Physics 2D Lecture Slides
Lecture 2

Jan. 5, 2010

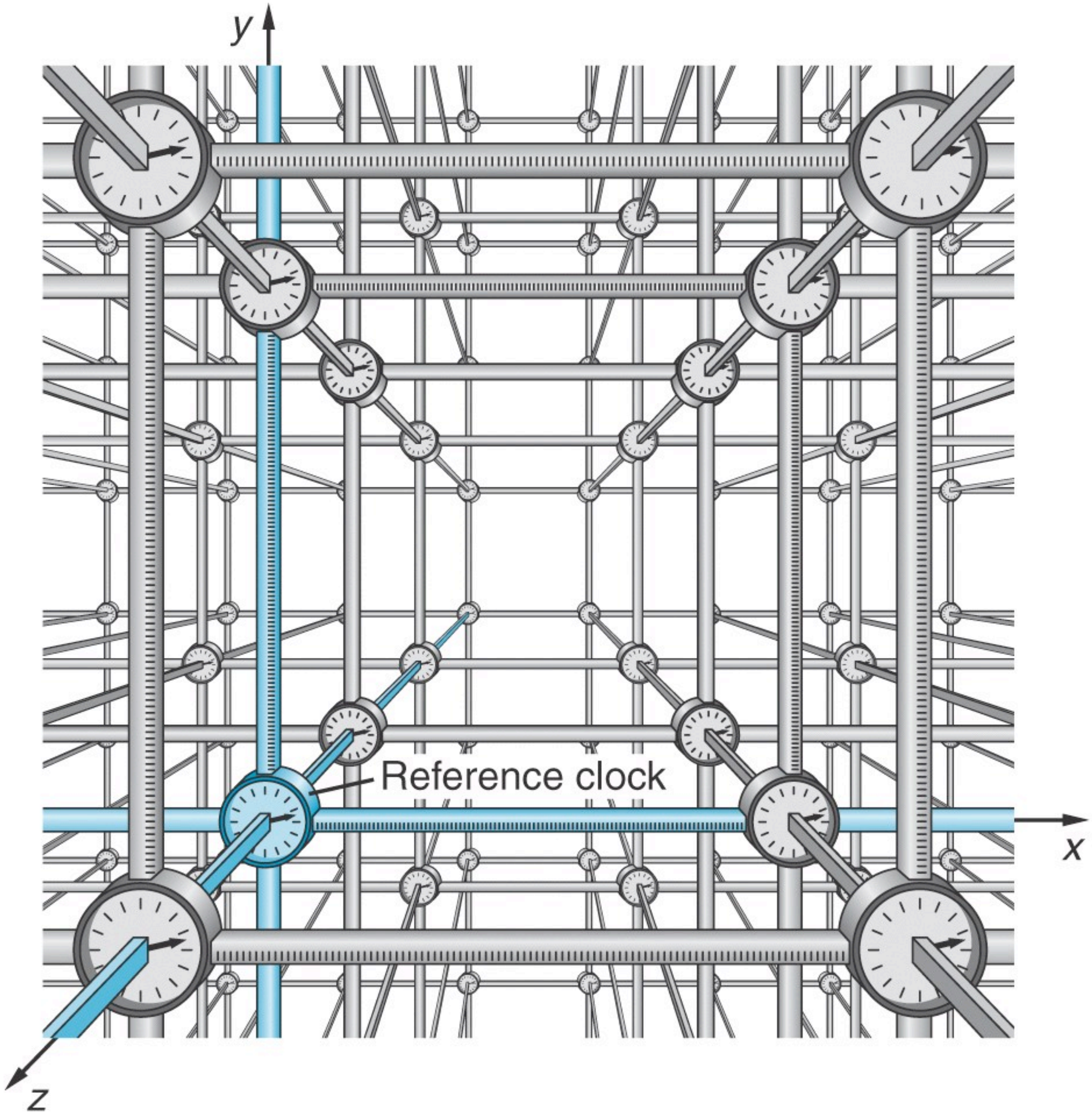
Lecture 1: Relativity

- Describing a Physical Phenomenon
  - Event (s)
  - Observer (s)
  - Frame(s) of reference (the point of View !)
    - Inertial Frame of Reference
    - Accelerated Frame of Reference

Described on Black board

- Newtonian Relativity and Inertial Frames
  - Laws of Mechanics and Frames of Reference
  - Galilean Transformation of coordinates
    - Addition law for velocities
- Maxwell's Equations & Light
  - Light as Electromagnetic wave
  - Speed of Light is not infinite !
  - Light needs no medium to propagate

The Universe as a Clockwork of Reference Frames



Event, Observer, Frame of Reference

- Event : Something happened => (x,y,z,t)
  - Same event can be described by different observers
- Observer(s) : Measures event with a meter stick & a clock
- Frame of Reference :observer is standing on it
  - Inertial Frame of reference 🡐 constant velocity, no force
- An event is not OWNED by an observer or frame of reference
- An event is something that happens, any observer in any reference frame can assign some (x,y,z,t) to it
- Different observers assign different space & time coordinates to same event
  - S describes it with : (x,y,z,t)
  - S' describes same thing with (x',y',x',t')

