Ch. 19 Solutions

These are the problems from the chapter I've worked out so far. I've gone through the solutions and corrected several mistakes. For questions, email me at hosam@ucsd.edu

Monday 3:20p: Corrected problem 45.

2.

a) The number of moles of silver is the number of grams divided by the molar mass, which is $10/107.87 = 0.09$ moles. Each mole contains $6.02 \cdot 10^{23}$ atoms, each of which has 47 electrons. Multiplying it all out gives a total of $2.55 \cdot 10^{24}$ electrons in the pin.

b) The total charge from all the electrons already on the pin is $2.55 \cdot 10^{24} \times -1.6 \cdot 10^{-19} = -4.07 \cdot 10^5 C$. Keep in mind that for each electron there is a proton with equal and opposite charge: The pin is *neutral* overall. If we add electrons until the total charge is $-1mC$, then we must add $\frac{1mC}{1.6 \cdot 10^{-19} C/elec} = 6.25 \cdot 10^{15}$ electrons total, or 2.38 new electrons per 10^9 electrons already present.

3.

An order of magnitude estimate essentially means a ballpark figure, so:

Two persons at arms length is approximately 2 meters. The number of atoms in the body can be roughly guessed by **approximating** the body as being made up entirely of water (not a terrible approximation). A water *molecule* has around 20 protons and electrons, and a *mole* of water weighs about 20 grams; a typical adult human weighs around 100kg. This gives 50 moles per kilogram, equivalent to about 5000 moles of water per person. Crunching these numbers says that the average individual has a total of $\approx 100,000$ moles of both protons and electrons, totalling up to about 10^9 (a billion) Coulombs. The net charge so far is still zero, since the electrons and protons balance each other out. If we remove 1% of the charge from both, then the net charge on each is about 10^7 C. Coulomb's law can now be used:

$F_e = \frac{kqQ}{r^2} = \frac{(9 \cdot 10^9)(10^7)(10^7)}{2^2} \approx 2.25 \cdot 10^{23}$ Newtons. The mass of the earth is $6 \cdot 10^{24} kg$, which would require (via $F = mg$) $\approx 10^{25}$ N to lift – not too far off for such crude approximation.

4.

The charge of a proton is $+1.6 \cdot 10^{-19} C$, so using Coulomb's law (above) gives 57.6N. They would repel each other.

5.

Remember that forces are vectors and thus require vector addition. The x components will add, while the y components will substract. Compute the magnitude of the force between the 2-7 pair (0.5N), and then the magnitude between the (-4)-7 pair (1.01N). Then add the x and y components separately. $F_x = 0.76N$ and $F_y = -0.44N$. The net force is then $|F| = \sqrt{F_x^2 + F_y^2} = 0.87N$. The direction is given by $\theta = \arctan(\frac{F_y}{F_x}) =$ 30.1 degrees clockwise from the positive x-axis.

6.

Newton's 3rd law guarantees that the force of A on B is the same as that of B on A at all times. Knowing the force and the distance, you can work backwards with Coulomb's law to figure out what $k \cdot q \cdot Q$ is (you can't figure out what each charge is separately, and theres no point in punching in the value of k). Knowing what that value is, write down Coulomb's law again substituting the above number for $k \cdot q \cdot Q$ and use the new distance $r = 17.7mm$. The final result is $1.56 \cdot 10^{-6}$ N. Since particle B is to the right of particle A, its force must point left (opposite the direction of F_{AonB}).

7.

The first part is straight Coulomb's law. Using Gauss' Law you'll see that even though the spheres are not point charges, you can still pretend that all the charge is concentrated at the center. I'll talk about this later (Problem 40). The answer to (a) is $2.16 \cdot 10^{-5} N$, and the force is attractive.

In part b, connecting the wire will cause current to flow in order to balance out as much charge as possible. The *net* charge in this problem is $-6nC$, which remains the same even after connecting the wire. After connecting the wire, each sphere will then have a total of $-3nC$. Using Coulomb's law again with this new figure gives $F = 9 \cdot 10^{-7} N$. Since they both have negative charge after connecting, the spheres would repel each other.

