Physics 130B Course Outline Fall 2011

Day boundaries are approximate. Don't worry about keeping up. Go at the pace the class can understand.

Week 0, Day 1

Teaching philosophy

Like our first date: Let's get to know each other

"Now in the further development of science, we want more than just a formula. First we have an observation, then we have numbers that we measure, then we have a law which summarizes all the numbers. But the real *glory* of science is that we can *find a way of thinking* such that the law is *evident*."
— Richard Feynman, The Feynman Lectures on Physics, p26-3

I'm Eric Michelsen: I was an electrical engineer for a few decades

BSEE, PhD Physics UCSD, June 2010, research: Lunar Laser Ranging

All questions are good. There are no dumb questions. Every question asked is good. Every question people learn from is a *great* question. We *never* laugh at questions or incorrect answers. They are tools for learning. Smart people ask questions. Smart people volunteer reasoned answers, even if they might be wrong.

My job is to provide a safe environment for questions and learning, and I take that seriously. Take chances. Make mistakes. It's how we learn. Better to think, risk, and learn, than to do nothing and learn nothing.

Who are you? What majors? What have you taken?

Classical mechanics? Lagrangians? Hamiltonians? E&M? 100C (radiation)?

You must keep me honest! Point out my mistakes. Special recognition for those who point out errors in class, slides, and on exams. Can improve your final grade if borderline.

During our class, please open computers for this class only.

This is only fair to those around you: please don't distract them

No cell phones.

If you *must* take a call (e.g., family matter), sit on the aisles, and step outside when it comes.

Course administrative overview

There should be a text on reserve in the library; they also have an e-copy you can read online

I have corrections and clarifications to the book on the web page.

Late HW: bring it to Casey's office and slide it under the door if he's not there.

I'm very concept oriented: I want you to *understand* what you're doing, not just blindly plug numbers into formulas.

Quizzes: HW and class discussion are your guide to what to study.

The book and Funky Quantum Concepts includes most of what's in class

Grading: roughly equal parts HW, midterm exam, final exam, plus 5-15% class participation

I will adjust exact percentages based on class performance.

Cheating: I *really* don't like it. Will be prosecuted.

When you send me an email, do not assume I remember what we talked about. Tell me the full story in writing, as if you had never talked to me.

You must check the course web page every day before class for changes.

I sometimes forget h-bars, and c.

Learn the Greek alphabet: you're physicists.

Let me know if you want to be on my Funky list.

Please remind me to repeat audience questions, so everyone can hear

Please suggest to me anything else I should be doing

Topics

Prerequisites: Is everyone comfortable with the course pre-requisites?

Scientific notation, computer scientific notation, radians, trigonometry, complex numbers

what is 10e6 in scientific notation? It's 1×10^7.

complex numbers (what do $\exp(-i\omega t)$ and $\exp[i(\mathbf{k}\cdot\mathbf{x} - \omega t)]$ mean?)

Phasors

Differential equations, wave-equation, traveling waves

Fourier Transforms

Classical mechanics, classical E&M

Statistics and PDFs.

130A: basic wave-mechanics: Conditions on ψ?

square integrable (implies $\rightarrow 0$ at ∞), cont., cont. derivatives

Particle in a box: Why is its derivative not continuous?

Linear algebra

Please tell me what you want to learn. Perhaps we can accommodate it.

Week 1, Day 1

Take chances; make mistakes.

Review: wave-function has physical restrictions

> "We are all agreed that your theory is crazy. The question that divides us is whether it is crazy enough to have a chance of being correct." − *Niels Bohr*

We will mostly follow the book, but often with a different viewpoint, plus phasors. You will need to read the book, as I won't cover in class all you need to know. Please read ahead, and start the homework while we are still discussing the topics in class.

Units: mostly cgs (aka gaussian), but lots of SI, as well.

How much is a joule? Throw a dime against a wall as hard as you can: that's about 1 joule.

Public web sites can be useful, but are unreliable for physics: First sentence and fundamental principle of Wikipedia "Lagrangian Mechanics" is dangerously wrong. "Schrodinger Equation" also has some errors.

Pre-req: Gauge invariance? Magnetic vector potential?

This is Chapter 10 in book, but book uses Dirac notation

Similarities between classical electromagnetic radiation and quantum mechanics:

Schrödinger equation is a *classical* wave equation.

Axioms of QM: motivate and derive momentum operator

osc. in space, osc. in time, linear superpositions, consistency, mag^2 (real-ampl)^2 probability

Be careful to distinguish real and complex amplitudes:

E-field of light is like a wave fn: $I \propto E^2 \;\rightarrow\; Pr \propto \psi^2$

Analogy of E and p of photons: same derivatives.

Photons: both simple and complicated: simple: massless; complicated: relativistic, vector field

Phasors and sinusoids: see Funky Electromagnetic Concepts (FEMC)

sinusoids: mathematical form: $A \cos(-\omega t + \phi)$, with a $-\omega t$ for physics (engineers use $+\omega t$)

Three parameters uniquely determine a sinusoid: amplitude, frequency, and phase.

Real amplitude = |complex amplitude|

Week 1, Day 2

Did 130A do any perturbation theory or scattering? How much Fourier Transforms?

Smart people ask questions.

Review: We're still discussing only *wave mechanics*. We'll get to matrix mechanics in a few weeks.

Going slowly now so we can go faster later, and it will make sense

Axioms of QM: see Funky Quantum Concepts (FQC)

Phasors: rotating stick projects a sinusoid; length of stick = amplitude, starting position = $-\phi$

QM: energy provides ω.

Finish phasors: Mathematically: start with complex # (complex vector), $\times\, e^{-i\omega t}$, and Re{ }.

Demonstrate that sum of two phasors is the phasor of the sum

Implications of axioms: superposition → operators, consistency → collapse, $|\psi|^2$ allows bilinear products.

Time and space frequencies also relativistically true (since true for photons).

$\omega \propto T + V$, but V has arbitrary offset $\Rightarrow \omega$ does too $\Rightarrow \psi(t, x)$ somewhat arbitrary.

Weird? It'll get weirder before we're done

Classically, can only observe Energy *differences*

QM: Every *average* observable comes down to an inner product: $<\psi|o_{op}|\psi>$.

$\Rightarrow$ Can only observe phase *differences*, which accumulate due to ω *differences*

Wave-function can be thought of as a phasor-valued function of space.

Briefly mention: Gauge dependence of ψ, and canonical momentum.

Review stationary (& quasi-stationary) states: *measurable* property statistics constant in time, not $\psi(t, x)$

$\Rightarrow E$ is constant everywhere, so time evolution is just phase evolution

Non-stationary SHO state: how does it behave?

Movie on web: http://www.youtube.com/watch?v=VWxMPjDo3Ak&feature=related

Any Questions about QM? From last quarter?

Continuous operators:

what local operators do: multiply wave-function by local value of operator

Linear operators: what is defn? are x, x^2, $\cos(x)$, etc. linear operators? How about arbitrary $V(x)$?

Local observables, e.g. energy and momentum. Generalizes to discrete spaces.

Derive momentum and energy operators: local momentum and energy

Week 1, Day 3

Review: Phasors: So far we're covering Chapter 10 in book, but book uses Dirac notation

EM propagation uses phasor-valued functions of space for the E-fields.

prove addition of phasors theorem

What do operators do? show local values: $o(x) = o_{op}\psi(x)/\psi(x)$.

Also: sum and composition (aka "product") of linear operators are linear

Recap momentum and energy operators

Units of ψ, units of operators

Chapter 11: Derive Schrödinger equation from axioms: p^2 operator. Define H_{op}. S-E is EOM of ψ.

start momentum representation: Fourier transforms

Implication of consistency axiom: collapse of wave-function

The real uncertainty principle? It's not that I can't know; it's that the particle can't have ...

energy-time uncertainty is totally different

Week 2, Day 1

How is the pace of class? Am I going to slowly? Covering chaps 10, 11, and some of App B.

Review: collapse of wave-function; uncertainty principle.

The act of observation is a nonlinear operation: it chooses an eigenstate from a superposition.

