Physics 2D Lecture Slides
Lecture 3

January 8, 2010

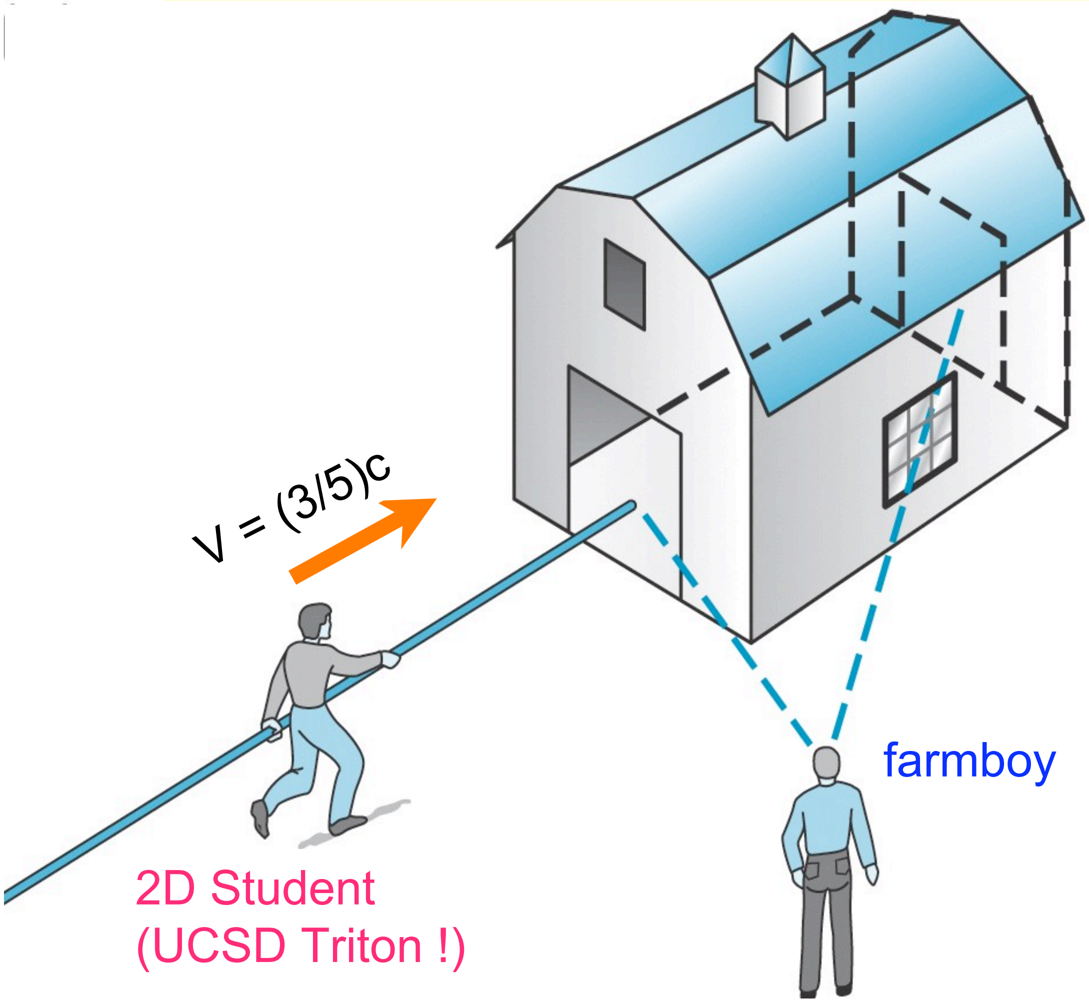
Immediate Consequences of Einstein's Postulates: Recap

- Events that are simultaneous for one Observer are not simultaneous for another Observer in relative motion
- Time Dilation : Clocks in motion relative to an Observer appear to slow down by factor γ
- Length Contraction : Lengths of Objects in motion appear to be contracted in the direction of motion by factor γ^{-1}
- New Definitions :
  - Proper Time (who measures this ?)
  - Proper Length (who measures this ?)
  - Different clocks for different folks !

Fitting a 5m pole in a 4m Barnhouse ?

Student attends 2D lecture (but does no HW) …banished to a farm in Iowa ! Meets a farmboy who is watching 2D lecture videos online. He does not do HW either!

There is a Barn with 2 doors 4m apart ; There is a pole with proper length = 5m
Farm boy goads the student to run fast and fit the 5m pole within 4m barn
The student tells the farmboy: “Dude you are nuts!” …who is right and why ?



Sequence of Events

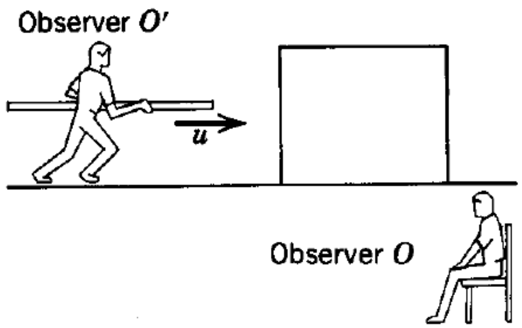
A: Arrival of right end of pole at left end of barn

B: Arrival of left end of pole at left end of barn

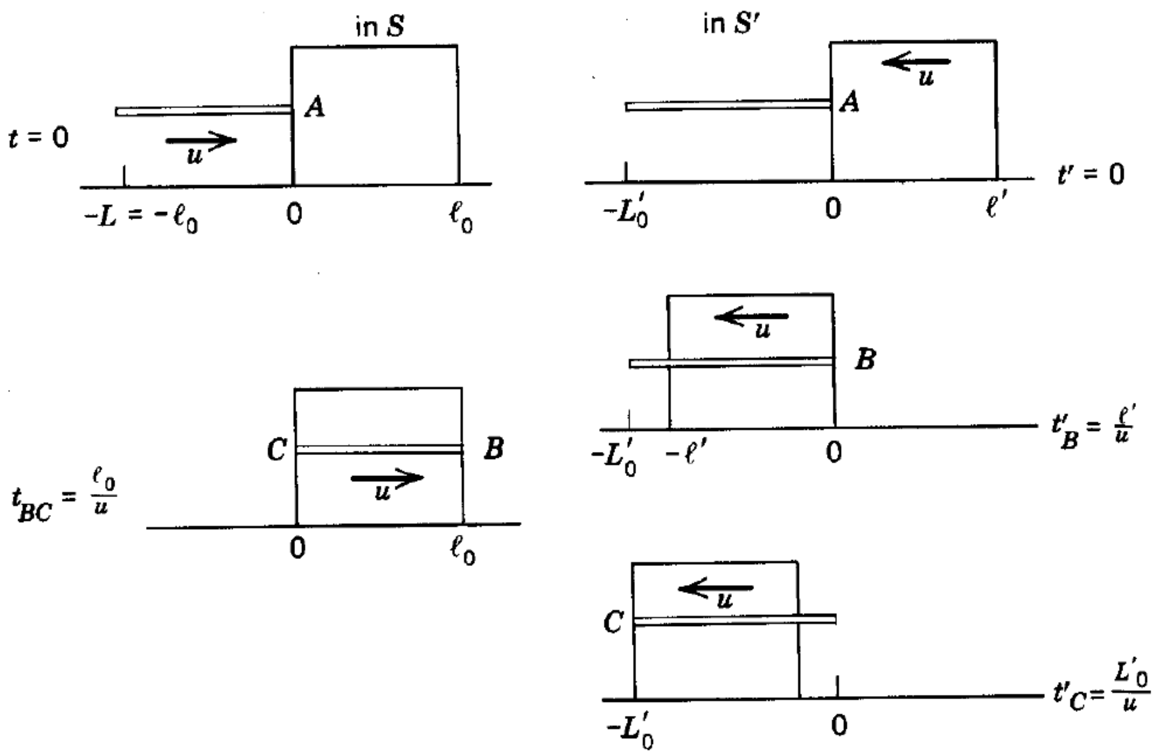
C: Arrival of right end of pole at right end of barn

Think Simultaneity !