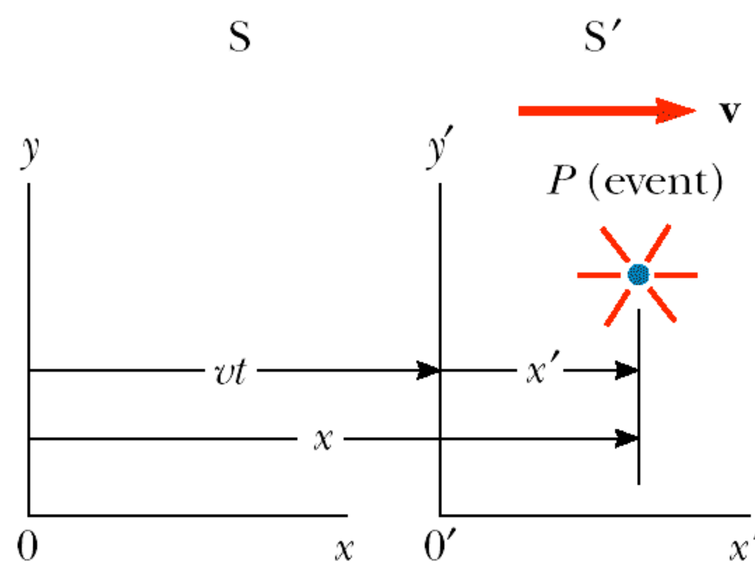
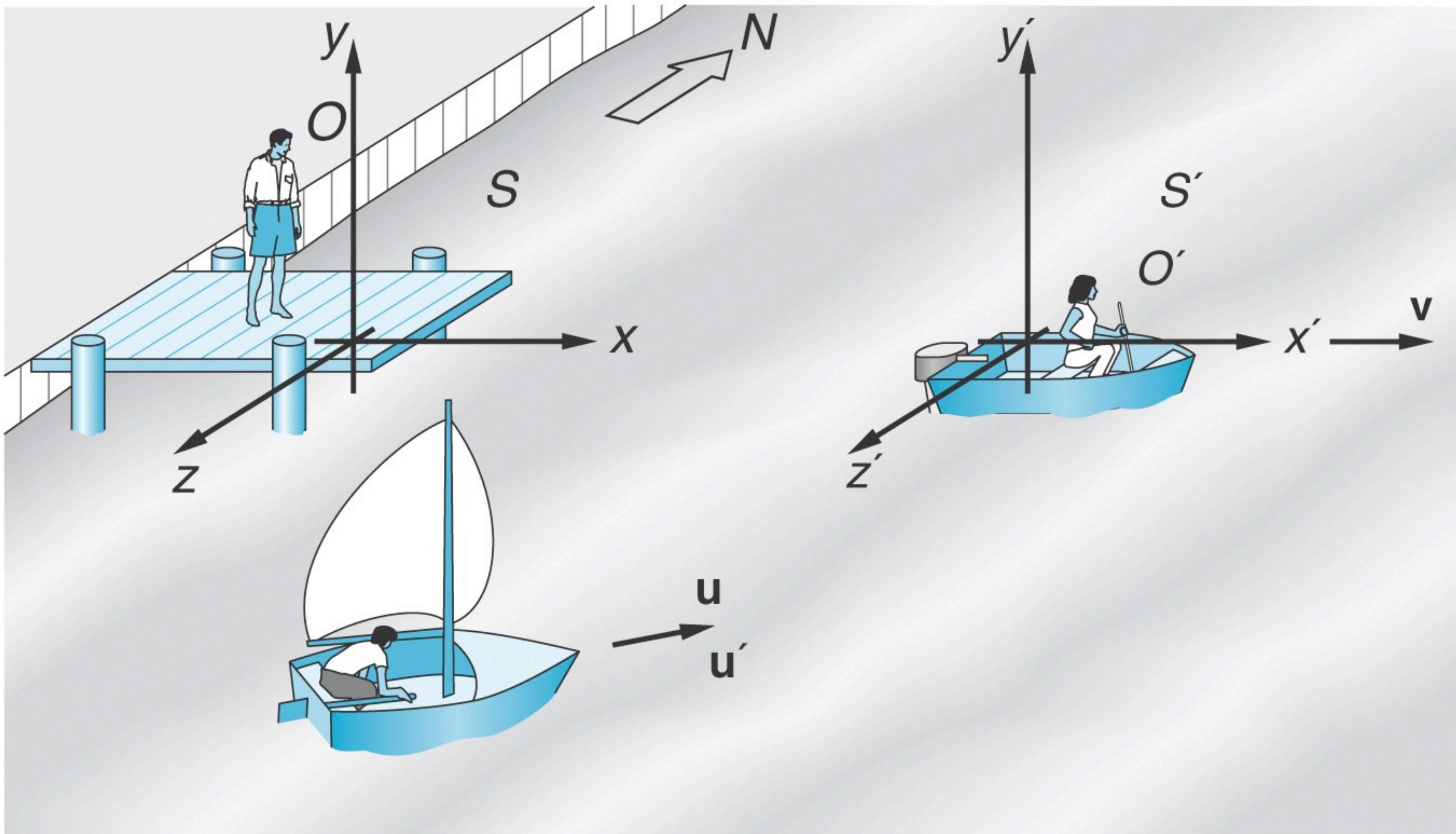


Figure 39.2 An event occurs at a point *P*. The event is seen by two observers in inertial frames S and S′, where S′ moves with a velocity **v** relative to S.

"Imagining" Ref Frames And Observers



Galilean Transformation of Coordinates

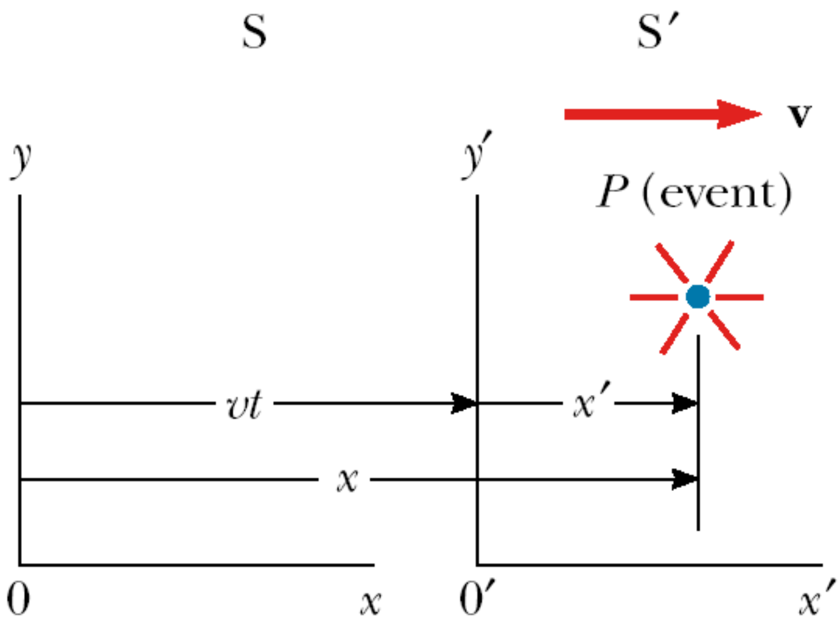


Figure 39.2 An event occurs at a point P. The event is seen by two observers in inertial frames S and S′, where S′ moves with a velocity **v** relative to S.