Leads to uncertainty in measurements:

Nature is not mean: a particle can't *have* well-defined position *and* momentum

Correct/clarify normalization of momentum eigenstate: Convention appropriate for QM (unitary transform): $f(x)=\int_{-\infty}^{\infty}\tilde{f}(k)\,e^{ikx}\,dk \qquad \Rightarrow \qquad \tilde{f}(k)=\frac{1}{2\pi}\int_{-\infty}^{\infty}f(x)\,e^{ikx}\,dx$, (unlike the book)

but what matters: $\delta(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{ikx}\,dk \qquad \Rightarrow \qquad p=\frac{1}{\sqrt{2\pi\hbar}}e^{ipx/\hbar}$

NB: There is no separate concept of wave-particle duality: no more axioms

just superposition and collapse of wave-function

wave/particle duality is a heuristic, not new physics

Later we'll look at: Quantum eraser, Wheeler's delayed-choice experiment (requires multi-particle QM)

Why are there 3 quantum numbers? Compare particle-in-box to free-particle: what's the difference?

Meaning of ψ: statistics, pdfs; avgs from pdfs: discrete first, then pdf( )

Correspondence principle: sketch wave-function of step-potential in a box

Continuous and discrete states: position and momentum are **continuous**: (define)

Dirac notation: independent of basis: not specific to x, p, or any other representation

all states are kets, but not all kets are states.

inner products: $<\alpha|\beta>$ is **conjugate bilinear**

Week 2, Day 2

Review: No separate concept of wave-particle duality: contained in axioms and SE.

SHO energy and angular momentum are discrete

particle in step-bottom box: E is fixed: both sides have same E (no "lowest E"): correspondence principle

Boltzmann distribution is independent of system: depends only on heat bath

SHO ground state is completely contrary to classical view (unlike particle in step-bottom box).

Dirac Notation: $|\psi> \Rightarrow \psi(x)$ or $\phi(p)$, $<\psi| \Rightarrow \psi^*(x)$ or $\phi^*(p)$, etc.

Note: 6^{th} axiom: The Schrödinger Equation for cases of $T < 0$, and thus p imaginary.

Meaning of ψ: statistics, pdfs; avgs from pdfs: discrete first, then pdf( )

For 1-particle QM: can imagine the particle is truly spread out according to ψ

The decomposition theorem: $\left|\psi\right\rangle = c_a\left|a\right\rangle + c_b\left|b\right\rangle + ...$ (*orthonormal basis*) $\Rightarrow$ $c_a = \left\langle a|\psi\right\rangle$

$\Rightarrow$ <a|ψ> is the "amount" of |a> in |ψ>, and its phase. Then we can write:

$$\left|\psi\right\rangle = c_1\left|b_1\right\rangle + c_2\left|b_2\right\rangle + ... = \sum_j \left\langle b_j|\psi\right\rangle\left|b_j\right\rangle = \sum_j \left|b_j\right\rangle\left\langle b_j|\psi\right\rangle$$

inner products: <α|β> is amount of |α> in |β>, and its phase. conjugate bilinear: $\langle a|b\rangle = \langle b|a\rangle^*$

bases: analogous to $\mathbf{a} = (a_x, a_y, a_z) = (\mathbf{a}\cdot\mathbf{e}_x, \mathbf{a}\cdot\mathbf{e}_y, \mathbf{a}\cdot\mathbf{e}_z)$, i.e. $a_x = \mathbf{a}\cdot\mathbf{e}_x$, etc.

Week 2, Day 3

HW should be stapled, and no ragged edges.

Review: inner products: <α|β> is amount of |α> in |β>, and its phase. conjugate bilinear: $\langle a|b\rangle = \langle b|a\rangle^*$

$$\left|\psi\right\rangle = c_1\left|b_1\right\rangle + c_2\left|b_2\right\rangle + ... = \sum_j \left\langle b_j|\psi\right\rangle\left|b_j\right\rangle = \sum_j \left|b_j\right\rangle\left\langle b_j|\psi\right\rangle \quad \text{(show both orders of writing)}$$

bases: analogous to $\mathbf{a} = (a_x, a_y, a_z) = (\mathbf{a}\cdot\mathbf{e}_x, \mathbf{a}\cdot\mathbf{e}_y, \mathbf{a}\cdot\mathbf{e}_z)$, i.e. $a_x = \mathbf{a}\cdot\mathbf{e}_x$, etc.

Continue Dirac Notation:

Variable kets, e.g. $|x\rangle$. Normalization: $\langle x|x'\rangle = \delta(x' - x)$, $\langle p|p'\rangle = \delta(p' - p)$.

Units of δ( ). $\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx}\, dk, \qquad \psi(x) = \left\langle x|\psi\right\rangle, \qquad \phi(p) = \left\langle p|\psi\right\rangle$

Just because we didn't bother to show it's rigorous, doesn't mean it's not

$\mathbf{0}_v$ (zero vector) = null vector = null ket, but is not |0>,

Momentum representation: The decomposition theorem in continuous space:

Fourier decomposition: "amount" of |p> in |ψ> is:

$$\left\langle p|\psi\right\rangle = \frac{1}{\sqrt{2\pi\hbar}}\int_\infty dx\left(e^{ipx/\hbar}\right)^* \psi(x) = \frac{1}{\sqrt{2\pi\hbar}}\int_\infty dx\, e^{-ipx/\hbar}\psi(x)$$

Week 3, Day 1

Review: A while ago, I noted that I can write the position wave-function $\psi(x)$ as a sum of momentum eigenstates. We recognized that as a Fourier Transform, and I appealed to that prior knowledge. Now, I've shown by the decomposition theorem that the formula for $\phi(p)$ is as given.

$Y_{lm}(\theta, \phi)$ are dimensionless since Y^2 is density per steradian. Everyone know what a steradian is?

Variable kets, bras, δ-fn, $\mathbf{0}_v$ = null = |null>

continuous **eigenstate** is one where local value is constant everywhere: $E_{op}\, u_n(x) = E_n\, u_n(x)$.

Operators: continuous and discrete: SHO energy and orbital ang. momentum L are **discrete** states

Recursion relations and Hermite polynomials: Chapter 13. Basis of all QFT.

Simple Harmonic Oscillator (SHO): ladder operators: crucial step for all of QM and QFT.

The position basis interpretation of a and $a^\dagger$. All(?) books give lo-and-behold, and Dirac algebra.

See FQC for ladder operator derivation from $H_n(x)$ recursion relations.

Week 3, Day 2

We're in Chapter 13.

Book uses Arial font (non serif) for operators.

Review: Recursion relations lead to ladder operators.

That $a^+ = a^\dagger$ is shown better in FQC directly from $<m|a^\dagger|n> = <m|a^+|n>$ (we drop a^+ from here on)

What are the units of a and $a^\dagger$?

For HW: Completeness operator: operator on vector = sum of operator on components:

$$\left|\psi\right\rangle = \sum_j \left|b_j\right\rangle\left\langle b_j\middle|\psi\right\rangle \qquad \Rightarrow \qquad \sum_j \left|b_j\right\rangle\left\langle b_j\right| = \mathbf{1}_{op} \qquad aka \qquad \left(\sum_j \left|b_j\right\rangle\left\langle b_j\right|\right)\left|\psi\right\rangle = \left|\psi\right\rangle$$

Then: $$\mathcal{O}\left|\psi\right\rangle = \sum_j \mathcal{O}\left|b_j\right\rangle\left\langle b_j\middle|\psi\right\rangle. \qquad For\ SHO: \qquad \left\langle\psi\right|\hat{n}\left|\psi\right\rangle = \sum_{n=0}^{\infty}\left\langle\psi\right|\hat{n}\left|n\right\rangle\left\langle n\middle|\psi\right\rangle$$

May be used for computing inner products, especially average values.

Insert *after bra* or *before ket*.

