdot r$$

$$L^2 = M_{el}^2\, a r^3$$

$$a = \frac{L^2}{M_{el}^2 r^3}$$

$$\frac{G\, M_{prot}}{r^2} = \frac{L^2}{M_{el}^2 r^3} \Rightarrow r = \frac{L^2}{G\, M_{prot}\, M_{el}^2}$$

using $M_{prot} = 1{,}67 \cdot 10^{-27} kg$ we get

$$\boxed{r = 1{,}2 \cdot 10^{27}\ m}$$