8.

Equilibrium, as in mechanics, means that the net force on the object is zero. Here there are two forces, one from each of beads on the end. Coulomb's law needs the distance between the objects involved, which means we have to figure out where to measure from. I will choose $x = 0$ to be where the $+3q$ charge is, and $x = d$ to be where the $+q$ charge is. We are looking to solve for the position of the grey bead, which I will label x. Imagine an arrow pointing from $x = 0$ to wherever the grey bead is, that distance is x (it is a variable right now since we don't know where the bead is). Now we can write down Newton's 2nd law as:

$$\sum F = \frac{3kq^2}{x^2} - \frac{kq^2}{(d-x)^2} = 0$$

Notice that in the denominators, x, and $d - x$ are the distances from the grey bead to each of the orange beads (refer to diagram). This becomes an algebra problem for x. Notice that q and k both cancel. I am subtracting the two forces because the bead on the left pushes to the *right*, whereas the bead on the right pushes to the *left*. Since these directions are opposite, the forces will subtract. Play with the algebra to get $2x^2 - 6xd + 3d^2 = 0$, which you can solve with the quadratic formula (treat d as if it were a number) or with some clever algebra. The answer is $x = 0.634d$. You can understand this problem as follows: Each of the orange charges push the grey charge in opposite directions, and the strength of that push depends on how far away the grey charge. There must be a place inbetween the two charges that the pushes are equal: this is the equilibrium position. This equilibrium is NOT stable for negative charges because if you put the grey charge at the equilibrium spot and poked it slightly, you'll see that the net force is AWAY from the equilibrium position causing it to fly off. Both forces point inwards for a positive charge, so it is stable in that case.

9.

The force between a proton and electron is given by Coulomb's law. Plug in their charges and the distance to find $F = 8.23 \cdot 10^{-8} N$. For the second part you'll need to remember that the centripetal force (needed for circular motion) is $F_c = \frac{mv^2}{r}$ where v is the velocity of the orbiting object and m is it's mass. Since the only force is the electric force between the two and the motion is circular, the net force must equal the centripetal force:

$$F_{net} = \sum F = \frac{ke^2}{r^2} = \frac{m_{elec} v^2}{r} \text{ with } e \text{ being the charge of an electron/proton.}$$

Solving this for v gives $v = 2.19 \cdot 10^6$ m/s. Thats 5 million miles per hour.

10.

By *weight*, they mean the force of gravity on earth ($F = mg$). Looking up the masses one finds the weights to be $F_p = 1.64 \cdot 10^{-26} N$ and $F_e = 8.93 \cdot 10^{-30} N$. In terms of the electric field, the electric force is given by $F = qE$. q is the same for both the electron and the proton, but remember that negative charges travel *opposite* the direction of the electric field, whereas positive charges travel *along* the field. Setting each of the F's above equal to qE we find that for the proton we need $E = 1.02 \cdot 10^{-7} N/C$ and for the electron one needs $E = 5.58 \cdot 10^{-11} N/C$. With the above rules for the directions, we know that the electric field will have to point up for the proton, and down for the electron.

11.

This is just like problem 8, except the two charges on the ends are of opposite sign. The set up and solution are identical to problem 8; however be careful with the signs when writing down the net electric field. I choose $x = 0$ to be where the $q = -2.5\mu C$ charge is, and $x = 1$ where the $Q = 6\mu C$ charge is. The point where the E field is zero I will call x, again. The equation to look at is then:

$$\sum E = -\frac{kq}{x^2} + \frac{kQ}{(1-x)^2} = 0$$

Here k cancels again, and you can substitute in numbers for q and Q. You can again use the quadratic formula, as in 8, or some clever algebra tricks can simplify the math. The two solutions should be $x = 0.39m$ and $x = -1.82m$. The first one is between q and Q, while the second one (since it is negative) is out to the left of both q and Q. If you consider the direction E points at $x = 0.39m$ for each charge, you will see that they cant possibly add up to zero, which leaves you with $x = -0.1.82m$. (Check where it points there too, for practice.)