Galilean Rules of Transformation

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

Quote from Issac Newton Regarding Time

Absolute, true and mathematical time, of itself, and from nature, flows equably without relation to anything external

$$t = t'$$

There is a universal clock

Or

All clocks are universal

Galilean Addition Law For Velocities

$$dx' = dx - v\,dt$$

$$dt = dt'$$

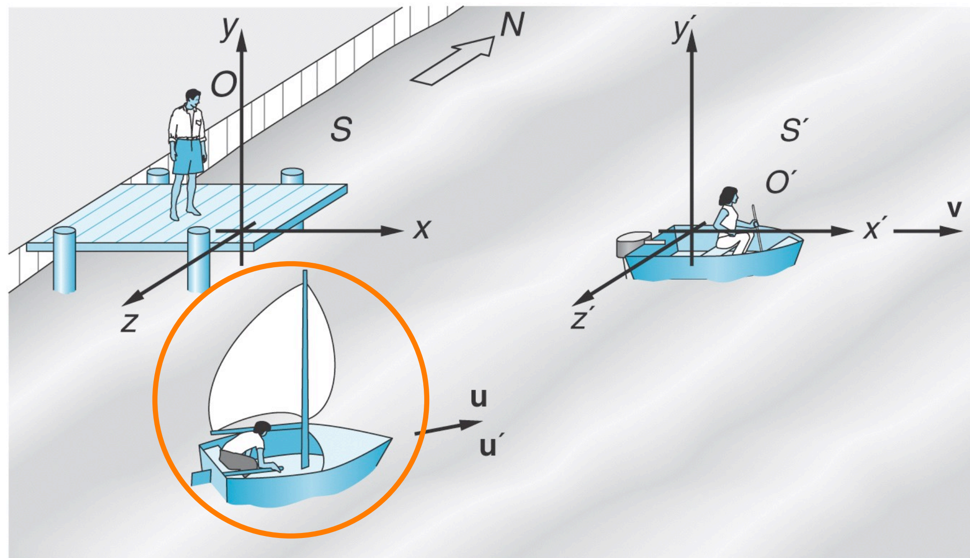
⇩

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v$$

⇩

$$u'_x = u_x - v$$

This rule is used in our everyday observations (e.g. driving a car) and is consistent with our INTUITIVE notions of space and time



But what happens when I drive a car very fast !!

How fast: (v = ?)

- As fast as light can travel in a medium !!!

Newton's Laws and Galilean Transformation !

- But Newton's Laws of Mechanics remain the same in All frames of references !

$$\frac{d^2x'}{dt^2} = \frac{d^2x'}{dt^2} - \frac{dv}{dt}$$

$\Rightarrow$

$$a' = a \qquad \Rightarrow \vec{F}' = \vec{F}$$

Description of Force does not change from one inertial frame of reference to another

Newtonian/Galilean Relativity

Inertial Frame of Reference is a system in which a free body is not accelerating

Laws of Mechanics must be the same in all Inertial Frames of References

⇒Newton's laws are valid in all Inertial frames of references

⇒No Experiment involving laws of mechanics can differentiate between any two inertial frames of reference