Farmboy Vs 2D Student

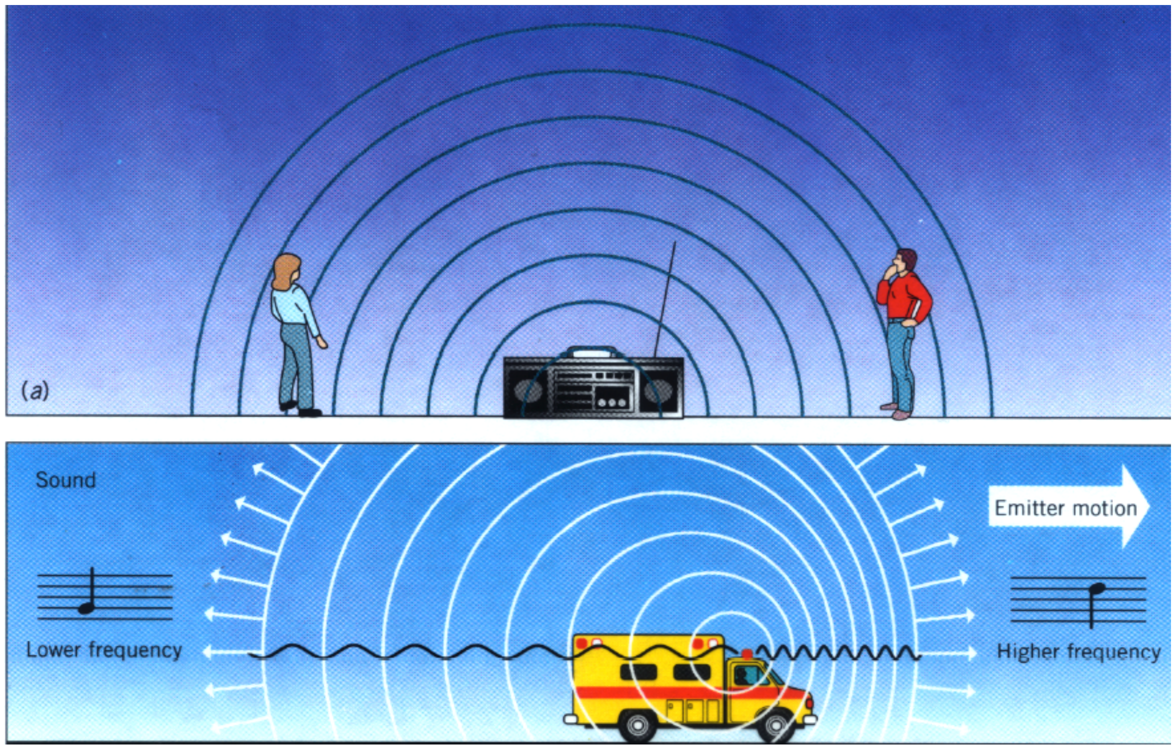


Pole and barn are in relative motion u such that lorentz contracted length of pole = Proper length of barn



In rest frame of pole, Event B precedes C

Doppler Effect In Sound : Reminder from 2C

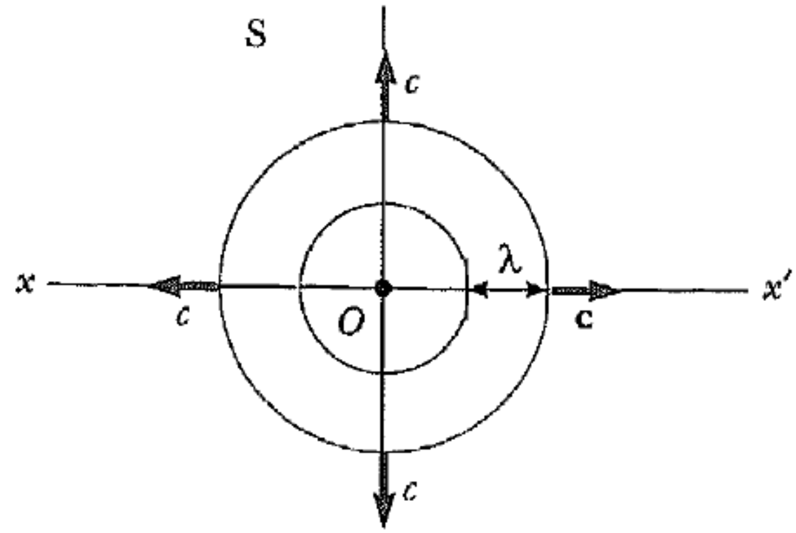


Observed Frequency of sound INCREASES if emitter moves towards the Observer
Observed Wavelength of sound DECREASES if emitter moves towards the Observer