SHO: Length scale: $\alpha \equiv \sqrt{\dfrac{\hbar}{m\omega}}$ $\qquad z \equiv \dfrac{x}{\alpha}$ (from Dr. Abarbanel)

Example of ladder operator use: (1) quantized energy:

Time Independent Schrodinger Equation (for stationary states): $\hat{H}\left|\psi\right\rangle = E\left|\psi\right\rangle$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 \qquad \hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(a^\dagger + a\right) \qquad \hat{p} = i\sqrt{\frac{m\hbar\omega}{2}}\left(a^\dagger - a\right) \qquad [7.43\ p156]$$

$$\hat{H} = -\frac{m\hbar\omega}{4m}\left(a^\dagger - a\right)^2 + \frac{m\omega^2\hbar}{4m\omega}\left(a^\dagger + a\right)^2 = \frac{\hbar\omega}{4}\left(a^{\dagger 2} + a^\dagger a + aa^\dagger + a^2 - a^{\dagger 2} + a^\dagger a + aa^\dagger - a^2\right)$$

$$= \frac{\hbar\omega}{2}\left(a^\dagger a + aa^\dagger\right) = \frac{\hbar\omega}{2}\left(a^\dagger a + a^\dagger a + 1\right) = \hbar\omega\left(\hat{n} + \frac{1}{2}\right) \quad \Rightarrow \qquad \hat{H}\left|n\right\rangle = \hbar\omega\left(n + \frac{1}{2}\right)\left|n\right\rangle$$

$$\Rightarrow \qquad E_n = n + \frac{1}{2}$$

(2) $<x^2>$ in SHO states. By symmetry $<x> = 0$ in all SHO stationary states.

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(a^\dagger + a\right) \qquad \Rightarrow \qquad \hat{x}^2 = \frac{\hbar}{2m\omega}\left(a^\dagger + a\right)^2$$

$$\langle n|\hat{x}^2|n\rangle = \frac{\hbar}{2m\omega}\langle n|\left(a^\dagger + a\right)^2|n\rangle = \frac{\hbar}{2m\omega}\langle n|\left(a^\dagger a^\dagger + a^\dagger a + aa^\dagger + aa\right)|n\rangle$$

$For\ |0\rangle: only\ aa^\dagger\ survives$

$For\ |all\ others\rangle:\ a^\dagger a + aa^\dagger\ survive$

Week 3, Day 3

Please staple, be neat, no ragged edges.

Review: We're in chap 13. <a| o |b> ≡ **matrix element**

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(a^\dagger + a\right) \qquad (similarly) \qquad \hat{p} = i\sqrt{\frac{m\hbar\omega}{2}}\left(a^\dagger - a\right) \qquad [7.43\ p156]$$

$$\Rightarrow \qquad any\ f(x) = polynomial(a, a^\dagger)$$

$\langle m|\left(a^\dagger a^\dagger + a^\dagger a + aa^\dagger + aa\right)|n\rangle$ *has many terms that cancel*

$For\ |0\rangle: only\ aa^\dagger\ survives,$ $\qquad For\ |all\ others\rangle:\ a^\dagger a + aa^\dagger\ survive$

Discrete operator: associates value with each component of ket, & each pair of components (interaction).

Eigenvectors: e.g., SHO E|n>, or

(Chapter 12) ang. mom. $L^2|l, m>$, or $L_z|l, m>$

L (like **p**) is a vector operator ≡ $L_x\mathbf{e}_x + L_y\mathbf{e}_y + L_z\mathbf{e}_z$ (a collection of 3 related numerical operators)

definition of **scalar**: a number which is the same in any coordinate system.

Orbital angular momentum: $2l + 1$ dimensional space. Not just discrete, but finite. $\Phi(\phi) = \exp(im\phi)$

Vector spaces, bases: Definition of vector space: commutative group of vectors with +,

"field" scalars with + and ×, distributive properties: (**u** + **v**), and (a + b)**u**

Hilbert spaces: VecSpa + inner prod & ops. distinction between physicist's and mathematician's

dimensionality of the space: finite, countably infinite, uncountably infinite

Why spherical coordinates are *not* components of a vector (they *are* parameters).

Generators: will lead to angular momentum ladder operators, and ultimately spin

linear momentum is generator of translations: shift to the right (forward): Is it linear?

$$\psi(x) \to \psi(x - dx) = \psi - \frac{\partial\psi}{\partial x}dx = \left(1 - \frac{i}{\hbar}\hat{p}\,dx\right)\psi \qquad \Rightarrow \qquad \hat{t}(dx) \equiv 1 - \frac{i}{\hbar}\hat{p}\,dx$$

$$\hat{T}(a) = \lim_{N\to\infty}\hat{t}(a/N)^N = \lim_{N\to\infty}\left(1 - \frac{i}{\hbar}\hat{p}\frac{a}{N}\right)^N = \exp\left(-\frac{i}{\hbar}\hat{p}a\right) \qquad \to \qquad \exp\left(-\frac{i}{\hbar}\hat{\mathbf{p}}\bullet\mathbf{a}\right)$$

Week 4, Day 1

Moving into chapter 12: Rotations

Review: Vector spaces: basis vectors, decomposition theorem.

Can solve linear eqs. in vectors: even Kramer's rule. Will use this in perturbation theory.

Hilbert spaces: VecSpa + inner prod & ops. distinction between physicist's and mathematician's

dimensionality of the space: finite, countably infinite, uncountably infinite

Why spherical coordinates are *not* components of a vector (they *are* parameters).

Orbital angular momentum: $2l + 1$ dimensional space. Not just discrete, but finite.

For spherically symmetric potentials, $V(r)$:

$$\psi_{nlm}(r,\theta,\phi) = R_{nl}(r)P_{lm}\left(\cos\theta\right)e^{im\phi}, \qquad \textit{only } R_{nl}(r) \textit{ depends on } V(r).$$

$$\psi(x) \to \psi(x-dx) = \psi - \frac{\partial\psi}{\partial x}dx = \left(1 - \frac{i}{\hbar}\hat{p}\,dx\right)\psi \qquad \Rightarrow \qquad \hat{t}(dx) \equiv 1 - \frac{i}{\hbar}\hat{p}\,dx$$

Not time evolution: relation between momentum and space

Finite displacements. Then: nothing special about x direction, so in 3D:

$$\hat{T}(a) = \lim_{N\to\infty}\hat{t}(a/N)^N = \lim_{N\to\infty}\left(1 - \frac{i}{\hbar}\hat{p}\frac{a}{N}\right)^N = \exp\left(-\frac{i}{\hbar}\hat{p}a\right) \qquad \to \qquad \exp\left(-\frac{i}{\hbar}\hat{\mathbf{p}}\bullet\mathbf{a}\right)$$

angular momentum is generator of rotations: show it in z-direction.

Finite angles. Nothing special about z-direction.

$$\textit{rot about } z: \qquad L_z = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$$

$$\psi_{nlm}(r,\theta,\phi) \to \psi_{nlm}(r,\theta,\phi - d\phi) = \psi - \frac{\partial\psi}{\partial\phi}d\phi = \left(1 - \frac{i}{\hbar}\hat{L}_z\,d\phi\right)\psi \qquad \Rightarrow \hat{r}_z(d\phi) \equiv 1 - \frac{i}{\hbar}\hat{L}_z\,d\phi$$

$$\hat{R}_z(\alpha) = \lim_{N\to\infty}\hat{r}(\alpha/N)^N = \lim_{N\to\infty}\left(1 - \frac{i}{\hbar}\hat{L}_z\frac{\alpha}{N}\right)^N = \exp\left(-\frac{i}{\hbar}\hat{L}_z\alpha\right) \qquad \to \hat{R}(\boldsymbol{\alpha}) = \exp\left(-\frac{i}{\hbar}\hat{\mathbf{L}}\bullet\boldsymbol{\alpha}\right)$$

Be careful with angular "vectors": $\boldsymbol{\alpha}$ is *not* a vector; it's 3 numbers: Rotates about $\hat{\mathbf{n}} \equiv \dfrac{\boldsymbol{\alpha}}{|\boldsymbol{\alpha}|}$.

Doesn't work as composition of 3 separate angular rotations.

Week 4, Day 2

Did HW partial solutions help? .