⇒Only the relative motion of one frame of ref. w.r.t other can be detected

⇒ Notion of ABSOLUTE motion thru space is meaningless

⇒There is no such thing as a preferred frame of reference

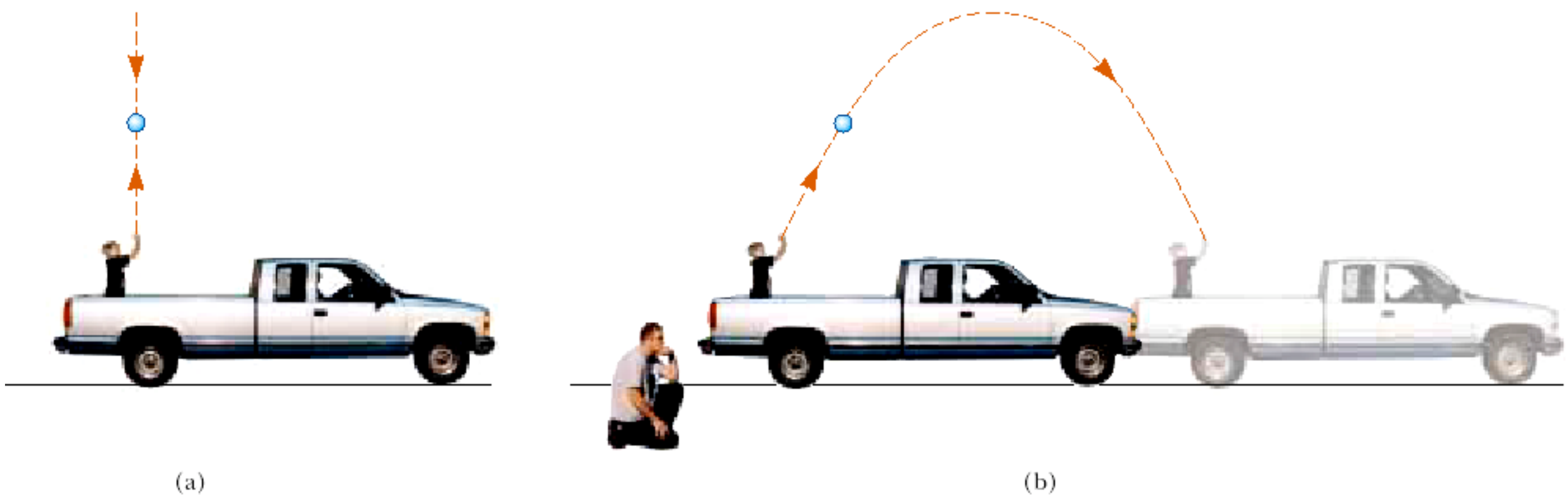


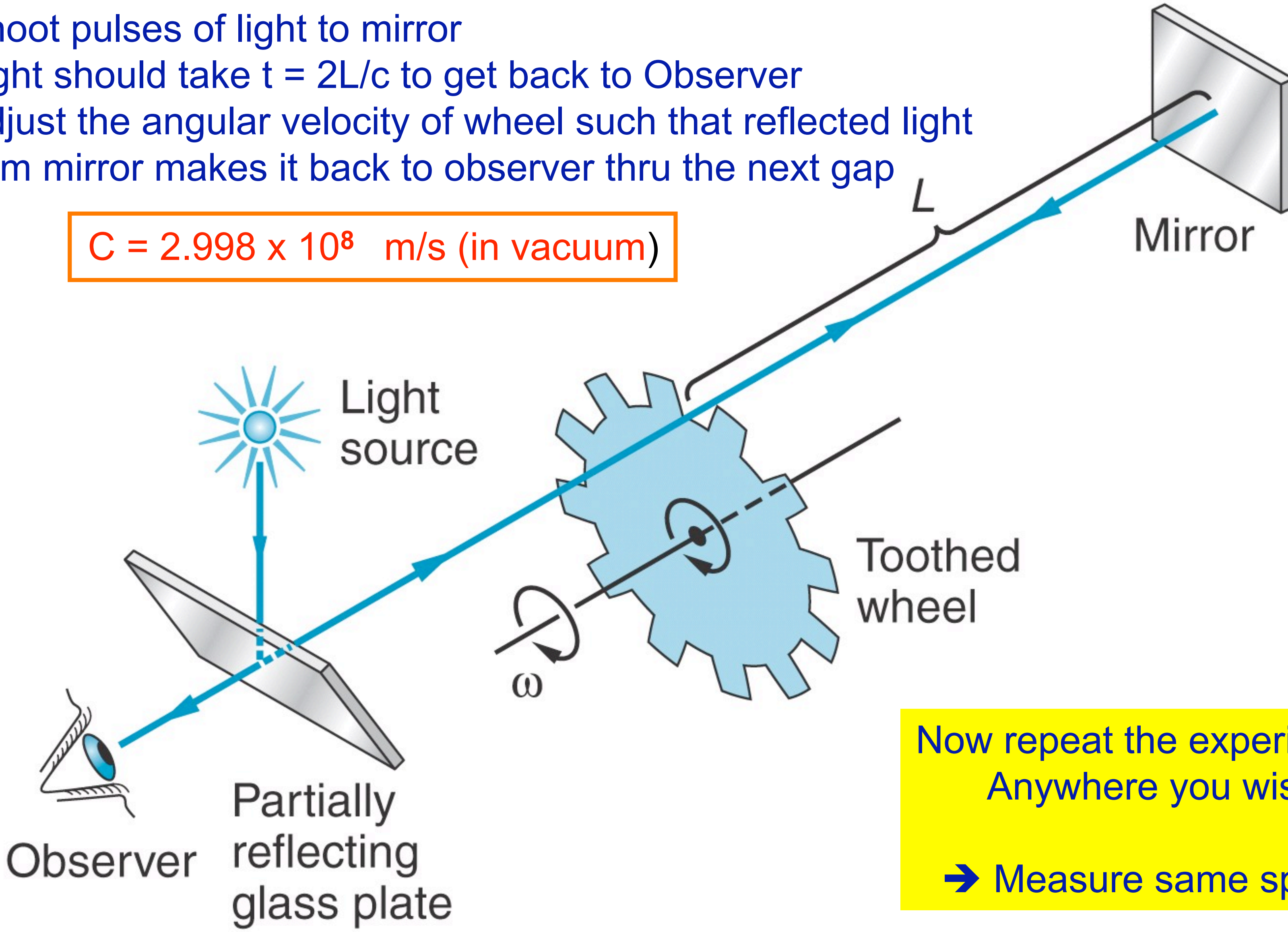
Figure 39.1 (a) The observer in the truck sees the ball move in a vertical path when thrown upward. (b) The Earth observer sees the path of the ball as a parabola.

Measuring The Speed Of Light

High Technology of 1880's: Fizeau's measurement of speed of light

1. Shoot pulses of light to mirror
2. Light should take t = 2L/c to get back to Observer
3. Adjust the angular velocity of wheel such that reflected light from mirror makes it back to observer thru the next gap

$C = 2.998 \times 10^8$ m/s (in vacuum)



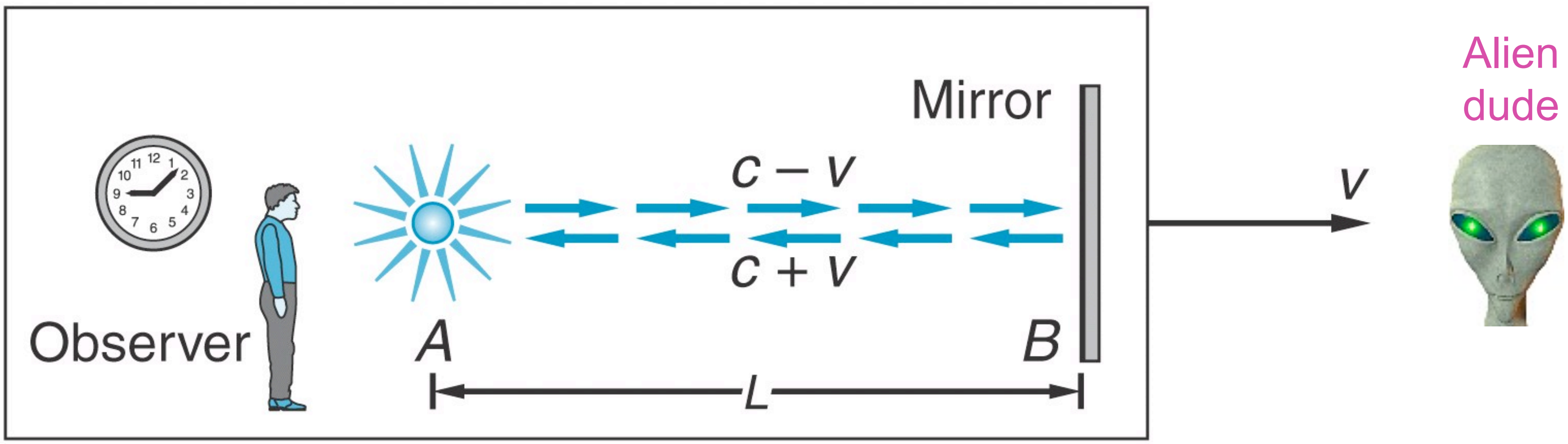
Now repeat the experiment
Anywhere you wish

➔ Measure same speed

Newtonian Relativity & Light !