12.

This is a similar set up to 8 and 11, except we're solving for charge instead of location. I'll call the unknown charge q and the known charge Q. The origin is where $x = 0$, and the electric field there is $2kQ/a^2$. Drawing a diagram will help with this problem. The net electric field at $x = 0$ is:

$$E = E_Q + E_q = \frac{kQ}{a^2} - \frac{kq}{(3a)^2} = \pm\frac{2kQ}{a^2}$$

The last equality is just setting the total field equal to the value in the book. The trick here is to realize that they gave you the *magnitude* of the field at $x = 0$, which means it could be both positive and negative. You'll have to solve the plus and minus cases separately. I get $q = -9Q$ and $q = 27Q$.

13.

The electric field will add as a vector, so we will work in components again. Luckily since each charge has only a horizontal or vertical component it is not so difficult.

6nC charge: $E_x = -600N/C, E_y = 0$. -3nC charge: $E_x = 0, E_y = -2700N/C$.

Adding them up is easy, you get total $E_x = -600N/C$ and total $E_y = -2700N/C$. The total electric field at the origin is then $|E| = \sqrt{E_x^2 + E_y^2} = 2765N/C$. The direction is again given by $\theta = \arctan(\frac{E_y}{E_x}) = 12.5$ degrees counterclockwise from the negative x axis. Finding the electric force on the $5nC$ charge uses the equation $\vec{F} = q\vec{E}$. The direction is the same, and the magnitude of the force is $F = 5 \cdot 10^{-9} \cdot 2765 = 1.38 \cdot 10^{-5}N$.

14.

Same procedure as 13, just not as nice. Drawing a diagram will help again. Use the pythagorean theorem to show that each of the two charges is $1.12m$ from the point of interest (x=0,y=0.5). This is the distance you'll use in the electric field equations. Notice that the problem is symmetric about the y-axis: This is more than just pretty, it means that the force along the x-axis has to be zero.

The electric field due to each charge is identical. Using the standard equation, the magnitude of the electric field for each of the charges is $E = 1.44 \cdot 10^4 N/C$. To find the total E-field, add the components of each vector separately. $\theta = \arctan(\frac{0.5}{1}) = 26.6$deg giving $|E_x| = |E|\cos\theta = 1.28 \cdot 10^4 N/C$ and $|E_y| = 6.44 \cdot 10^3 N/C$. The symmetry comes in here: Since $|E_x|$ is the same for both, but points in opposite directions (b/c the charges are on opposite sides of the y-axis), they sum to zero. The y components add directly to give total $E_y = 1.29 \cdot 10^4 N/C$, and the total E field points up. The force also points up and is given by $F = qE = -3.85 \cdot 10^{-2}N$.

15.

The electric field at q is not affected by the charge at q, so we include only the effects of the other three charges.

$$E_x = \frac{2kq}{a^2} + \frac{3kq}{(\sqrt{2}a)^2} \cdot \cos 45$$
$$E_y = \frac{4kq}{a^2} + \frac{3kq}{(\sqrt{2}a)^2} \cdot \sin 45$$

$|E| = \sqrt{E_x^2 + E_y^2} = 5.91\frac{kq}{a^2}$. The force is obtained by multiplying by q, the charge at the location of interest. The direction is given by $\theta = arctan(\frac{E_y}{E_x}) = 58.8$deg CCW from the +x axis.

16.