Review: We're in sec 12.3. We note the classical rotation operators (linear). You'll do similar in HW.

Show how going around the loop is related to the commutator.

Derive classical $R_z(\alpha)$

Angular momentum, both orbital and spin, are crucial to spectroscopy, which enables a huge amount of information about molecular structure, biochemistry, etc. NMR. Also, spintronics may someday be useful.

Classically: $\quad To\ O\left(\varepsilon^2\right): \qquad \mathbf{R}_x(-\varepsilon)\mathbf{R}_y(-\varepsilon)\mathbf{R}_x(\varepsilon)\mathbf{R}_y(\varepsilon) = \begin{pmatrix} 1 & -\varepsilon^2 & 0 \\ \varepsilon^2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \mathbf{R}_z\left(\varepsilon^2\right) + 1$

Quantum rotation operators must have the same rotation & thus commutation properties.

From commutation relations of rotations, we can infer the commutation relations of generators:

Assume some hermitian generator of rotations, and expand quantum rotations in it.

We could assume the commutation relations of angular momentum from orbital L, but using rotations is deeper (connects to group theory). (Eq. 12.48 p159 is really confusing.)

$$To\ O\left(\varepsilon^2\right): \qquad R_x(\varepsilon) = \mathbf{1} - i\varepsilon G_x - \frac{\varepsilon^2}{2}G_x^2, \qquad R_y(\varepsilon) = \mathbf{1} - i\varepsilon G_y - \frac{\varepsilon^2}{2}G_y^2$$

$$R_x(-\varepsilon)R_y(-\varepsilon)R_x(\varepsilon)R_y(\varepsilon) = [81\ terms]$$

$$= \left(\mathbf{1} + i\varepsilon G_x - \frac{\varepsilon^2}{2}G_x^2\right)\left(\mathbf{1} + i\varepsilon G_y - \frac{\varepsilon^2}{2}G_y^2\right)\left(\mathbf{1} - i\varepsilon G_x - \frac{\varepsilon^2}{2}G_x^2\right)\left(\mathbf{1} - i\varepsilon G_y - \frac{\varepsilon^2}{2}G_y^2\right)$$

$$\approx \mathbf{1} + \varepsilon^2\left(G_x^2 + G_y^2 - \frac{1}{2}G_x^2 \cdot 2 - \frac{1}{2}G_y^2 \cdot 2 - G_xG_y + G_yG_x\right) = \mathbf{1} - \left(G_xG_y - G_yG_x\right)\varepsilon^2$$

$$which\ must = R_z\left(\varepsilon^2\right) = \mathbf{1} - i\varepsilon^2 G_z. \quad \Rightarrow \qquad \left[G_x, G_y\right] = iG_z \quad (no\ \hbar\ in\ book's\ notation)$$

Everything there is to know about angular momentum is in these commutation relations.

Week 4, Day 3

Did the Dirac algebra homework solutions help? We're going to do Dirac algebra again, today.

Review: How to avoid my mistakes from Wednesday

The book does the rotations in the right order in Sec 12.3.

Notation terrible in 12.1 ("spin" instead of "angular momentum") & 12.2 ($|m,\alpha>$ instead of $|\alpha, m>$)

Read section "Rotating a Spin", p110. But now, 'j' isn't just spin, it's *any angular momentum*:

$$\left[j_z, j^2\right] = \left[j_z, j_x^2\right] + \left[j_z, j_y^2\right] + \cancel{\left[j_z, j_z^2\right]} = [j_z, j_x]j_x + j_x[j_z, j_x] + \left[j_z, j_y\right]j_y + j_y\left[j_z, j_y\right]$$

$$= i\hbar\left(j_y j_x + j_x j_y - j_x j_y - j_y j_x\right) = 0$$

$$Consider: \qquad [j_z, j_x] = i\hbar j_y$$

$$\left[j_z, j_y\right] = -i\hbar j_x \quad \text{if I stuck an '}i\text{' in there, these would be symmetric:}$$

$$[j_z, j_x] = i\hbar j_y$$

$$\left[j_z, ij_y\right] = \hbar j_x \qquad \Rightarrow \qquad \left[j_z, j_x + ij_y\right] = \hbar\left(j_x + ij_y\right). \qquad Define:\ j_+ \equiv j_x + ij_y \quad \left(\text{cf } a\ and\ a^\dagger\right)$$

$$[j_z, j_+] = \hbar j_+ \qquad \Rightarrow \qquad j_z j_+ = j_+ j_z + \hbar j_+ \qquad Suggests\ acting\ on\ j_z\ eigenstate:$$

$$j_z j_+ |j\ m\rangle = (j_+ j_z + \hbar j_+)|j\ m\rangle = (j_+ m\hbar + \hbar j_+)|j\ m\rangle = (m+1)\hbar j_+|j\ m\rangle \quad \Rightarrow \qquad j_+|j\ m\rangle \propto |j, m+1\rangle$$

$$j_+ \equiv j_x + ij_y, \qquad j_- \equiv j_x - ij_y \qquad \Rightarrow \qquad j_- = (j_+)^\dagger \qquad j_-|j\ m\rangle \propto |j, m-1\rangle$$

Week 5, Day 1

Already had trouble with 's' vs. 'j' notation: have to change to j.

Review: 'j' is any angular momentum: orbital, spin, sum of either or both

$$\left[j_z, j^2\right] = 0$$

$$\textit{Consider}: \qquad \left[j_z, j_x\right] = i\hbar j_y$$

$$\left[j_z, j_y\right] = -i\hbar j_x \quad \text{if I stuck an '}i\text{' in there, these would be symmetric:}$$

$$\left[j_z, j_x\right] = i\hbar j_y$$

$$\left[j_z, ij_y\right] = \hbar j_x \qquad \Rightarrow \qquad \left[j_z, j_x + ij_y\right] = \hbar\left(j_x + ij_y\right). \qquad \textit{Define}: \ j_+ \equiv j_x + ij_y \ \left(\text{cf } a \text{ and } a^\dagger\right)$$

$$\left[j_z, j_+\right] = \hbar j_+ \qquad \Rightarrow \qquad j_z j_+ = j_+ j_z + \hbar j_+ \qquad \textit{Suggests acting on } j_z \textit{ eigenstate}:$$

$$j_z j_+ \left| j\ m \right\rangle = \left(j_+ j_z + \hbar j_+\right)\left| j\ m \right\rangle = \left(j_+ m\hbar + \hbar j_+\right)\left| j\ m \right\rangle = (m+1)\hbar j_+ \left| j\ m \right\rangle \quad \Rightarrow \quad j_+ \left| j\ m \right\rangle \propto \left| j, m+1 \right\rangle$$

$$j_+ \equiv j_x + ij_y, \qquad j_- \equiv j_x - ij_y \qquad \Rightarrow \qquad j_- = \left(j_+\right)^\dagger \qquad\qquad j_- \left| j\ m \right\rangle \propto \left| j, m-1 \right\rangle$$

NB: we don't know the eigenvalues of j^2, we anticipate by calling them λ, which can be written as $j(j + 1)$.

With SHO, we found that a and $a^\dagger$ had an interesting property in $<\psi|a^\dagger a|\psi>$, proving e-val of $n = a^\dagger a \ge 0$.

$$\left\langle j\ m \right| j_- j_+ \left| j\ m \right\rangle \ge 0. \qquad \text{Use:} \quad j_- j_+ = j^2 - j_z^{\,2} - \hbar j_z$$

$$\Rightarrow \quad j(j+1) - m^2 - m = j(j+1) - m(m+1) \ge 0 \qquad \textit{or} \quad \lambda - m(m+1) \ge 0$$

Therefore, ladder must terminate for *both* pos and neg m. By symmetry, we expect that $m_{largest} = -m_{smallest}$.

But this symmetry is built-in to our $R_k(\alpha)$ operator assumptions (i.e. x, y, z, interchangeable).

Imagine that we don't know e-val of j^2, call it λ.

From the above, λ must $\to j(j + 1)$, (secret: j is some half integer)

Integer number of steps 'tween $m = -j$ and $m = +j$. What does that tell us about j?