Light source, mirror & observer moving thru some medium with velocity V
Galilean Relativity ➔

- If the alien measures velocity of light = c
- Then observer must measure speed of light = c-v when it is leaving him
  =c+v when it is reflected back

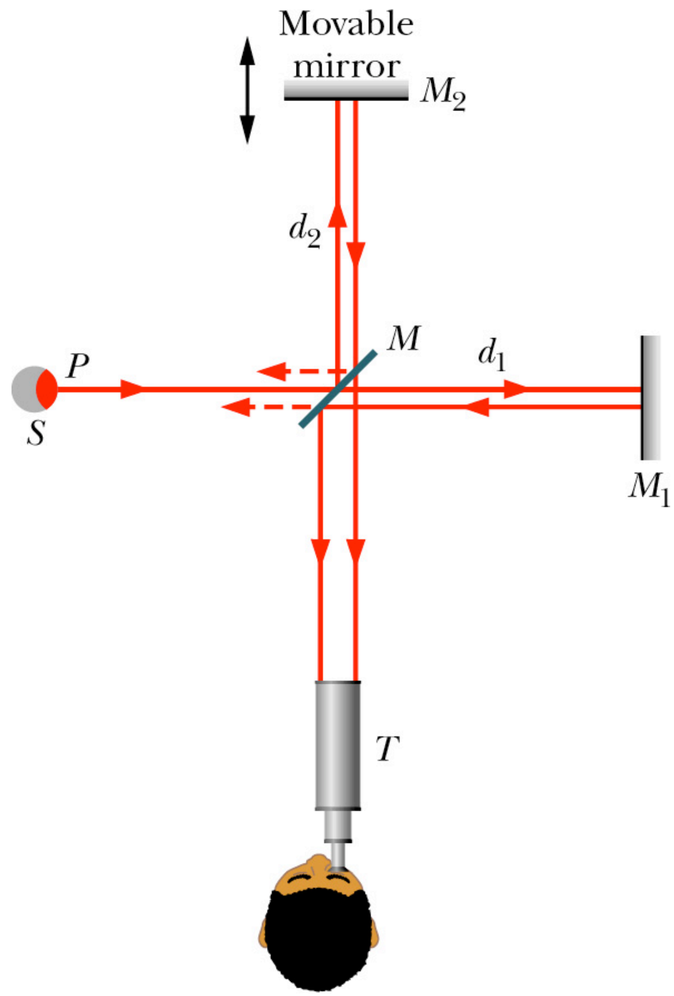


But Maxwell's Eq → speed of light is constant in a medium??

Must it be that laws of Mechanics behave differently from E&M in different inertial frames of references ? …if so how inelegant would nature be!

Does Light Need a Medium to Propagate ?

- EM waves are a different
  - What is the required medium of propagation ? Aether ??
    - How to verify whether Aether exists or not?
      - (Always) Do an Experiment !
- The Michelson-Morley Interferometer
  - Interferometer: device used to measure
    - Lengths or changes in lengths
  - Measured with great accuracy
    - Using interference fringes
- HW Reading : Section 1.3
  - If you don’t understand this, pl. review
    - Wave Phenomena
- Bottomline: Light needs no medium



Light Is An Electromagnetic Wave (2C)

- Maxwell's Equations:

$$\oint_S \mathbf{E}\cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

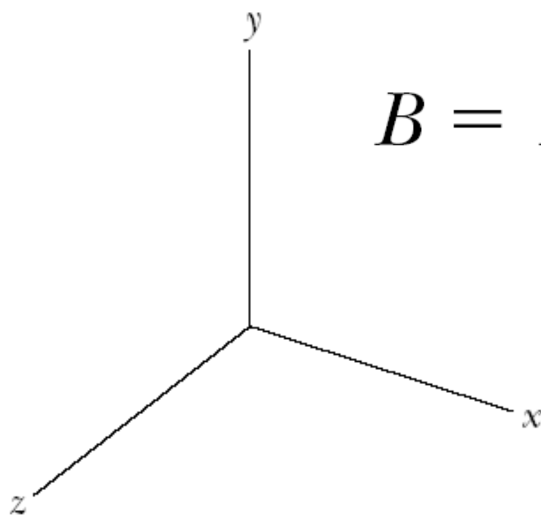
$$\oint_S \mathbf{B}\cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I + \mu_0\epsilon_0\frac{d\Phi_E}{dt}$$