The electric field far away will be due to the combined field of each of the two charges q. We know this will be

$$\sum E = E_- + E_+ = \frac{kq}{r_+^2} - \frac{kq}{r_-^2}$$

It is then convenient to write $r_+ = r + a$ and $r_- = r - a$ which one can deduce from the figure. Plugging this into the above equation gives

$$E = kq\left(\frac{1}{(r+a)^2} - \frac{1}{(r-a)^2}\right) = \frac{kq}{r^2}\left(\frac{1}{(1+\frac{a}{r})} - \frac{1}{(1-\frac{a}{r})^2}\right)$$

Since the point in consideration is very far along the x axis, $\frac{a}{r} \ll 1$. Using the Taylor expansion for $\frac{1}{(1\pm x)^2} \approx 1 \mp 2x$ gives $E = \frac{kq}{r^2} \cdot (1 - 2\frac{a}{r} - 1 - 2 \cdot \frac{a}{r}) = -\frac{kq}{r^2} \cdot (\frac{4a}{r})$ - the answer. Note the answer is negative since the negative charge is closer to the point of interest than the positive, so its electric field is slightly larger than that of the positive charge.

You won't be expected to solve this difficult of a problem on an exam.

17.

Let's put the center of the rod at $x = 0$, then we want to find the electric field at $x = 36$. The field due to a single point charge is $E = \frac{kq}{r^2}$, with r the distance away from that charge. Imagine the line to be lots of small positive charges lumped very close together. Each tiny piece of charge on the line dq will generate a field dE according to $dE = \frac{k}{r^2}dq$, with r the distance between that particular piece of the line and the origin. For each charge dq, the distance from $x = 36$ can be broken up into two pieces: the distance from $x = 36$ to the center of the rod $x_0 = 36$, plus the distance from the center of the rod to the charge in question, call it x. This latter distance changes depending on which charge along the rod we are talking about. To find the total field, we add up (integrate) along the entire string:

$$E = \int dE = k\int_{rod} \frac{dq}{r^2} = k\lambda \int_{-7}^{7} \frac{1}{r^2}dx = k\lambda_0 \int_{-7}^{7} \frac{1}{(x_0 + x)^2}$$

Since the rod is uniformly charged, $\lambda = \frac{Q}{L} = 1.57 \cdot 10^{-4} C/m$. I also used the chain rule $dq = \frac{dq}{dx}dx = \lambda_0 dx$ to change the integration variable to x. The u-substitution $u = x + x_0, du = dx$ (don't forget to change the limits) simplifies the integral to give $E = 1.59 \cdot 10^6 N/C$. Since the rod is negatively charged, the field points towards it.

18.

Refer to problem 17. Using the chain rule, $dq = \frac{dq}{dx}dx = \lambda_0 dx$ (by definition of line charge density). Then

$$E = \int dE = k\int_{line}\frac{dq}{r^2} = k\lambda_0\int_{x_0}^{\infty}\frac{1}{r^2}dx = k\lambda_0\int_{x_0}^{\infty}\frac{1}{x^2}dx$$

The last equality holds because the distance away from the origin of any particular charge along the line is given by it's x coordinate. Computing the integral and plugging in the two limits gives $E = \frac{k\lambda_0}{x_0}$. Since the line is made up of positive charges, the field will point away from them, meaning that it points left at the origin.

19.

The axis of a ring means the line passing perpendicular to it and through the center. Looking at the ring from the side (so that it looks like a horizontal line), the distance from the edge to the center is $R = 10cm$. As we go up along the axis, the distance between the charges and the point of interest increases: $r = \sqrt{R^2 + y^2}$, where y is the distance up along the axis. $\lambda = Q/L = 1.19 \cdot 10^{-4} C/m$, and we will again use $dq = \frac{dq}{d\ell}d\ell$

It just so happens that, at a given y, the distance from each charge to the observation point is exactly the same. However, the difficulty in this problem is that the direction of the electric field changes with each charge. To see this consider two charges on opposite points along the ring (draw a diameter, and consider the charges on the ends of the diameter). Both charges are positive meaning the electric field will point upwards and away from them; however, as in problem 14 the x components cancel by symmetry. The total field therefore points up, but its magnitude its reduced because only the y component survives, and $E_y = E\cos\theta$ where θ is the angle between the line pointing down from the observation point along the y axis to the center and the line pointing from the observation point to the ring edge. Using the diagram you have you can see $\cos\theta = \frac{y}{r}$. Write down the integral:

$$E_y = \int dE_y = k\int\frac{dq}{r^2}\cos\theta = k\lambda\int_{ring}\frac{1}{r^2}\frac{y}{r}d\ell = k\lambda\int_{ring}\frac{y}{\left(\sqrt{R^2+y^2}\right)^{3/2}}d\ell$$