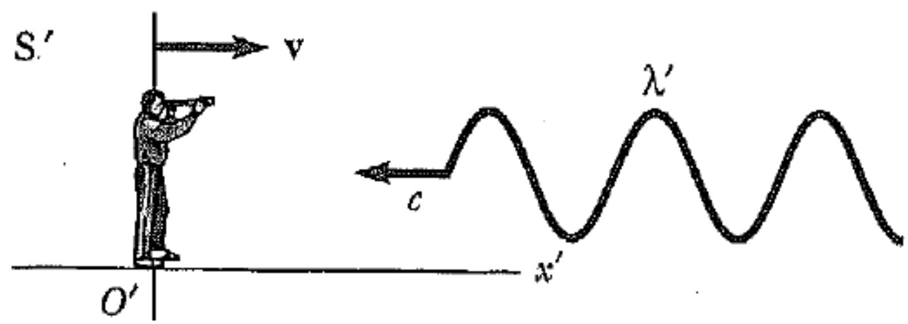
$$v = f \lambda$$

Time Dilation Example: Relativistic Doppler Shift

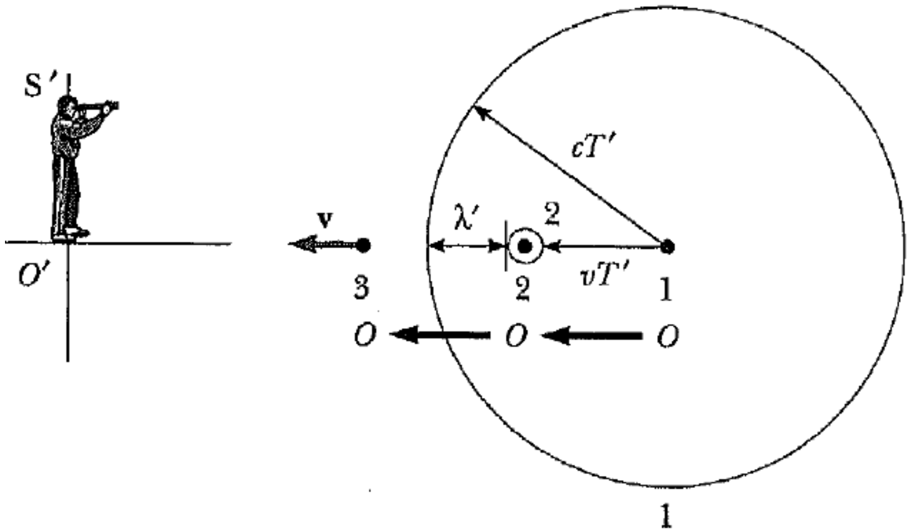
- Light : velocity $c = f\,\lambda$, $f=1/T$
- A source of light S at rest
- Observer S′ approaches S with velocity v
- S′ measures f′ or λ′, $c = f'\lambda'$
- Expect $f' > f$ since more wave crests are being crossed by Observer S’due to its approach direction than if it were at rest w.r.t source S



(a)



Relativistic Doppler Shift



Examine two successive wavefronts emitted by S at location 1 and 2

In S' frame, T' = time between two wavefronts

In time T', the wavefront moves by cT' w.r.t 1

Meanwhile Light Source moves a distance vT'

Distance between successive wavefronts

$$\lambda' = cT' - vT'$$

use $f = c/\lambda$

$$f' = \frac{c}{(c-v)T'} \quad , \; T' = \frac{T}{\sqrt{1-(v/c)^2}}$$

Substituting for T', use f=1/T

$$\Rightarrow f' = \frac{\sqrt{1-(v/c)^2}}{1-(v/c)} f$$

$$\Rightarrow f' = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}} f$$

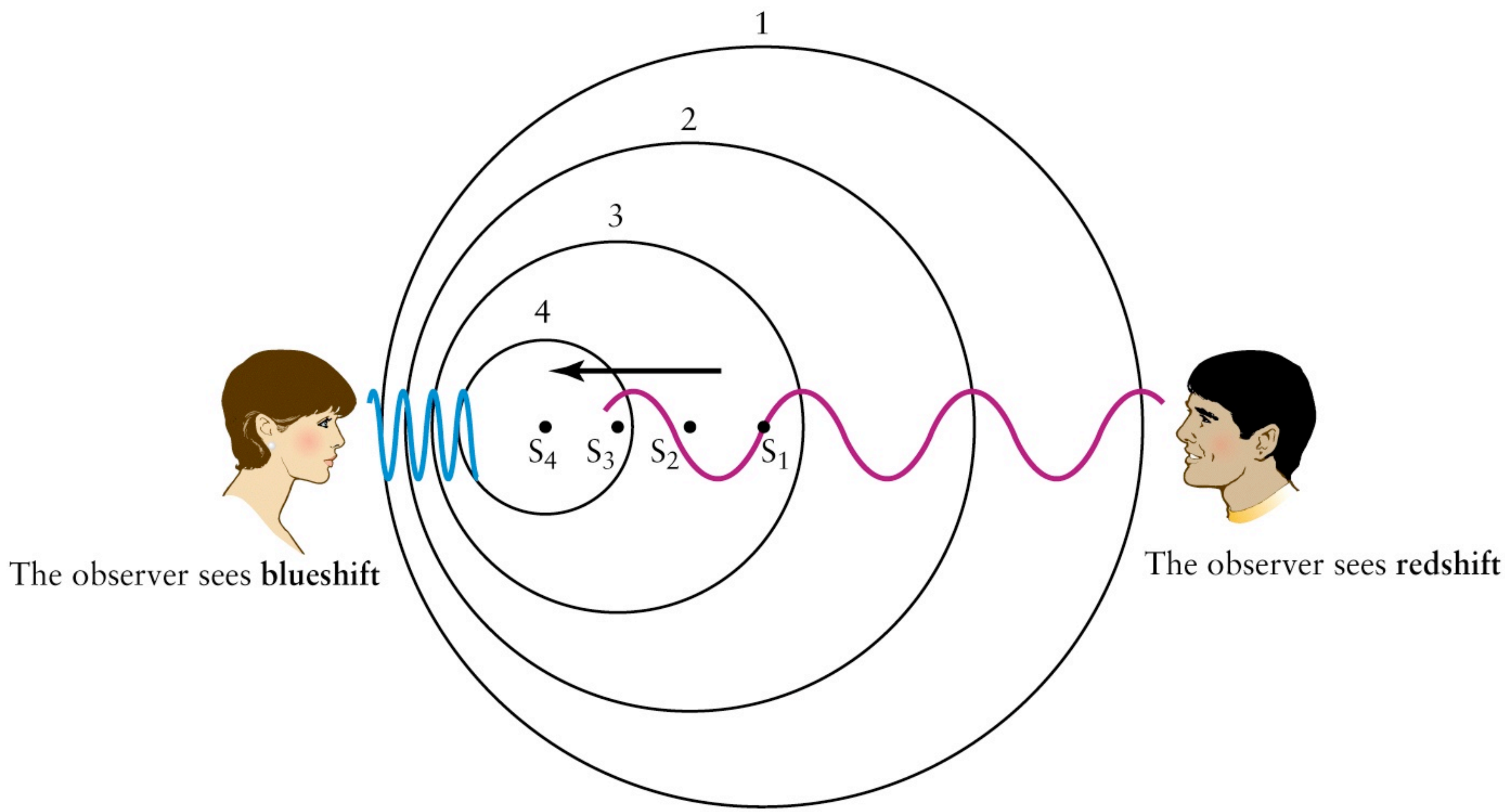
better remembered as:

$$f_{obs} = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}} f_{source}$$

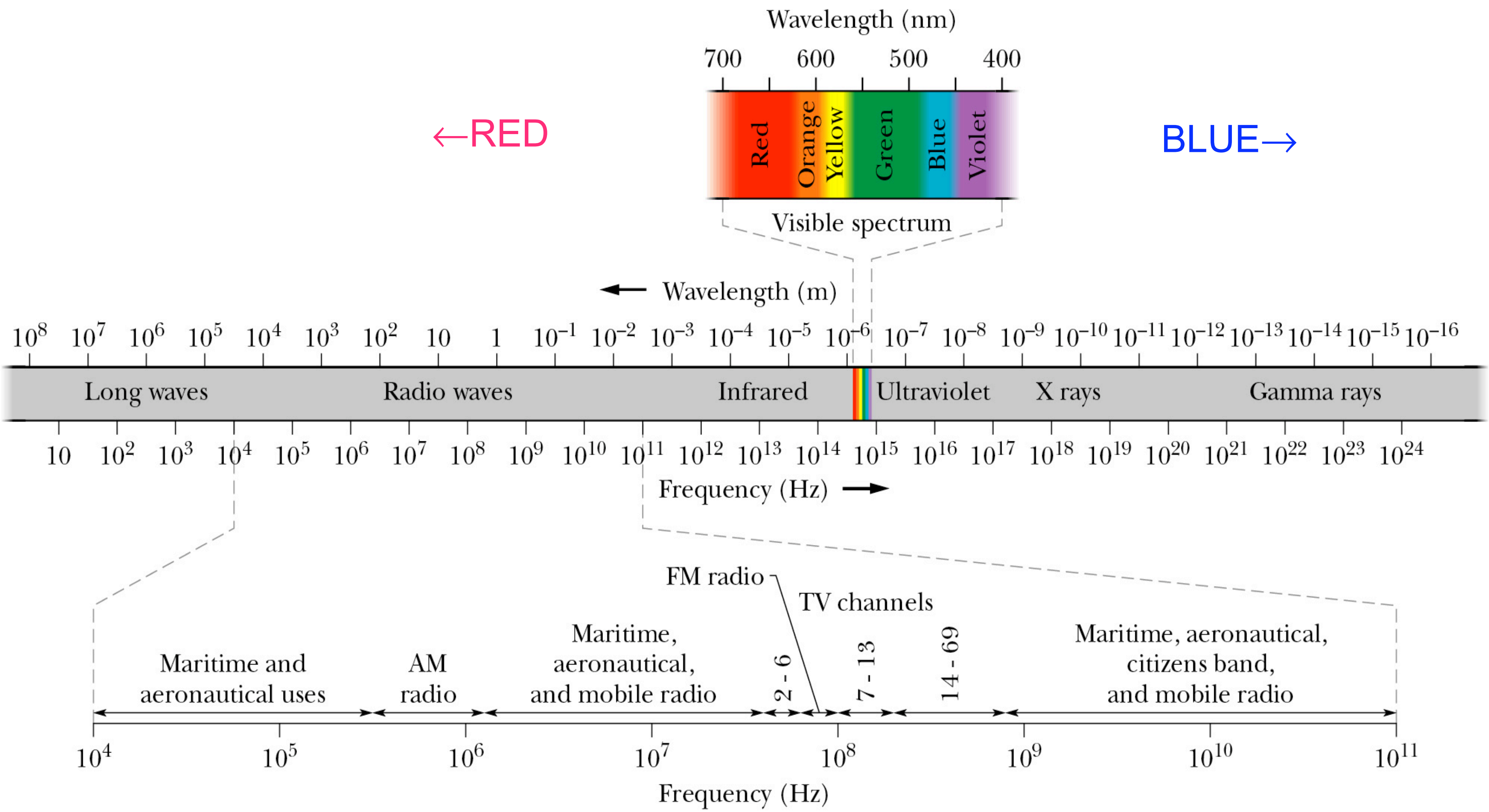
f_{obs} = Freq measured by observer approaching light source

Relativistic Doppler Shift

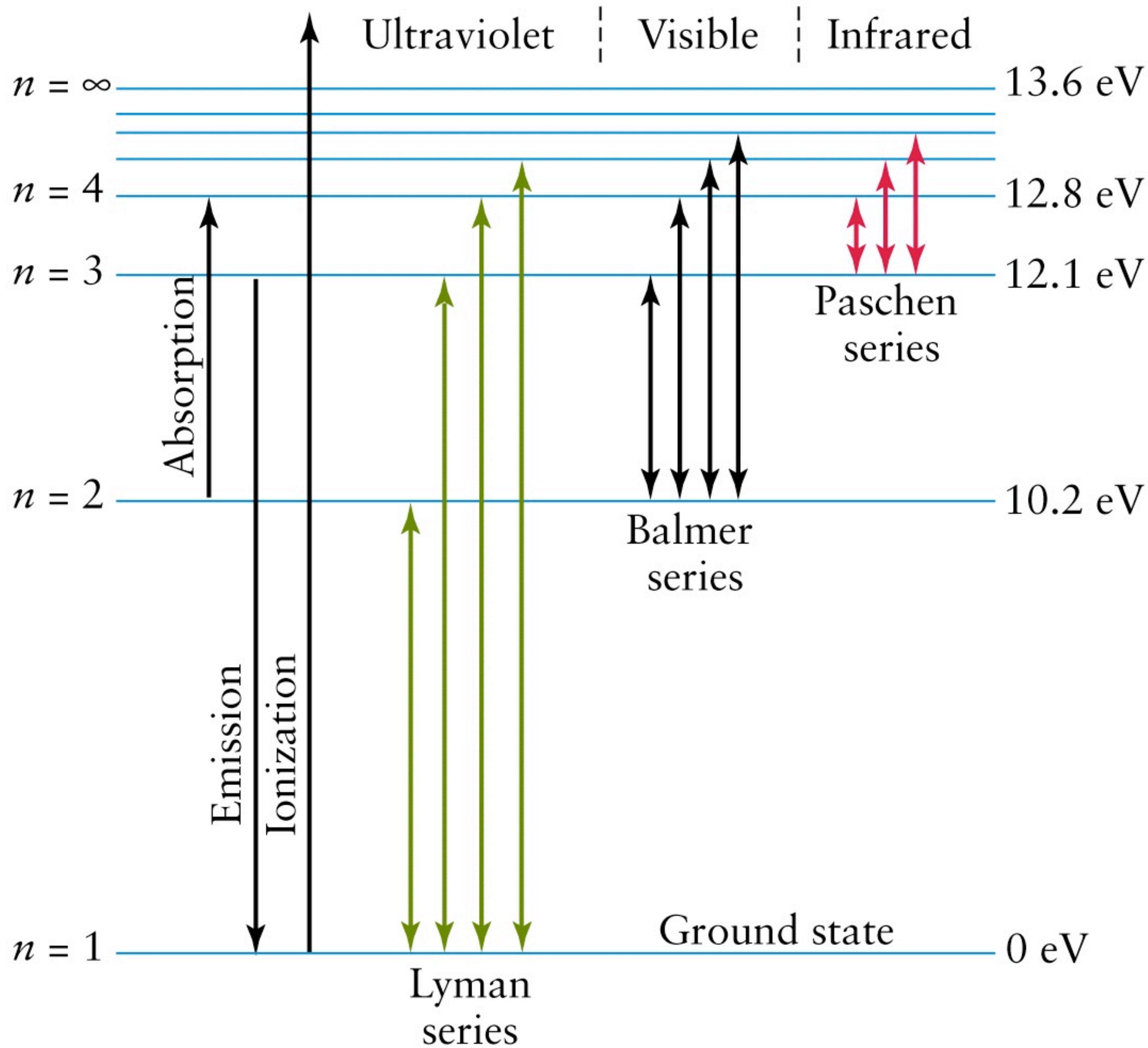
$$f_{obs} = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}} \; f_{source}$$