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 B}{\partial t^2}$$



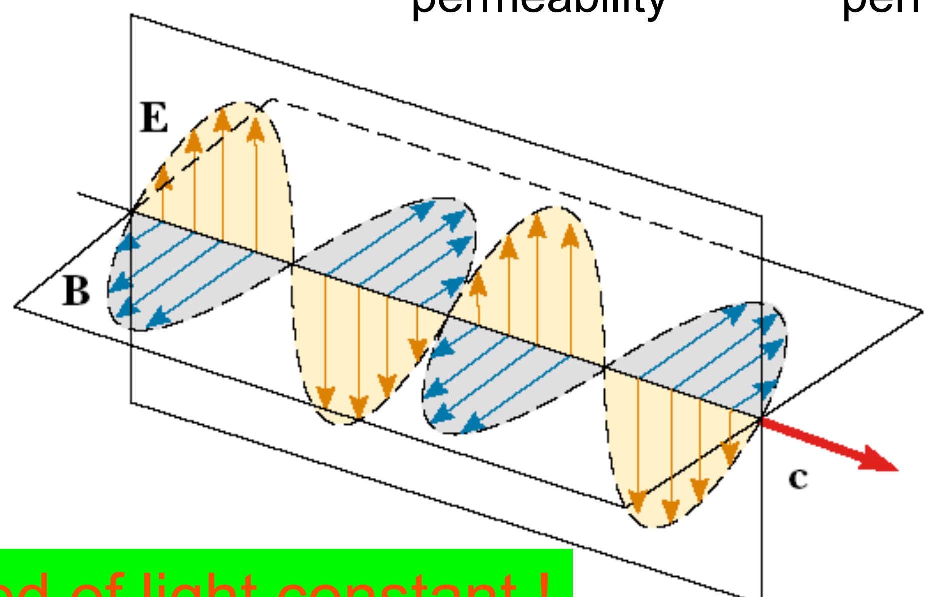
$$E = E_{\max}\cos(kx - \omega t)$$

$$B = B_{\max}\cos(kx - \omega t)$$

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

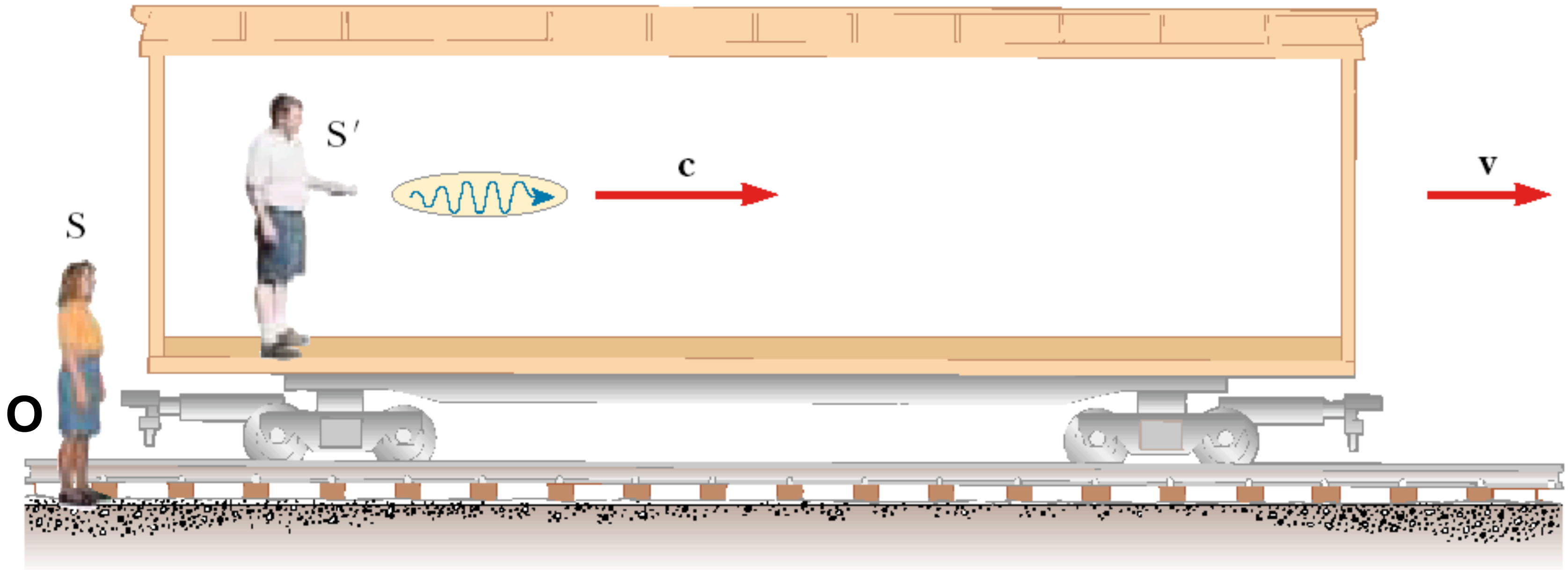
permeability

permittivity



Speed of light constant !

Galilean Relativity and EM Waves



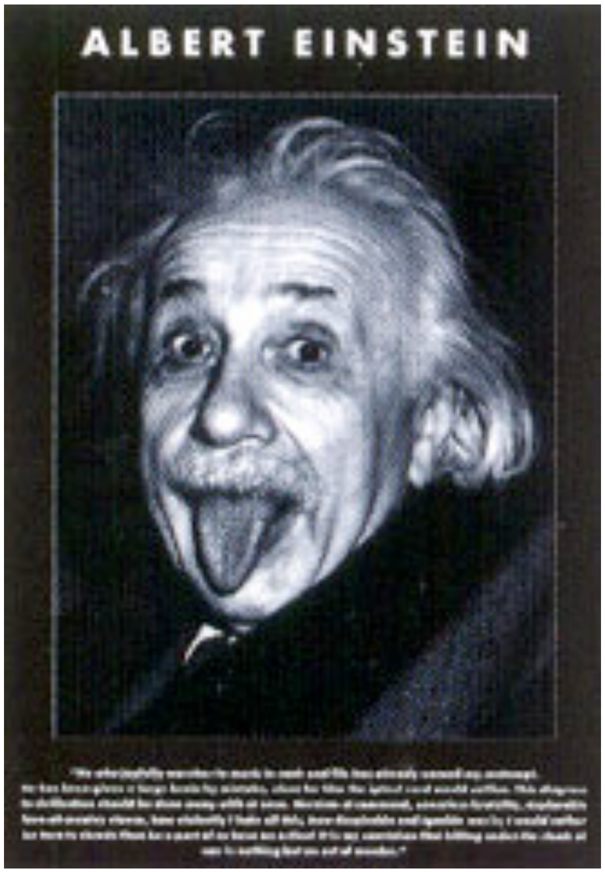
It would appear to Observer O in S frame that velocity of light

$$V_S = c + v > c$$

This contradicts Maxwell's theory of Light !

Are Newton's Laws and Maxwell's laws inconsistent??!!