The integral is therefore over a constant (for a given y), and so $E = k\lambda\frac{y}{(R^2+y^2)^{3/2}}\int_{ring}d\ell = k\lambda\frac{2\pi Ry}{(R^2+y^2}))^{3/2}$. Here, I used the fact that adding up the distance along a ring just gives you the ring's circumference. Now plug in values for y to solve the problem (all values in N/C):

a)$6.63 \cdot 10^6$
b)$2.41 \cdot 10^7$
c)$6.38 \cdot 10^6$
d)$6.63 \cdot 10^5$

20.

Above we showed that for a ring the electric field along at a distance y along the axis is $E(y) = k\lambda\frac{2\pi Ry}{(R^2+y^2)}^{3/2}$. Recall from calculus that the maximum of a function occurs when its derivative is equal to zero. Set the derivative (quotient rule) equal to zero and solve for the value of y:

$E'(y_{max}) = 2\pi k\lambda R\left(\frac{1}{(R^2+y_{max}^2)^{3/2}} - \frac{3y_{max}^2}{(R^2+y_{max}^2)^{5/2}}\right) = 0 \rightarrow y_{max} = \frac{R}{\sqrt{2}}$

Plug this into E(y) to find the the value of E.

21.

Notice the problem is symmetric about the x-axis. This immediately implies that $E_y = 0$, so I won't bother showing it. Once again, integrate over each charge on the half ring keeping only the x component. Define θ to be the angle between the horizontal line connecting the point O to the ring and the line pointing to the charge dq in question (diagram). Note also since O is the center of the circle, the distance from each charge to O is the same - namely $\frac{1}{2}C = \frac{1}{2}(2\pi r) \to r = \frac{.14}{\pi}cm = 4.46cm$ and so r **is a constant**. Also here, $d\ell = rd\theta$ (differential form for the arclength in terms of the angle $\ell = r\theta$).

$$E_x = \int dE_x = k\int \frac{dq}{r^2}\cos\theta = \frac{k\lambda r}{r^2}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta = \frac{k\lambda}{r}(1-(-1)) = 2.17\cdot 10^7 N/C$$

Since the ring is negatively charged, the field points towards the ring at O (directly left).

22.

The x component vanishes by symmetry as in the previous problems. The distance to each charge is no longer a constant as we move along the rod, so $r = \sqrt{y^2+x^2}$ won't fall out of the integral. Again, $dq = \lambda dx$

$$E_y = \int dE_y = k\int_{line}\frac{dq}{r^2}\cos\theta = k\lambda\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}}\frac{y}{(x^2+y^2)^{3/2}}dx$$

To do this integral, substitute $x = y\tan\theta$, $dx = y\sec^2\theta d\theta$ and note $1+\tan^2\theta = \sec^2\theta = \frac{1}{\cos^2\theta}$ to get

$$E_y = \frac{ky\lambda}{y^2}\int_{-\theta_0}^{\theta_0}\frac{\cos^3\theta}{\cos^2\theta}d\theta = \frac{2k\lambda\sin\theta_0}{y}$$

As the length of the rod goes to infinity, θ_0 gets closer and closer to 90 degrees, and so $\sin\theta_0 \to 1$. This gives the desired result.

23.

The two faces of the cylinder have a total area of $2\pi R^2 = 4.0\cdot 10^{-3}m^2$, while the curved part has an area $C\cdot\ell = 9.4\cdot 10^{-3}m^2$. The volume of the cylinder is $V = \pi R^2\ell = 1.12\cdot 10^{-4}m^3$. The charges therefore are given by $Q = \sigma A$ and $Q = \rho V$

a) $2.01\cdot 10^{-10}C$
b) $1.41\cdot 10^{-10}C$
c) $5.60\cdot 10^{-11}C$

24.