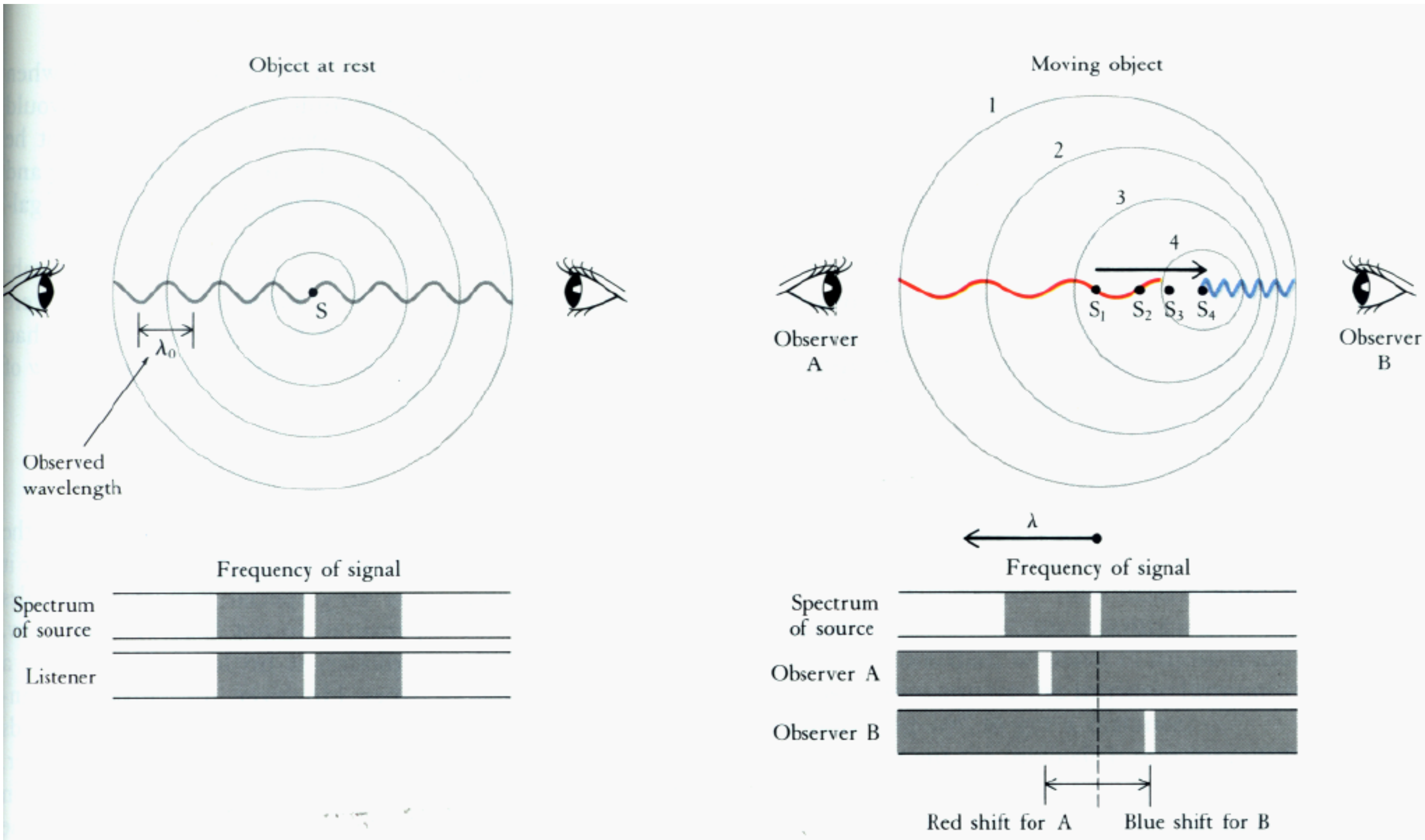
Doppler Shift & Electromagnetic Spectrum



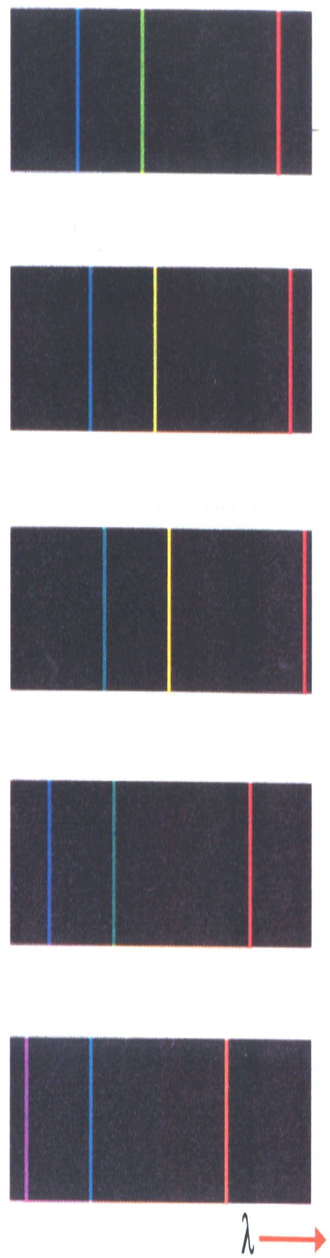
Fingerprint of Elements: Emission & Absorption Spectra



Spectral Lines and Perception of Moving Objects



Doppler Shift in Spectral Lines and Motion of Stellar Objects



Laboratory Spectrum, lines at rest wavelengths

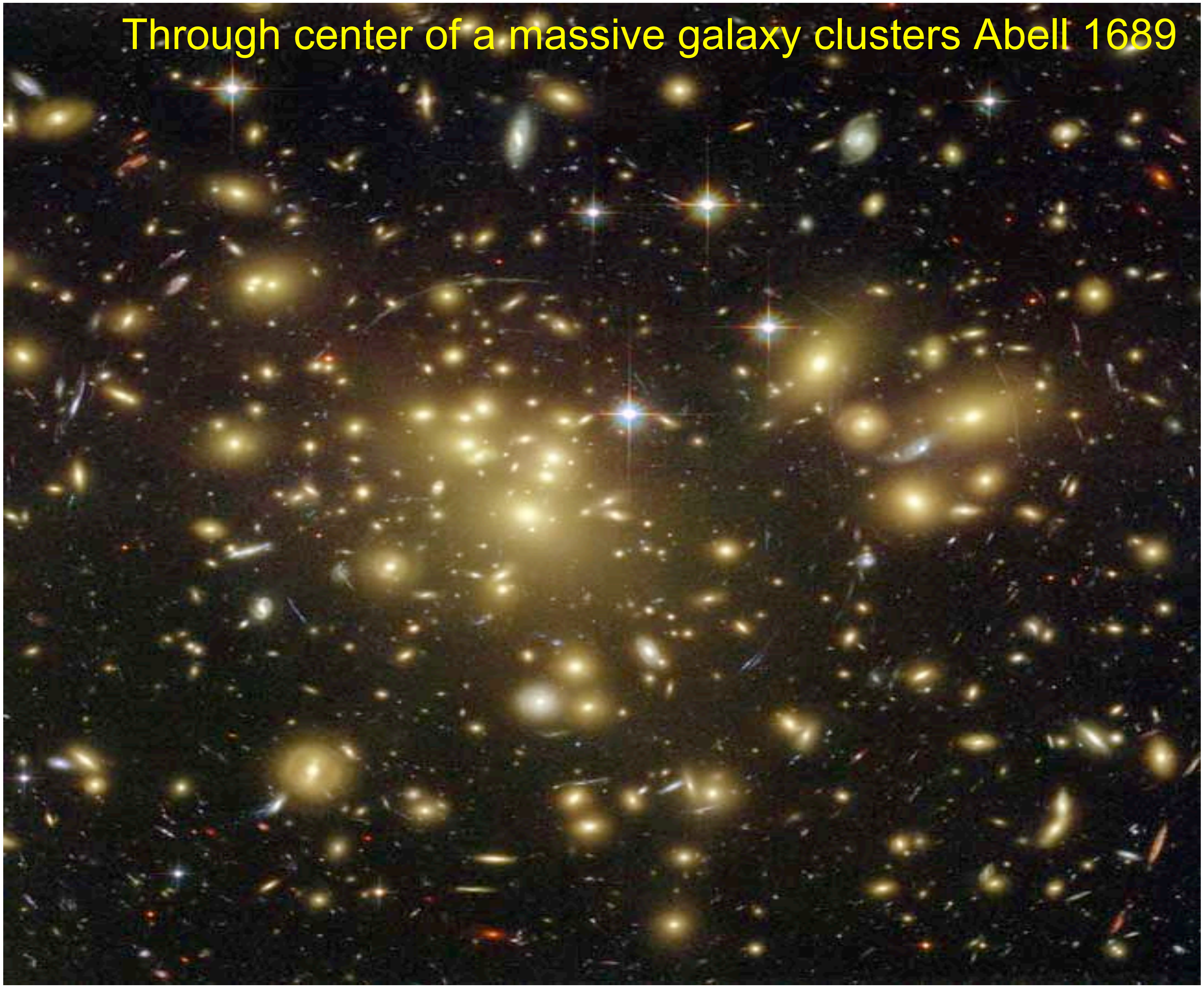
Lines Redshifted, Object moving away from me