All these results are based on the j-hats: hermitian, and satisfy the commutation relations

Spin ½: Stern-Gerlach: the photos. What do they mean?

2D Hilbert space (or state space). Need linear operators. What are they? 2x2 matrices.

$\to$ matrix mechanics

Spin operators must satisfy commutation relations. But which matrices?

Doesn't matter: but there's a standard: **Pauli matrices**. Universal, even in QFT.

Week 5, Day 2

Administrative: Midterm Wednesday, 11/2/2011

Review: Applications: spintronics, NMR. Sec 2.2, 5.4.

rotation generator commutation relations $\to$ ladder operators, quantization rules:

j is ½ integer, $m = j, \ldots -j$

We *chose* to write eigenvalues in terms of $\hbar$: $j(j+1)\hbar^2$, and $m\hbar$.

squared norm: $\left| j_+ \left| j\ m \right\rangle \right|^2 = \left\langle j\ m \right| j_- j_+ \left| j\ m \right\rangle = \hbar^2 \left[j(j+1) - m(m+1) \right]$

we speculate rotation generator operators to be angular momentum operators. True for orbital L.

Just like in classical mechanics.

Stern-Gerlach specializes us to *spin*

standard: **Pauli matrices**. Universal, even in QFT. Satisfy commutation relations.

Coefficients of raising and lowering operators: can choose them real, which then *defines* the Pauli matrices

$\left\langle j\ m \right| j_- j_+ \left| j\ m \right\rangle \quad \Rightarrow \qquad j_+ \left| j\ m \right\rangle = \hbar \sqrt{j(j+1) - m(m+1)}$ (within a phase factor)

Matrices for raising and lowering operators, $\Rightarrow$ matrices for s_x and s_y

Pauli matrices satisfy comm. relations, but need units, and nice eigenvalues. Propagate L-hat results:

s-hat$_k \equiv (\hbar/2)\sigma_k$. What are eigenvalues of σ_k?

Spin vectors. Degrees of freedom, compare to Euler angles, $\langle s_x \rangle = 0$ and $\langle s_y \rangle = 0$.

Week 5, Day 3

Review:

$\left\| \left| \psi \right\rangle \right\|^2 = \left\langle \psi | \psi \right\rangle \ or \ \left(\left| \psi \right\rangle \right)^\dagger \left| \psi \right\rangle \qquad \Rightarrow \qquad \left| \hat{o} \left| \psi \right\rangle \right|^2 = \left(\hat{o} \left| \psi \right\rangle \right)^\dagger \hat{o} \left| \psi \right\rangle = \left\langle \psi \right| \hat{o}^\dagger \hat{o} \left| \psi \right\rangle \ge 0$

$\left| j_+ \left| j\ m \right\rangle \right|^2 = \left\langle j\ m \right| j_+^\dagger j_+ \left| j\ m \right\rangle = \left\langle j\ m \right| j_- j_+ \left| j\ m \right\rangle \qquad \Rightarrow \qquad j_+ \left| j\ m \right\rangle = \hbar \sqrt{j(j+1) - m(m+1)}$

Note $\hbar$ in raising/lowering coefficients.

Write $\boldsymbol{\sigma}$. s-hat$_k \equiv (\hbar/2)\sigma_k$. (hermitian) What are eigenvalues of σ_k?

Spin vectors. Degrees of freedom, compare to Euler angles, $\langle s_x \rangle = 0$ and $\langle s_y \rangle = 0$.

Some magic properties of the Pauli matrices (more to come):

$$evals = \pm 1, \qquad \det = -1, \qquad \mathrm{Tr} = 0, \qquad \left[\sigma_x, \sigma_y \right] = 2i\sigma_z, \qquad \sigma_k{}^2 = \mathbf{1}_2$$

PDF of spin-1/2 ang. mom. eigenstate in ang. mom. space: halo (uniform circle at $z = \pm \hbar/2$).

Spin vectors. Eigenvectors of $s(\theta, \phi)$. Examples of z, x, and y.

$$\hat{s}(\theta, \phi) = \frac{\hbar}{2} \left[\sin\theta \left(\cos\phi \hat{s}_x + \sin\phi \hat{s}_y \right) + \cos\theta \hat{s}_z \right]$$

$$\sim \sin\theta\cos\phi \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \sin\theta\sin\phi \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \cos\theta \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{+i\phi} & -\cos\theta \end{pmatrix}$$

Eigenvector, e-val = 1, set $a = 1$:

$$\hat{s}(\theta,\phi)\begin{pmatrix} a \\ b \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} a \\ b \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{+i\phi} & -\cos\theta \end{pmatrix} \qquad \rightarrow \qquad \begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{+i\phi} & -\cos\theta \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\rightarrow \qquad \cos\theta + b\sin\theta e^{-i\phi} = 1, \qquad b = \frac{1-\cos\theta}{\sin\theta}e^{+i\phi} = \frac{2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2}e^{+i\phi} = \frac{\sin\theta/2}{\cos\theta/2}e^{+i\phi}$$

$$\textit{Normalize}: \qquad \left|s(\theta,\phi)+\right\rangle = \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2e^{+i\phi} \end{pmatrix}$$

Now do it by rotating |z+> (R operators unitary, det = 1, but need not be hermitian):

$$\left|s(\theta,\phi)+\right\rangle = \hat{R}_z(\phi)\hat{R}_y(\theta)\left|z+\right\rangle.$$

$$\hat{R}_y(\theta) = \exp\left(-\frac{i}{\hbar}\theta\hat{s}_y\right) = \exp\left(-i\frac{\theta}{2}\sigma_y\right) = 1 - i\frac{\theta}{2}\sigma_y - \frac{\left(\frac{\theta}{2}\sigma_y\right)^2}{2}...$$

$$= \cos\left(\frac{\theta}{2}\sigma_y\right) - i\sin\left(\frac{\theta}{2}\sigma_y\right) = \cos\theta/2\mathbf{1}_2 - i\sigma_y\sin\theta/2 = \begin{pmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{pmatrix}$$

Week 6, Day 1

Review:

Correction: $\left[\sigma_x,\sigma_y\right] = 2i\sigma_z$: $\quad$ *evals* $= \pm 1$, $\quad$ det $= -1$, $\quad$ Tr $= 0$, $\quad {\sigma_k}^2 = \mathbf{1}_2$

$$\hat{s}(\theta,\phi) = \frac{\hbar}{2}\left[\sin\theta\left(\cos\phi\hat{s}_x + \sin\phi\hat{s}_y\right) + \cos\theta\hat{s}_z\right] = \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{+i\phi} & -\cos\theta \end{pmatrix}$$

Solving e-vec equation: $\hat{s}(\theta,\phi)\begin{pmatrix} a \\ b \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} a \\ b \end{pmatrix} \qquad \rightarrow \qquad \begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{+i\phi} & -\cos\theta \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$

$$\rightarrow \qquad \cos\theta + b\sin\theta e^{-i\phi} = 1, \qquad b = \frac{\sin\theta/2}{\cos\theta/2}e^{+i\phi} \quad \textit{Normalize}: \qquad \left|s(\theta,\phi)+\right\rangle = \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2e^{+i\phi} \end{pmatrix}$$

Alternate method: $\left|s(\theta,\phi)+\right\rangle = \hat{R}_z(\phi)\hat{R}_y(\theta)\left|z+\right\rangle. \qquad \hat{R}_y(\theta) = \exp\left(-\frac{i}{\hbar}\theta\hat{s}_y\right) = \begin{pmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{pmatrix}$

Continuing rotation matrices: Note they are unitary.

$$\hat{R}_z(\phi) = \exp\left(-\frac{i}{\hbar}\phi\hat{s}_z\right) = \exp\left(-i\frac{\phi}{2}\sigma_z\right) = 1 - i\frac{\phi}{2}\sigma_z - \frac{\left(\frac{\phi}{2}\sigma_z\right)^2}{2}...$$

$$= \cos\left(\frac{\phi}{2}\sigma_z\right) - i\sin\left(\frac{\phi}{2}\sigma_z\right) = \cos(\phi/2)\mathbf{1}_2 - i\sigma_z\sin(\phi/2)$$

$$= \begin{pmatrix} \cos(\phi/2) - i\sin(\phi/2) & 0 \\ 0 & \cos(\phi/2) + i\sin(\phi/2) \end{pmatrix} = \begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{+i\phi/2} \end{pmatrix}$$

Note that σ_z is *diagonal*, which means f(σ_z) = diag[f($\sigma_{z,11)}$, f(σ_z,22)], which would have saved us time.