Einstein's Special Theory of Relativity



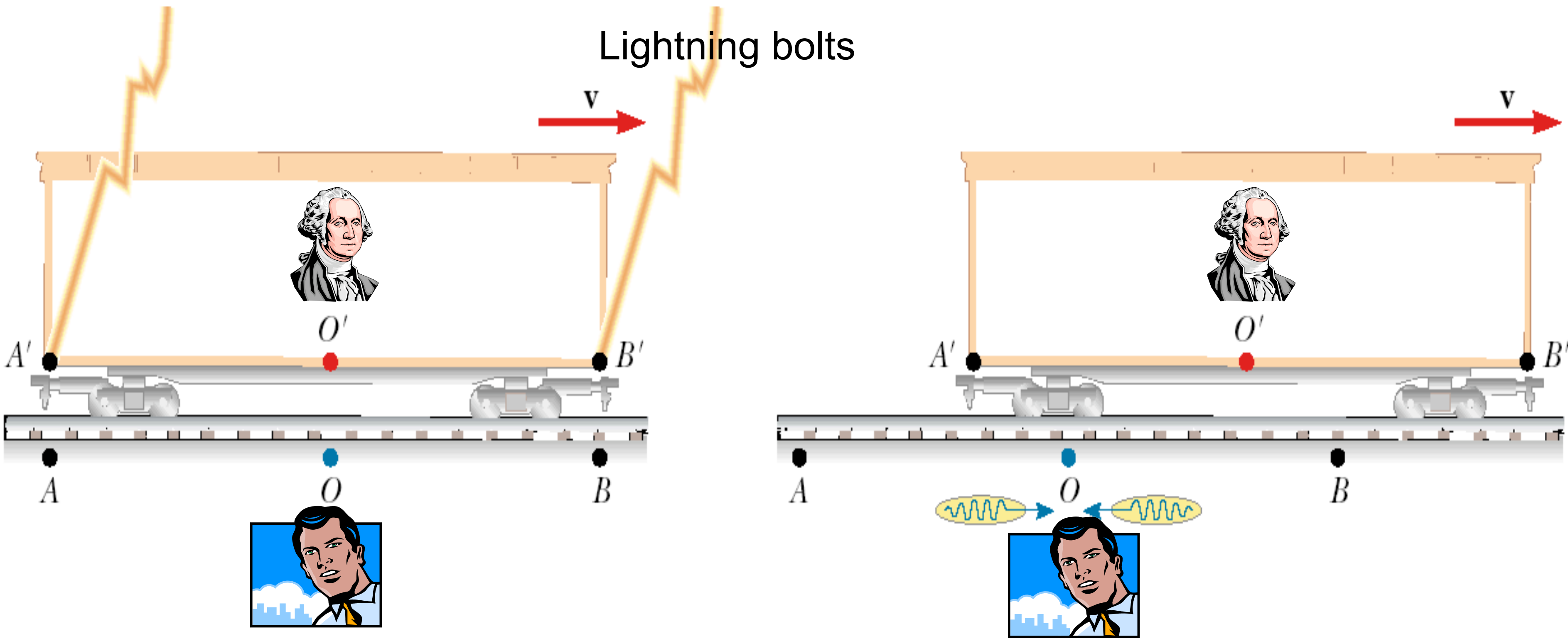
Einstein's Postulates of SR

– The laws of physics must be the same in all inertial reference frames

– The speed of light in vacuum has the same value (c = 3.0 x 10^8 m/s) , in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

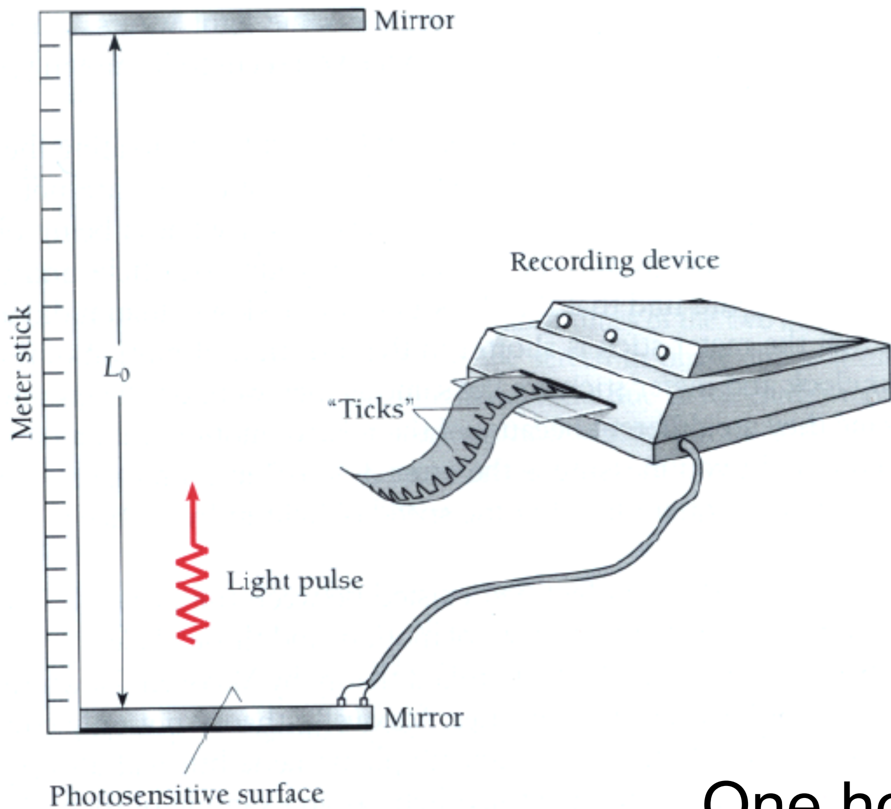
Consequences of Special Relativity: Simultaneity not Absolute

Simultaneity: When two events occur at same time, held absolute for Classical Phys



Events that are simultaneous for one Observer are
not simultaneous for another Observer in relative motion
Simultaneity is not absolute !!
Time interval depends on the Reference frame it is measured in