Larger Redshift, object moving away even faster

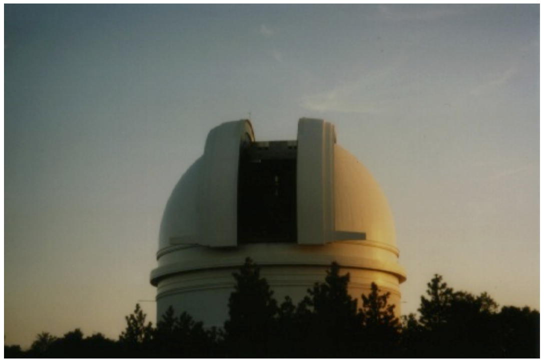
Lines blueshifted, Object moving towards me

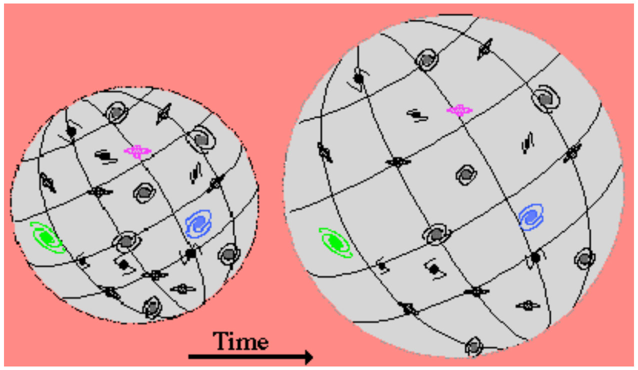
Larger blueshift, object approaching me faster