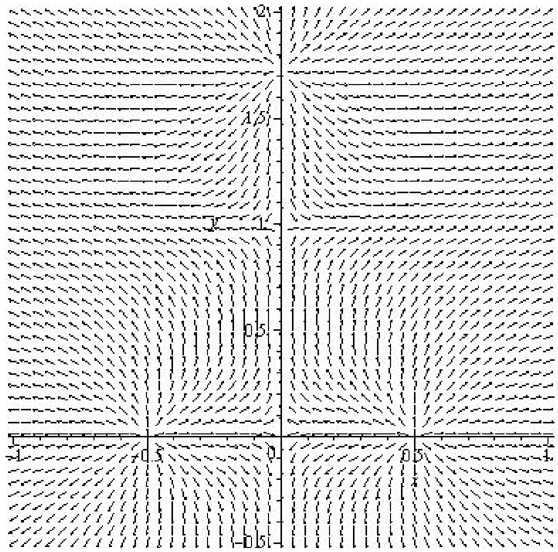
By Gauss' Law, the flux flowing through a surface is proportional to the charge enclosed by that surface. Imagine a gaussian surface surrounding only q_2: for this surface there are 18 flux lines that flow out. For a surface surrounding only q_1, there are 6 field lines, which means that q_2 must be three times larger than q_1, $\frac{q_1}{q_2} = \frac{1}{3}$. Since the field lines flow out of q_2 and into q_1, their charges must be positive and negative, respectively.

25.

Solution is drawn in the back.

26.

All coordinates in the graph are multiplied by a. The location where the field is zero is at (x=0,y=1), which is infact the centroid of the triangle (symmetry, again). The field at P due to both charges is the sum of the forces of each. The x components cancel by symmetry, and the y component is proportional to the cosine of the angle between the line pointing straight down from P to the midpoint between the two charges and the line connecting P to either charge. This angle is 30 degrees. Thus $E_y = \frac{2kq}{a^2}\cdot\frac{\sqrt{3}}{2}$.



27.

The acceleration is given by Newton's Law, $F_{net} = ma = qE \rightarrow a = qE/m = 6.13 \cdot 10^{10} m/s^2$. Since the proton is in a constant electric field, the acceleration is uniform and the standard kinematics equations give $v = at \rightarrow t = 19.5\mu s$, $x = \frac{1}{2}at^2 = 11.7m$, and $KE = \frac{1}{2}mv^2 = 1.2 \cdot 10^{-15} J$.

28.

The work done by a constant force is equal to $W = F\Delta x \cos\theta = eE\delta x \cos\theta$ where Δx is the distance travelled and θ is the angle between the force and displacement. Here, the force serves to slow down the particle, which means the force is opposite the displacement ($\theta = 180$deg). To stop the particle, all of its KE must vanish, meaning the work done equals the KE. To stop in a distance d one has $KE = eEd \rightarrow E = \frac{KE}{ed}$ where e is the electron charge.

29.

a) Since the field (hence the force) is vertical, the horizontal component of the velocity doesn't change. For constant velocity $d = vt \rightarrow t = \frac{d}{v} = 1.11 \cdot 10^{-7} s$.

b) In the vertical direction, the net force on the proton is $F = qE = 1.54 \cdot 10^{-15} N$. Dividing by the mass of the proton gives $a = 9.2 \cdot 10^{11}$m/s^2 so that $\Delta y = \frac{1}{2}at^2 = 5.67 \cdot 10^{-3} m$.

c) The horizontal velocity does not change since there is no force in the horizontal direction: $v_x = 4.5 * 10^5$ m/s. The vertical velocity increases since the proton is accelerating: $v_{fy} = v_{iy} + at = 0 + 1.02 \cdot 10^5$ m/s.

30.