Then: $\left|s(\theta,\phi)+\right\rangle = \hat{R}_z(\phi)\hat{R}_y(\theta)\begin{pmatrix}1\\0\end{pmatrix} = \hat{R}_z(\phi)\begin{pmatrix}\cos(\theta/2)\\ \sin(\theta/2)\end{pmatrix} = \begin{pmatrix}\cos(\theta/2)e^{-i\phi/2}\\ \sin(\theta/2)e^{+i\phi/2}\end{pmatrix} \to \begin{pmatrix}\cos(\theta/2)\\ \sin(\theta/2)e^{+i\phi}\end{pmatrix}$

Quantify magnetic dipole moment:

Stern-Gerlach says g-factor: ≈ 2 ≈ 2.0023 (Distinct from SM g = 2 exactly.)

Week 6, Day 2

Fun day: Midterm

Week 6, Day 3

Went over midterm exam Q1, and most of Q2.

Week 7, Day 1

Went over midterm exam: finished Q2, Q3, and part 1 of Q4.

Week 7, Day 2

Went over midterm exam: finish Q4.

Be careful to distinguish between rotations in real space, and rotations in spinor space, or any other angular momentum space. Recall:

$$\hat{R}_y(\theta) = \exp\left(-\frac{i}{\hbar}\theta\hat{s}_y\right) = \begin{pmatrix}\cos\theta/2 & -\sin\theta/2\\ \sin\theta/2 & \cos\theta/2\end{pmatrix}, \qquad \hat{R}_z(\phi) = \exp\left(-\frac{i}{\hbar}\phi\hat{s}_z\right) = \begin{pmatrix}e^{-i\phi/2} & 0\\ 0 & e^{+i\phi/2}\end{pmatrix}$$

Review: Quantify Stern-Gerlach: oops! Off by factor of 2.

Eigenvectors: |x+>, |x->,

Physical meaning of the matrix elements of σ_z consider the average in a state:

$$\langle\chi|\sigma_z|\chi\rangle = \begin{bmatrix}a^* & b^*\end{bmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix} \quad \to$$

	$a\|z+\rangle$	$b\|z-\rangle$
$a^*\langle z+\|$	1	0
$b^*\langle z-\|$	0	−1

Week 8, Day 1

Review: Note that $\left\|\psi\rangle\right\|^2 = \langle\psi|\psi\rangle$, *but* $\Pr\left(\textit{measuring } |\alpha\rangle\right) = \left|\langle\alpha|\psi\rangle\right|^2$

An inner product can be the square of a magnitude, but is the square root of a probability.

Section 2.2: Quantify Stern-Gerlach: oops! Off by factor of 2.

We are heading to quantum information and quantum encryption: Eigenvectors: |x+>, |x->

Aside: More properties of σ-matrices: (1) $\sigma_k^2 = \mathbf{1}_2$: We cannot derive this from the commutation relations.

How do I know this? 'Cuz For $j = 1$, we satisfy the commutation relations, but not $(L_k/\hbar)^2 = \mathbf{1}_3$.

Implies that every 2-spinor is an eigenstate of s_x^2, s_y^2, and s_z^2 with eigenvalue $\hbar^2/4$.

(2) They anti-commute: $\{\sigma_i, \sigma_j\} = 0, \qquad i \ne j$

leads to the Pauli Hamiltonian, which predicts g = 2 exactly.

Physical meaning of the matrix elements of σ_z: Consider the average in a state $|\chi\rangle$:

$$\langle\chi|\sigma_z|\chi\rangle = \begin{bmatrix} a^* & b^* \end{bmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} \quad \rightarrow \quad \begin{array}{c|cc} & a|z+\rangle & b|z-\rangle \\ \hline a^*\langle z+| & 1 & 0 \\ b^*\langle z-| & 0 & -1 \end{array}$$

diagonal elements give average due to one component; there is no interaction

Physical meaning of the matrix elements of σ_x: Consider the average in a state $|\chi\rangle$:

$$\langle\chi|\sigma_x|\chi\rangle = \begin{bmatrix} a^* & b^* \end{bmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} \quad \rightarrow \quad \begin{array}{c|cc} & a|z+\rangle & b|z-\rangle \\ \hline a^*\langle z+| & 0 & 1 \\ b^*\langle z-| & 1 & 0 \end{array}$$

diagonal elements give average due to one component: e.g., $|z+\rangle$ alone contributes nothing.

Off-diagonal give average due to *interactions*: $|z+\rangle$ with same sign as $|z-\rangle$ contributes + to $\langle s_x\rangle$.

s_x is hermitian, so we know its eigenvalues (and therefore average) are real.

Physical meaning of the matrix elements of σ_y. Note that $|y+\rangle = \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$ *and* $|y-\rangle = \begin{pmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{pmatrix}$.

We know imaginary results cancel, so focus on real:

$$\langle\chi|\sigma_y|\chi\rangle = \begin{bmatrix} a^* & b^* \end{bmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} \quad \rightarrow \quad \begin{array}{c|cc} & a|z+\rangle & b|z-\rangle \\ \hline a^*\langle z+| & 0 & -i \\ b^*\langle z-| & i & 0 \end{array}$$

Mixes Re(a) with Im(b), and Im(a) with Re(b). If same sign, contributes + to $\langle s_y\rangle$.

Hermitian matrix splits the work evenly between $a_j^* b_k$ and $b_j^* a_k$. Only real parts survive, 'cuz the imaginary pieces of $a^* b\sigma_{12}$ and $b^* a\sigma_{21}$ cancel.

Basis changes: Suppose we want to write $|\chi\rangle$ in the x-basis instead of z (similar to position and momentum bases):

$$|\chi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}_z = \begin{pmatrix} c \\ d \end{pmatrix}_x \qquad \Rightarrow \qquad |\chi\rangle = a|z+\rangle + b|z-\rangle = c|x+\rangle + d|x-\rangle$$

Like writing a vector **a** in x-y or rotated x'-y' basis.

Then

$$c = \langle x+|\chi\rangle, \qquad d = \langle x-|\chi\rangle \qquad or \qquad \begin{pmatrix} c \\ d \end{pmatrix}_x = \underbrace{\begin{pmatrix} (x+bra\rightarrow)_z \\ (x-bra\rightarrow)_z \end{pmatrix}}_{2\times 2 \text{ matrix}}\begin{pmatrix} a \\ b \end{pmatrix}_z = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}_z$$

But just as well: $|\chi\rangle = \begin{pmatrix} c \\ d \end{pmatrix}_x = \underbrace{\begin{pmatrix} \begin{pmatrix} z+ \\ \downarrow \end{pmatrix}_x & \begin{pmatrix} z- \\ \downarrow \end{pmatrix}_x \end{pmatrix}}_{2\times 2 \text{ matrix}} \begin{pmatrix} a \\ b \end{pmatrix}$

Week 8, Day 2

Review: We considered an average value $<\chi|s_j|\chi>$. More generally, a (conjugate) bilinear inner product <a|o|c> of an operator o is a sum of terms. To be conjugate bilinear, each term must include exactly 1 factor from <a| and 1 factor from |b>; there are n^2 such terms. Each term may also be weighted by a constant, given by the operator o, which has n^2 elements. The inner product is then given by:

$$\langle a|o|c\rangle = \sum_{i,j=1}^{n} a_i^* c_j o_{ij} \qquad E.g., \langle a|o|c\rangle = a_1^* c_1 o_{11} + a_1^* c_2 o_{12} + a_2^* c_1 o_{21} + a_2^* c_2 o_{22}$$

$$or, e.g. \quad \langle a|\sigma_x|c\rangle = \begin{bmatrix} a_1^* & a_2^* \end{bmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \quad \rightarrow \quad \begin{array}{c|cc} & c_1|z+\rangle & c_2|z-\rangle \\ \hline a_1^*\langle z+| & 0 & 1 \\ a_2^*\langle z-| & 1 & 0 \end{array}$$

The diagonal elements of an operator matrix are the weights given to the components of the vector by themselves. The off-diagonal elements are the weights given to *interactions* between the components.