Seeing Distant Galaxies Thru Hubble Telescope

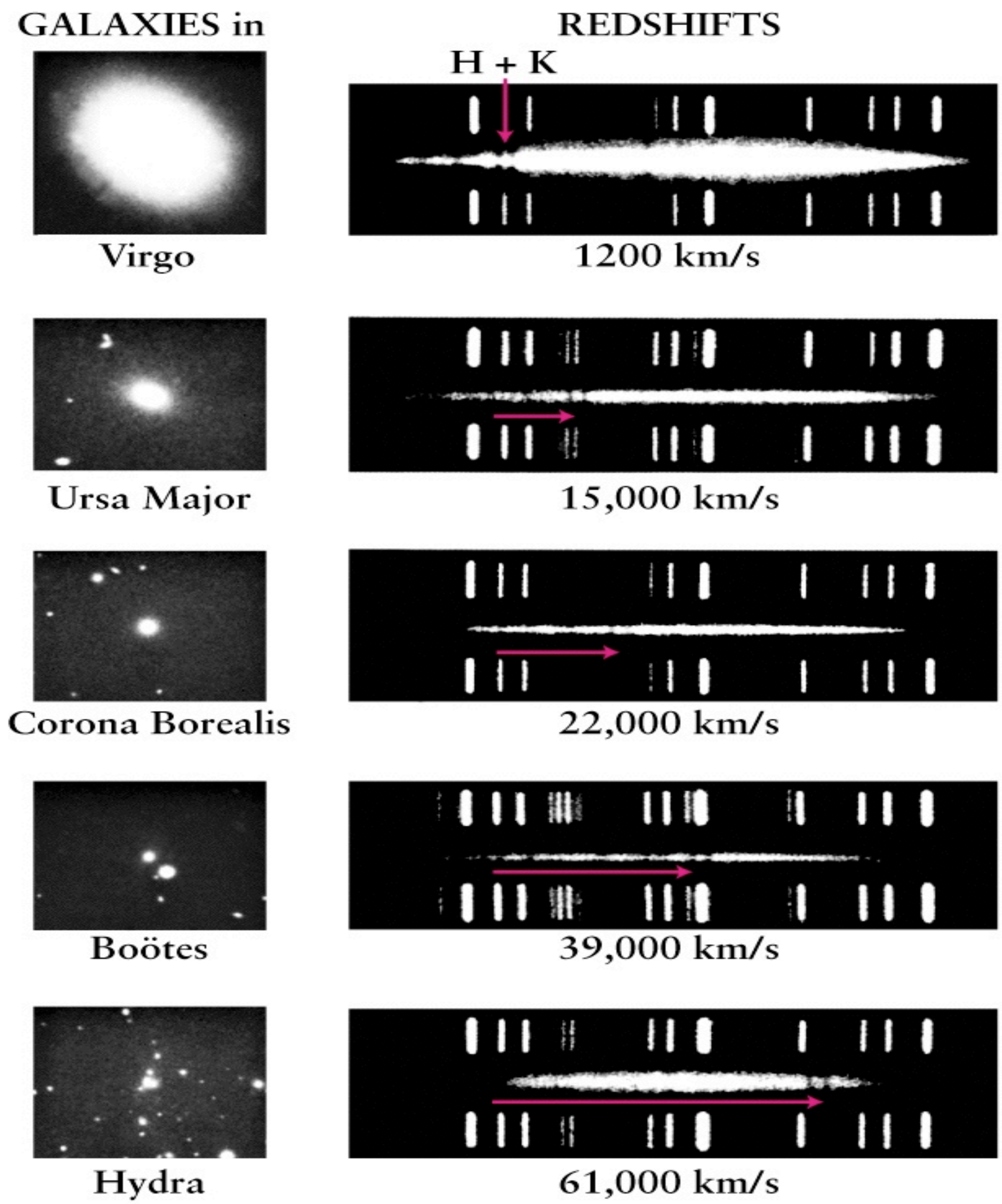


Expanding Universe, Edwin Hubble & Mount Palomar



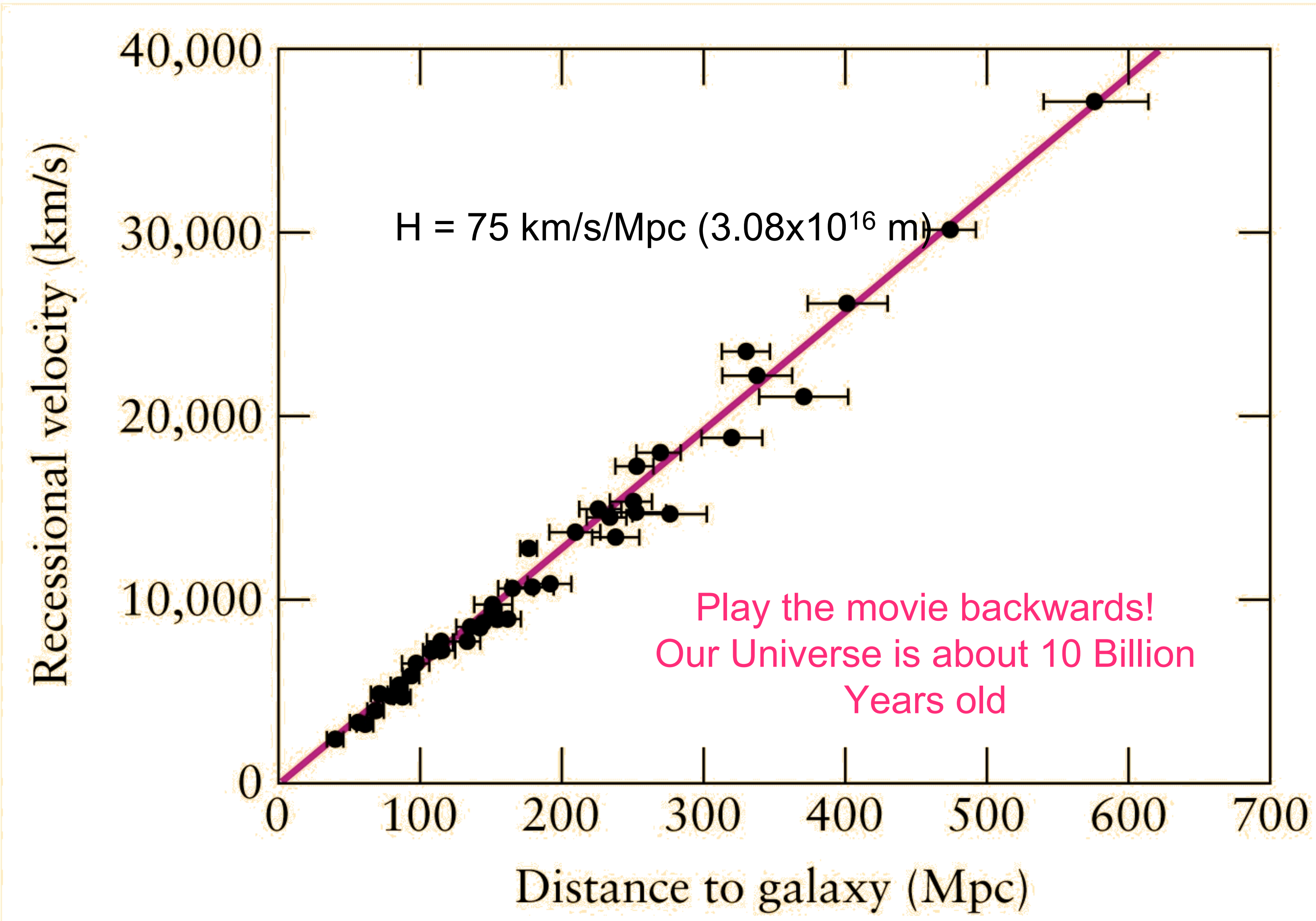


Galaxies at different locations in our Universe travel at different velocities



Hubble's Measurement of Recessional Velocity of Galaxies

V = H d : Farther things are, faster they go



New Rules of Coordinate Transformation Needed

- The Galilean/Newtonian rules of transformation could not handles frames of refs or objects traveling fast
  - $V \approx C$ (like v = 0.1 c or 0.8c or 1.0c)
- Einstein's postulates led to
  - Destruction of concept of simultaneity ($\Delta t \neq \Delta t'$)
  - Moving clocks run slower
  - Moving rods shrink
- Lets formalize this in terms of <u>general rules</u> of coordinate transformation : Lorentz Transformation
  - Recall the Galilean transformation rules
    - $x' = (x\text{-}vt)$
    - $t' = t$
      - These rules that work ok for Ferraris now must be modified for rocket ships with $v \approx c$

Discovering The Correct Transformation Rule

$$x' = x - vt \qquad \text{guess} \rightarrow \quad x' = G(x - vt)$$

$$x = x' + vt' \qquad \text{guess} \rightarrow \quad x = G(x' + vt')$$

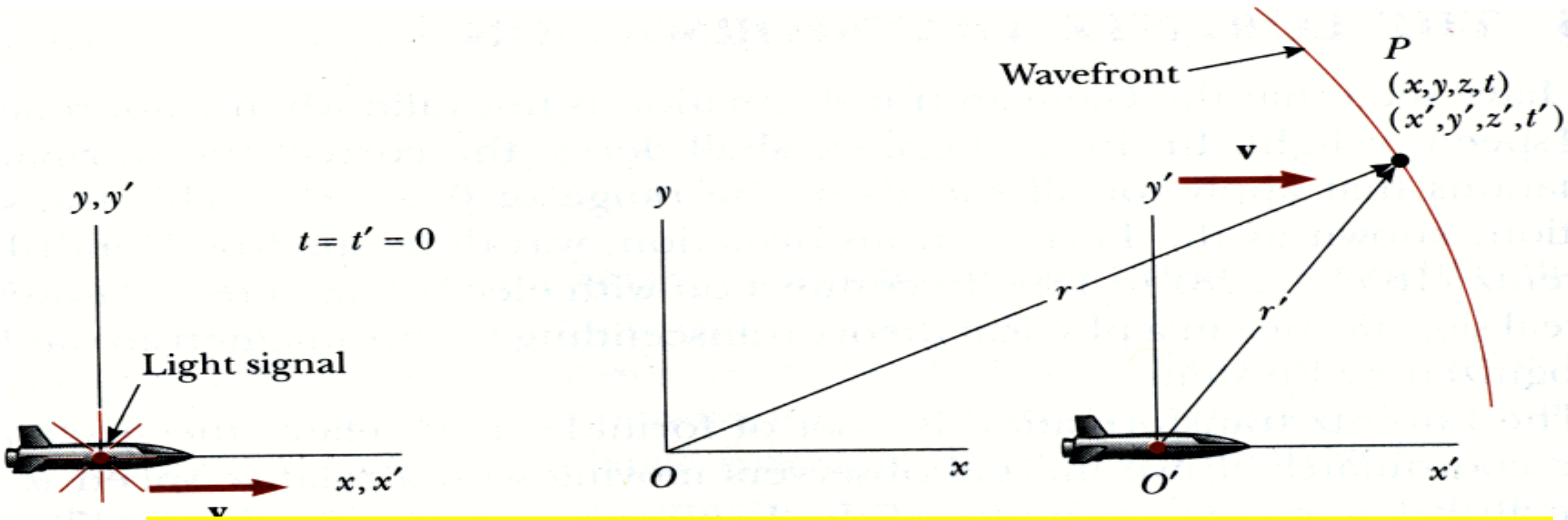
Need to figure out the functional form of G !