The expression for electric flux is given by $\Phi_e = \vec{E} \cdot \vec{A} = |E||A|\cos\theta$ where θ is the angle between the electric field vector and the vector perpendicular to the area (perpendicular to the car). Plugging in gives $\Phi_e = 3.55 \cdot 10^5 Nm^2/C$.

31.

The electric flux is given by Gauss' Law: $\Phi_e = \vec{E} \cdot \vec{A} = |E||A|\cos\theta$. This quantity is maximum when $\cos\theta = 1$, which means that $\Phi_e = EA \rightarrow E = 4.14 \cdot 10^6$ N/C after having used $A = \pi R^2$. Don't forget to change the diameter (given) into a radius.

32.

Gauss' law says $\Phi_e = E \cdot A = \frac{Q_{enc}}{\epsilon_0}$. A is the surface area of the sphere, $A = 4\pi R^2 = 7.1m^2$ which tells us that $Q_{enc} = 5.58 \cdot 10^{-8}C$.

33.

Since all the flux entering the flat surface must exit *somewhere* (flux always flows), we know that the total flux going out of the curved surface must equal that going in through the flat. This is also seen by noting that since the blue semisphere contains no charge, Gauss' law says the net flux exitting through its surface must be zero: all flux inside must flow out. Now, if the semisphere were instead a full sphere, it would fully enclose the charge and we know by Gauss' Law that the flux is then $\frac{q}{\epsilon_0}$. However, symmetry requires that half the flux go through the bottom of this sphere and half through the top. If we then remove the top, the flux through the bottom remains half the total: $\Phi_e = \frac{q}{2\epsilon_0}$. The remainder of the flux just shoots out towards where the top sphere previously was.

34.

A cube has six identical sides. With the charge in the center, there's no reason more flux should come out of one side than any other, so the total flux splits six ways through each face. The total flux is $\Phi_e = \frac{q}{\epsilon_0} = 1.92 \cdot 10^7 Nm^2/C$ and the flux through each face is $\frac{1}{6}$ that amount. If the charge were not in the center, the total flux through the cube remains the same (the cube still encloses the charge) but the flux through each face depends on the new position. Imagine you moved the charge very close to one side. Flux still flows outwards from the charge as before, but much more will flow through the side closest to the charge: The flux no longer splits evenly between the six sides.

40.

Note: This problem is conceptually important but difficult. opioo

A general note on Gaussian surfaces: Gauss' law takes the form of Coulomb's Law when the surfaces involved are all concentric and spherical. Concentric means the centers of each spherical shape is in the same spot as the others. The reason is because when everything is spherically symmetric, the gaussian surface takes the shape of a shell. Solving $\Phi_e = E(4\pi R^2) = \frac{q_{enc}}{\epsilon_0}$ for E gives $E = \frac{kq_{enc}}{R^2}$ where R is the radius of the gaussian shell. Using $F = qE$ you get Coulomb's Law. This is also why in problem 7 we didnt have to worry about the charges not being points.

Imagine the insulating solid sphere is glass, and the imaginary gaussian surface was made of mesh and spherically shaped. We apply Gauss' law to the spherical mesh shell. Call the radius of the glass ball R_{st}, and the radius of the mesh shell R_m. Now, since the mesh is imaginary we can shrink and expand it as we please – R_m is a variable; on the other hand R_{st}, the size of the see-through ball, is fixed at a constant value. As we increase the radius of the mesh, it will enclose more charge until we reach the surface of the glass ball, at which point increasing the size of the mesh surface doesn't enclose any more charge (since making it bigger only encloses more empty space). So, while the mesh grows but is still smaller than the glass ball, Q_{enc} grows with the volume (since the volume charge density is constant $Q = \rho V$): $Q_{enc} = \rho \frac{4}{3}\pi R_m^3$. Then $\Phi_e = \frac{Q_{enc}}{\epsilon_0} = \frac{\rho}{\epsilon}\frac{4}{3}\pi R_m^3$ where $\rho = Q/V$. However, the moment the size of the mesh shell surpasses the size of the glass ball, an increase in mesh size no longer encloses more charge. This means that when $R_m > R_{st}$, the flux no longer increases: $\Phi_e = const = \frac{Q}{\epsilon_0}$.