Basis changes: from $a_j|b_j>$ to $c_j|n_j>$. The transformation matrix can be viewed two ways.

(1) Since $c_1 = \langle n_1|\psi\rangle$, *etc.*

$$\begin{pmatrix} c_1 \\ c_2 \\ : \end{pmatrix}_{new} = \underbrace{\begin{pmatrix} (\mathbf{n}_1 \; bra \rightarrow)_{old} & \cdots \\ (\mathbf{n}_2 \; bra \rightarrow)_{old} & \cdots \\ : & \end{pmatrix}}_{n\times n \text{ matrix}} \begin{pmatrix} a_1 \\ a_2 \\ : \end{pmatrix}_{old}$$

(2) Simultaneously, the vector equation $|\psi\rangle = a_1|b_1\rangle + a_2|b_2\rangle + ... + a_n|b_n\rangle$ is true when written in any basis, and a matrix multiplied by a vector produces a weighted sum of the matrix columns:

$$\begin{pmatrix} c_1 \\ c_2 \\ : \end{pmatrix}_{new} = \underbrace{\begin{pmatrix} \begin{pmatrix} \mathbf{b}_1 \\ \downarrow \end{pmatrix}_{new} & \begin{pmatrix} \mathbf{b}_2 \\ \downarrow \end{pmatrix}_{new} & \cdots \end{pmatrix}}_{n\times n \text{ matrix}} \begin{pmatrix} a_1 \\ a_2 \\ : \end{pmatrix}_{old}$$

Stacks of SG devices: Eigenvectors |x+>, |x->, |y+>, |y->.

Measuring a 2nd time in a different axis *erases* all prior state information.

2D quantum state, with only 2 eigenvectors, cannot "store" any prior state information

Week 8, Day 3

Review: Section 2.2, and chapter 6.

Any bilinear inner product $<a|o|c>$ can be written as a sum of n^2 terms.

Each term must contain exactly 3 weight factors:

1 factor from <a|, 1 from |c>, and a weight factor from o

Operator matrices: diagonal contains weight factors for each component by itself

off-diagonal contains weight factors for *interactions* between components

may have overstated the 2 dof in a 2-spinor.

No history of prior superpositions is stored after taking a measurement. E.g., for l=1, there are only 3 dof, because there are only 3 eigenstates. For any l, there's no room to store any history.

Stern-Gerlach for $l = 2$, or higher: splits into $(2l + 1)$ beams.

Draw axes: z-up, y- travel, x- out of board.

Back to: Going through z, x (but no measurement), and z: preserves the z direction.

Compare going into SGx with |z+> to |z->: both cases preserve the z direction.

Measuring the x-spin collapses the wave-function, and loses z history.

Week 9, Day 1

Review: (Draw Friday's diagram: SGz, SGx (w/o measurement), SGz.)

A measuring device causes a system to time-evolve into a superposition of entangled states.

A measurement is made when *you* perceive it (and no one else).

This collapses the quantum state: chooses a component from a superposition

This violates Schrodinger's Equation. (We'll come back to EPR.)

A real cat-like experiment: two atoms in a box: with measurement, and without.

Experimental confirmation of "quantum weirdness."

Coupling position and spin: Simple tensor product states: $\psi(x)|\chi> \equiv \psi(x) \otimes |\chi>$.

Don't need to know anything about tensors to understand tensor products: they are more general.

E.g., state of an H atom: tensor product of space and spin. Complete quantum state requires both.

"Entangled" tensor product states: Before you look, the state of particle coming out of SG is:

superposition (of tensor products), entangled: $\psi_+(x)|x+>$ and $\psi_-(x)|x->$

This shows that the tensor product space is bigger than the set of tensor products.

We need new form of wave-function that includes entangled spin: $\psi(x) = [\psi_+(x), \psi_-(x)]^T$.

Common abuse of term "correlation". They mean "dependence".

Probabilities of measuring position, and measuring spin. Normalization.

$$pdf(x) = |\psi_+(x)|^2 + |\psi_-(x)|^2 \qquad \Pr(\uparrow\ at\ x) = \frac{|\psi_+(x)|^2}{|\psi_+(x)|^2 + |\psi_-(x)|^2},$$

$$\int_\infty \left(|\psi_+(x)|^2 + |\psi_-(x)|^2\right) dx = 1$$

Week 9, Day 2

Review: Chapter 6:

A **tensor product state** is a combination of states from two (or more) different Hilbert spaces.

The tensor product space is bigger than the set of tensor products, 'cuz it includes superpositions of tensor product states.

The output of a Stern-Gerlach device (before we measure it) is in the tensor product *space*

But is not in the tensor product *set*, because it's a superposition of two tensor products

Leads to a new kind of wave-function: the 2-component wave-function

$\psi(x) = [\psi_+(x), \psi_-(x)]^T \equiv \psi_+(\mathbf{r})|z+\rangle + \psi_-(\mathbf{r})|z-\rangle$

Probabilities of measuring position, and measuring spin. Normalization.

$$pdf(x) = |\psi_+(x)|^2 + |\psi_-(x)|^2 \qquad \Pr(\uparrow\ at\ x) = \frac{|\psi_+(x)|^2}{|\psi_+(x)|^2 + |\psi_-(x)|^2},$$

$$\int_\infty \left(|\psi_+(x)|^2 + |\psi_-(x)|^2\right) dx = 1$$

Show diagram of SG time evolution.

These imply $\psi_+(x)$ cannot interfere with $\psi_-(x)$: there are no cross terms in the pdf's.

Two-particle states: tensor product of two 1-particle states, or superposition thereof.

Another entangled example: EPR: classic (but not classical) example. draw diagram:

μ-μ+ pair, in $l = 0$ (s-state), s = 0, decay into e-e+: |↑↓> + |↓↑> (demanded by symmetry)

Or is it |↑↓> – |↓↑>? We'll come back to this.

When I measure my particle ↑, it collapses the wave-function, so I know the other will measure ↓.

Inner products of tensor products: see book for this: not done in class

$<\phi|<\chi_1|\cdot|\psi>|\chi_2> = <\phi|\psi><\chi_1|\chi_2>$

If *either* factor-pair is orthogonal, then the inner product is zero

Distributive property holds: $\left(|a\rangle + |b\rangle\right) \otimes \left(|c\rangle + |d\rangle\right) = |a\rangle \otimes |c\rangle + |a\rangle \otimes |d\rangle + |b\rangle \otimes |c\rangle + |b\rangle \otimes |d\rangle$

Sec. 12.4: Combining two spins: raising and lowering

$$S_x = s_{1x} + s_{2x}, \qquad etc.$$

$$S_+ \equiv S_x + iS_y = s_{1x} + s_{2x} + i\left(s_{1y} + s_{2y}\right) = s_{1+} + s_{2+}, \qquad similar\ for\ S_-$$

How can we distinguish between total S = |1 0>, and S = |0 0>? By raising or lower the state.

Combining angular momentum in general: Coupled and uncoupled basis

Week 10, Day 1

Review: Ch 12: Correction: $s_+\left|\downarrow\right\rangle = \sqrt{\frac{1}{2}\left(\frac{3}{2}\right) - \left(-\frac{1}{2}\right)\frac{1}{2}}\left|\uparrow\right\rangle = 1\left|\uparrow\right\rangle \qquad \left(not\ \frac{1}{\sqrt{2}}\left|\uparrow\right\rangle\right)$

We derived the total spin for a 2-spin system: |↑↓> + |↓↑> = |1 0>, |↑↓> – |↓↑> = |0 0>

Tensor products: see Section 6.1 for this, not covered in class.