- G must be dimensionless
- G does not depend on x,y,z,t
- But G depends on v/c
- G must be symmetric in velocity v
- As $v/c \rightarrow 0$, $G \rightarrow 1$

Guessing The Lorentz Transformation

Do a Thought Experiment : Watch Rocket Moving along x axis

Rocket in S' (x',y',z',t') frame moving with velocity v w.r.t observer on frame S (x,y,z,t)
Flashbulb mounted on rocket emits pulse of light at the instant origins of S,S' coincide
That instant corresponds to t = t' = 0 . Light travels as a spherical wave, origin is at O,O'



Speed of light is c for both observers: Postulate of SR

Examine a point P (at distance r from O and r' from O') on the Spherical Wavefront

The distance to point P from O : r = ct
The distance to point P from O′ : r' = ct'

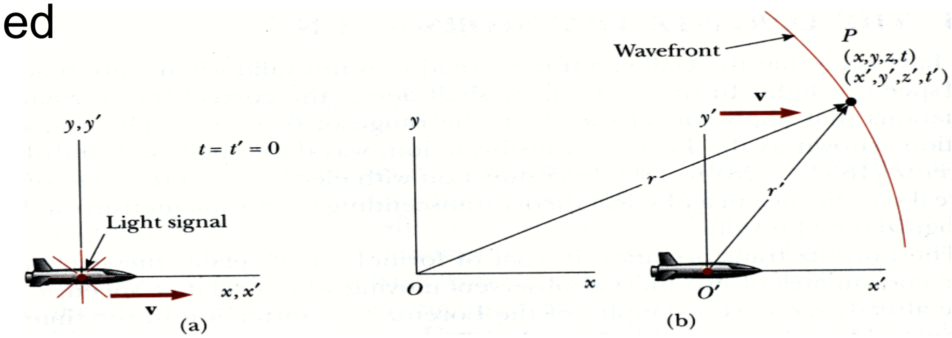
Clearly t and t' must be different
$t \neq t'$

Discovering Lorentz Transfromation for (x,y,z,t)

Motion is along x-x' axis, so y, z unchanged

y'=y,   z' = z

Examine points x or x' where spherical wave
crosses the horizontal axes: x = r , x' =r'



$$x = ct = G(x' + vt')$$

$$x' = ct' = G(x - vt) ,$$

$$\Rightarrow t' = \frac{G}{c}(x - vt)$$

$$\therefore x = ct = G(ct' + vt')$$

$$\therefore ct = G^2\left[(ct - vt) + vt - \frac{v^2}{c}t\right]$$

$$\Rightarrow c^2 = G^2[c^2 - v^2]$$

$$\text{or } G = \frac{1}{\sqrt{1-(v/c)^2}} = \gamma$$

$$\therefore \quad x' = \gamma(x - vt)$$

$$x' = \gamma(x - vt) , \; x = \gamma(x' + vt')$$

$$\Rightarrow \quad x = \gamma(\gamma(x - vt) + vt')$$

$$\therefore \quad x - \gamma^2 x + \gamma^2 vt = \gamma vt'$$

$$\therefore \quad t' = \left[\frac{x}{\gamma v} - \frac{\gamma^2 x}{\gamma v} + \frac{\gamma^2 vt}{\gamma v}\right] = \gamma\left[\frac{x}{\gamma^2 v} - \frac{x}{v} + t\right]$$

$$\therefore \quad t' = \gamma\left[t + \frac{x}{v}\left(\frac{1}{\gamma^2} - 1\right)\right], \text{ since } \left(\frac{1}{\gamma^2} - 1\right) = -\left(\frac{v}{c}\right)^2$$

$$\Rightarrow t' = \gamma\left[t + \frac{x}{v}[1 - \left(\frac{v}{c}\right)^2 - 1\right] = \gamma\left[t - \left(\frac{vx}{c^2}\right)\right]$$

Lorentz Transformation Between Ref Frames

<u>Lorentz Transformation</u>

$x' = \gamma(x - v t)$

$y' = y$

$z' = z$

$t' = \gamma(t - v x/ c^2)$

<u>Inverse Lorentz Transformation</u>

$x = \gamma (x' + v t)$

$y = y'$

$z = z'$

$t = \gamma (t' + v x/ c^2)$

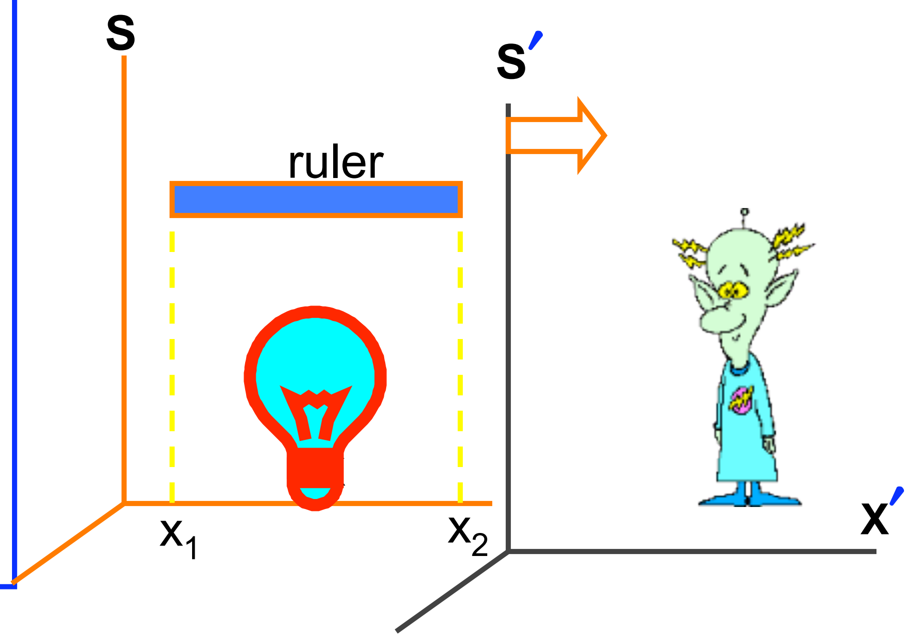
As $v \rightarrow 0$, Galilean Transformation is recovered, as per requirement

Notice : SPACE and TIME Coordinates mixed up !!!

Lorentz Transform for Pair of Events

$$\left.\begin{aligned}\Delta x' &= \gamma(\Delta x - v\,\Delta t)\\ \Delta t' &= \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)\end{aligned}\right\} S \rightarrow S'$$

$$\left.\begin{aligned}\Delta x &= \gamma(\Delta x' + v\,\Delta t')\\ \Delta t &= \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right)\end{aligned}\right\} S' \rightarrow S$$



Can understand Simultaneity, Length contraction & Time dilation formulae from this

Time dilation: Bulb in S frame turned on at t_1 & off at t_2 : What $\Delta t'$ did S′ measure ?

two events occur at same place in S frame => $\Delta x = 0$

$$\Delta t' = \gamma\,\Delta t \quad (\Delta t = \text{proper time})$$

Length Contraction: Ruler measured in S between x_1 & x_2 : What $\Delta x'$ did S′ measure ?

two ends measured at same time in S′ frame => $\Delta t' = 0$

$$\Delta x = \gamma\,(\Delta x' + 0) \Rightarrow \Delta x' = \Delta x / \gamma \qquad (\Delta x = \text{proper length})$$