A graph of this would look like $f(r) = r^3$ for a little while, then suddenly change into $f(r) = constant$ (a horizontal line) at the value where $r = R_{st}$.

45.

If you liked the mesh/glass analogy you can use it here. It will definitely help to draw 3 concentric circles labeled 3,4, and 5. You can add the mesh with a dotted circle as needed. The equation I'm using is Gauss' law for spherical shapes: $E = \frac{kQ_{enc}}{r^2}$

a) Any **solid** conductor has *all* of its charge on the surface of that conductor. This is a property of conductors only. If we draw a gaussian sphere anywhere within the solid ball, it must enclose no charge. $Q_{enc} = 0$ means that $E = 0$.

b) At $r = 3cm$, our gaussian sphere encloses all of the charge on the inner ball, and none of the charge contained on the shell. Q_{enc} is then the charge contained on the inner ball, $8\mu C$. Plugging in these numbers gives $E = 8 \cdot 10^7 N/C$. Since Q_{tot} is positive, we know the net field must point away from the center.

For c and d:

Since the outer conductor is a shell, it has both an inner surface and an outer surface. All of the charge in *any* conductor resides on the surface, so the $-4\mu C$ on this shell will be spread between both of these surfaces. Now, since we are looking for the electric field *inside* a conductor, we know that E = 0 there. That means that Q_{enc} must also be zero. Since any gaussian shell with $r > 2cm$ automatically encloses the $-8\mu C$ on the inner ball, drawing a gaussian shell with $4 < r < 5$ would show that there must be $+8\mu C$ on the inner surface of the actual shell. Since the total charge on the shell is $-4\mu C$, there must be $4\mu C$ on the outer ($r = 5cm$) surface.

c) At $r = 4.5cm$, we now acquire some charge from the spherical shell since the dotted circle is between 4 and 5. $Q_{enc} = 0$ as I showed above, which means that $E = 0$. This shouldn't be a surprise, $r = 4.5cm$ is located within a conductor after all.

d) At $r = 7cm$ the gaussian surface encloses all of the charge in the problem, $Q_{enc} = 4\mu C$ and $E = 7.36 \cdot 10^6 N/C$. Positive net charge means the electric field points outwards at this location.

50.

Since the electric field points down, it means that the charge on the surface is negative. Electric field points *from* positive *to* negative. To find the charge density, recall that for a planar conductor (flat surface), $E = \frac{\sigma}{\epsilon_0} \rightarrow \sigma = 1.06 \cdot 10^{-9} C/m^2$. The charge density represents how much charge there is per unit area. If we want to find out how much charge would be on the entire earth's surface, $Q = \sigma A = 4\pi\sigma R_{earth}^2$. $R_{earth} = 6 \cdot 10^6 m$ so $Q = 4.79 \cdot 10^5 C$. The number of electrons needed is $2.99 \cdot 10^{24}$.

54.

The ball is in equilibrium, so we know the net force is zero. There are three forces in this problem. Gravity is trying to pull the ball downards, the electric field exerts a force pulling the ball to the right. The string's tension serves to keep the ball in place. Write down the x and y components of Newton's 2nd law:

$$\sum F_x = qE - T_x = 0$$
$$\sum F_y = -mg + T_y = 0$$

If you dont know why the signs are this way, draw a force diagram for the three forces and compare the relative directions of the arrows to the corresponding +/- in the equations. I chose up to be positive and chose right to be positive. $\theta = 15$ degrees measured from the *y-axis*, so in this case $T_x = T\sin\theta$ and $T_y = T\cos\theta$. Plug numbers into the second equation to solve for T, then plug T into the first equation to solve for q. I get $q = 5.25\mu C$, positive since it is being pulled *along* the field.