We started EPR (but there's no paradox). When I measure mine, it collapses the state:

$$\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \qquad \to \qquad either\left|\uparrow\downarrow\right\rangle \quad or \quad \left|\downarrow\uparrow\right\rangle$$

HW 8 is on arbitrary two-state systems.

Photon polarization is a two-state system: H/V or R/L. QM uses different convention for R/L than optics.

Addition of angular momentum: Basis changes, Clebsch-Gordon coefficients. E.g.,

2-spin states: 4D. Two bases: the "product basis" or "uncoupled basis" $|j_1,m_1; j_2,m_2>$, and

the "coupled" basis: $|J, M>$.

Relationship between the two: | ½, ½; ½, ½> = |↑↑> = |1 1>, and |↓↓> = |1 –1>.

Symmetric combination is |↑↓> + |↓↑> = |1 0>. Can see this from symmetry:

s_- is symmetric, and |↑↑> is, too. So, all $J = 1$ states are symmetric.

Anti-symmetric must be the only choice left: |↑↓> – |↓↑> = |0 0>.

Clebsch-Gordon coefficients (CGs) relate the two bases of two-angular-momentum states.

(Not in book) **Decoherence:** In EPR: does the *other* observer collapse my wave-function?

Unknowable *and* unreproducible phases are indistinguishable from classical uncertainty:

No possibility of interference. Revisit the 2-atom "Schrödinger's cat" experiment.

Numerical example: to fully decohere, we approximate $\Delta E \equiv \sigma_E = 2\pi$, mixes all the phases well.

Let τ = .01 ps (my detectors resolve 25 ps). $\exp(-i(E/\hbar)t) \Rightarrow (\sigma_E\tau)/\hbar = 2\pi.$, $\sigma_E = h/\tau$.

$6.6e-34/1e-14 = 7e-20 < 1\ eV$ (about 1 chemical bond).

In the decoherence model, the other observer does *not* collapse my wave-function

But that's indistinguishable from "yes he does" for *macroscopic* bodies

But the decoherence model scales seamlessly down through mesoscopic to quantum scale

Again, Schrödinger's cat: decoherence turns it into a classical uncertainty.

Decoherence means no macroscopic body can ever be *measured* as in a coherent superposition:

essentially, it behaves as if it is in a classical uncertain state (a state of ignorance)

Hidden variables?

HW shows that the very form of Pr(spin ↓) = $\sin^2(\theta/2)$ is inconsistent with hidden variables.

Amazingly, it wasn't discovered 'til 1964, almost 4 decades after QM was developed.

Week 10, Day 2

Review: Clebsch-Gordon coefficients relate the two bases of two-angular-momentum states:

"uncoupled" or "product basis": $|j_1,m_1; j_2,m_2>$, and "coupled basis": $|J, M>$.

CG's can be written in standard inner-product form: $<j_1,m_1; j_2,m_2|J, M>$.

$$|J\ M\rangle = \sum_{m_1=-j_1}^{j_1} \langle j_1 m_1; j_2, M-m_1 | JM\rangle | j_1 m_1; j_2, M-m_1\rangle \qquad \textit{subject to } |M-m_1| \le j_2, \quad \textit{and}$$

$$|j_1 m_1; j_2, m_2\rangle = \sum_{J=|j_1-j_2|}^{j_1+j_2} \langle J, m_1+m_2 | j_1 m_1; j_2, m_2\rangle | J, m_1+m_2\rangle \quad \textit{or}$$

$$= \sum_{J=|j_1-j_2|}^{j_1+j_2} \langle J, M | j_1 m_1; j_2, m_2\rangle | J, M\rangle \; \textit{where} \quad M \equiv m_1+m_2$$

In all cases, j_1 and j_2 are known and fixed. Either m_1 and m_2 can vary, or J can vary; always $M = m_1 + m_2$.

CG's are real and symmetric: $<j_1,m_1; j_2,m_2|J, M> = <J, M|j_1,m_1; j_2,m_2>$

The lack of hidden definite states ("hidden variables") is actually embedded in the form of a spinor: $\begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)e^{+i\phi} \end{pmatrix}$, 'cuz it violates combining independent probabilities of error from a hypothetical z-axis.

Decoherence means no macroscopic body can ever be measured as in a coherent superposition:

essentially, it behaves as if it is in a classical uncertain state

Quantum decoherence is a *different model* from classical uncertainty, but leads to the same predictions for macroscopic bodies. However, decoherence scales seamlessly down to mesoscopic and quantum sizes, with the decoherence time smoothly increasing to observable times.

Ch 17: Stationary state Perturbation theory: will be on Final exam

QM can exactly solve only 3 problems: particle in a box, SHO, and the hydrogen atom. (QFT solves only the SHO.) Therefore, we need numerical approximation methods. The most important is PT. (QM PT is actually easier than classical mechanics PT.)

$$H = H_0 + H_p \qquad \to \qquad H = H_0 + \lambda H_p, \qquad\qquad H|\psi_n\rangle = E_n|\psi_n\rangle$$

$$\left|\psi_n\right\rangle = |n\rangle + \sum_{k\ne n} c_{kn}|k\rangle \quad [17.4], \qquad \left(H_0 + \lambda H_p\right)\left|\psi_n\right\rangle = E_n\left|\psi_n\right\rangle \quad [17.3] \qquad \Rightarrow$$

$$\left(H_0 + \lambda H_p\right)\left(|n\rangle + \left[\sum_{k\ne n}\left(\lambda c_{kn}^{(1)} + \lambda^2 c_{kn}^{(2)} + ...\right)\right]|k\rangle\right) \qquad [\text{S\&W 17.6 p350, corrected}]$$

$$= \left(E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + ...\right)\left(|n\rangle + \left[\sum_{k\ne n}\left(\lambda c_{kn}^{(1)} + \lambda^2 c_{kn}^{(2)} + ...\right)\right]|k\rangle\right)$$

"Small" expansion parameter is some physical property of the system, such as a magnetic or electric field.

I follow the book's notation and method, which is fairly standard. We consider only non-degenerate PT.

Week 10, Day 3

Review: Stationary state Perturbation Theory: Simply expand H and ψ, and plug and chug

No new physics, no QM. Just Dirac algebra.

"Small" expansion parameter is some physical property of the system,

e.g. magnetic or electric field.

"λ" is just a tool for counting powers of "small".

Important: Correction to Eq. 17.6.

$$E_n^{(1)} = \langle n|H_p|n\rangle \qquad [17.10\text{ p350}]$$

$$c_{kn}^{(1)} = \frac{\langle k|H_p|n\rangle}{\varepsilon_n - \varepsilon_k} \qquad [17.11\text{ p351}] \qquad \Rightarrow \qquad \left|\psi^{(1)}\right\rangle = \sum_{k\ne n}\frac{\langle k|H_p|n\rangle}{\varepsilon_n - \varepsilon_k}|k\rangle \qquad [17.12\text{ p351}]$$

$$E_n^{(2)} = \sum_{k\ne n}\frac{\left|\langle k|H_p|n\rangle\right|^2}{\varepsilon_n - \varepsilon_k}$$

Note the positions of n and k in the equations. Note the units of equations.

Quantum eraser: clear demonstration of quantum principles

Practical application: continuously variable mutual coherence between beams, with perfect intensity

95% of all Quantum Eraser stuff "out there" is bogus, even professional papers

t_0	$\left|1_{uv}\right\rangle$	1-photon state
t_1	$\sqrt{\frac{1}{2}}|u\rangle + \sqrt{\frac{1}{2}}|d\rangle$	2 1-photon states
t_2	$\sqrt{\frac{1}{2}}\left|s_1 i_1\right\rangle + \sqrt{\frac{1}{2}}|d\rangle$	entangled 2-photon + 1-photon
t_3	$\sqrt{\frac{1}{2}}\left|s_1 i\right\rangle + \sqrt{\frac{1}{2}}\left|s_2 i\right\rangle$	idlers recombine: 2 2-photon states
t_4	$\left(1 + e^{i\delta}\right)|s,i\rangle$	2-photon state, with interference