A Simple Clock Measuring a Time Interval

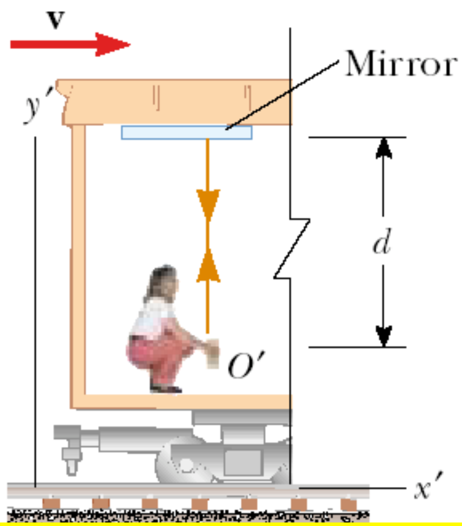


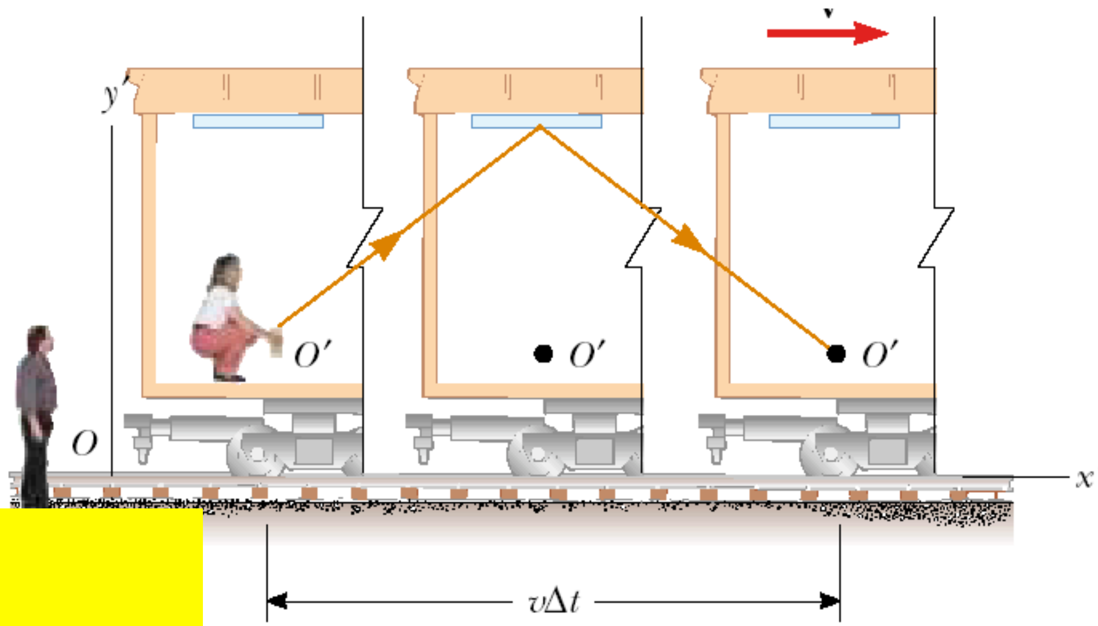
$$t = \int \Delta t$$

<u>One hour = 60 x 1 minute time intervals</u>

Time Dilation and Proper Time

Watching a time interval (between 2 events) with a simple clock





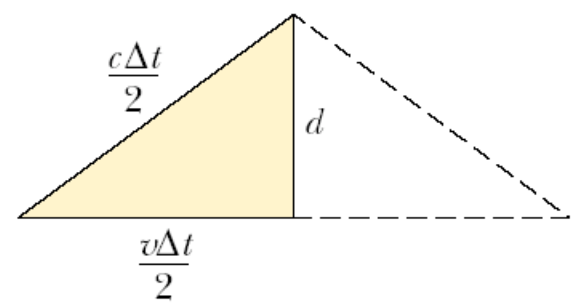
Observer O′ : $\Delta t' = \dfrac{2d}{c}$

Observer O : Apply Pythogoras Theorem

$$\left(\frac{c\Delta t}{2}\right)^2 = (d)^2 + \left(\frac{v\Delta t}{2}\right)^2, \text{ but } d = \left(\frac{c\Delta t'}{2}\right)$$

$$\therefore \quad c^2(\Delta t)^2 = c^2(\Delta t')^2 + v^2(\Delta t)^2$$

$$\therefore \quad \Delta t = \frac{\Delta t'}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma\,\Delta t',\ \Delta t > \Delta t'$$

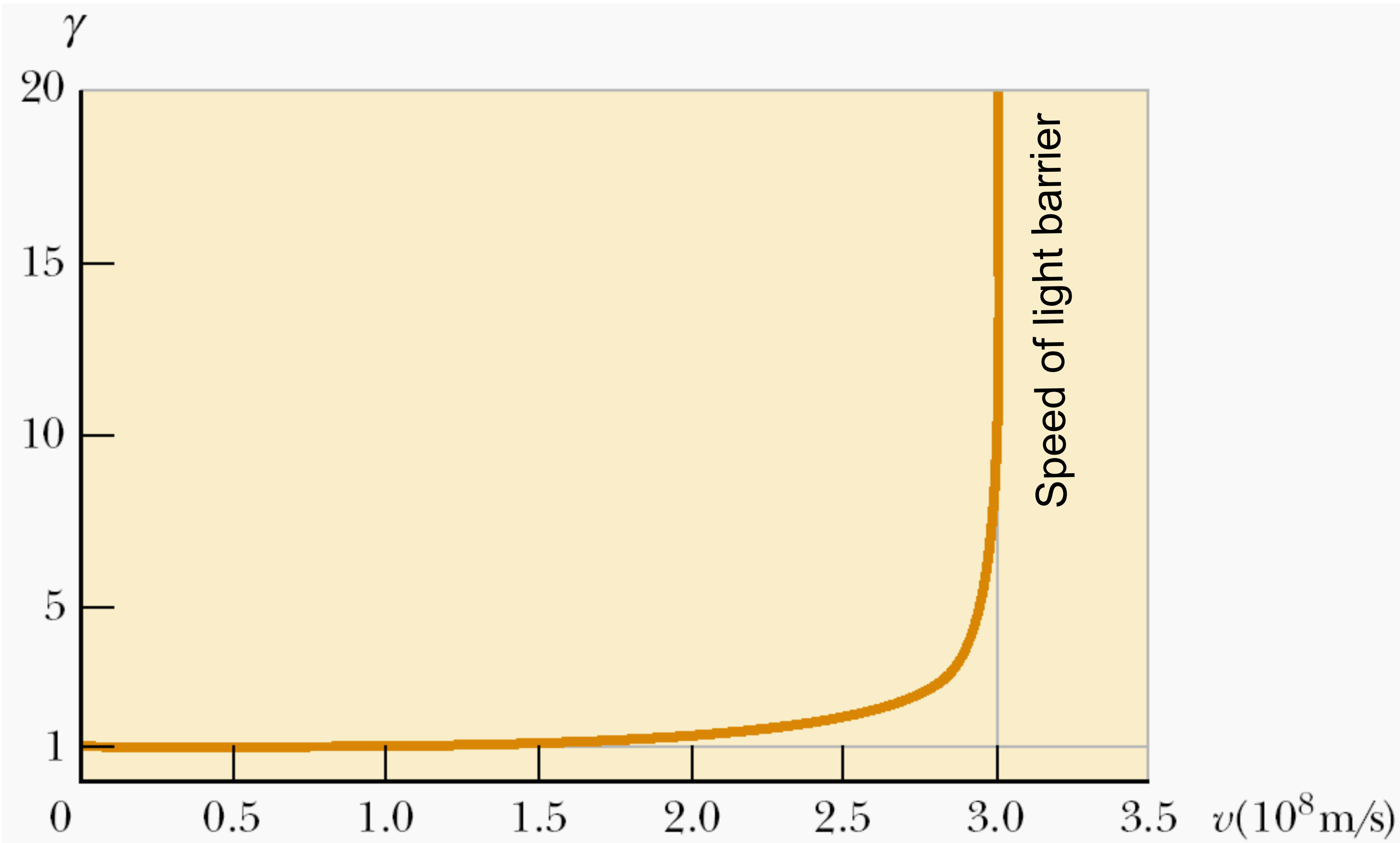


The γ factor

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

as $v \to 0,\ \gamma \to 1$

as $v \to c,\ \gamma \to \infty$



Measuring Time: Period of a Pendulum

- Period of a pendulum is 3.0 s in the rest frame of the pendulum
- What is period of the pendulum as seen by an observer moving at v=0.95c

Answer:

- Proper time T' = 3.0s
- Since motion is relative and time dilation does not distinguish between
    - relative motion →→ (V) from relative motion ← ← (-V)
- lets reformulate the problem like this (??)
    - A pendulum in a rocket is flying with velocity V =0.95c past a stationary observer
    - Moving clocks runs slower [w.r.t clock in observer's hand (rest)] by factor γ
    - → Period T measured by observer = γ T'

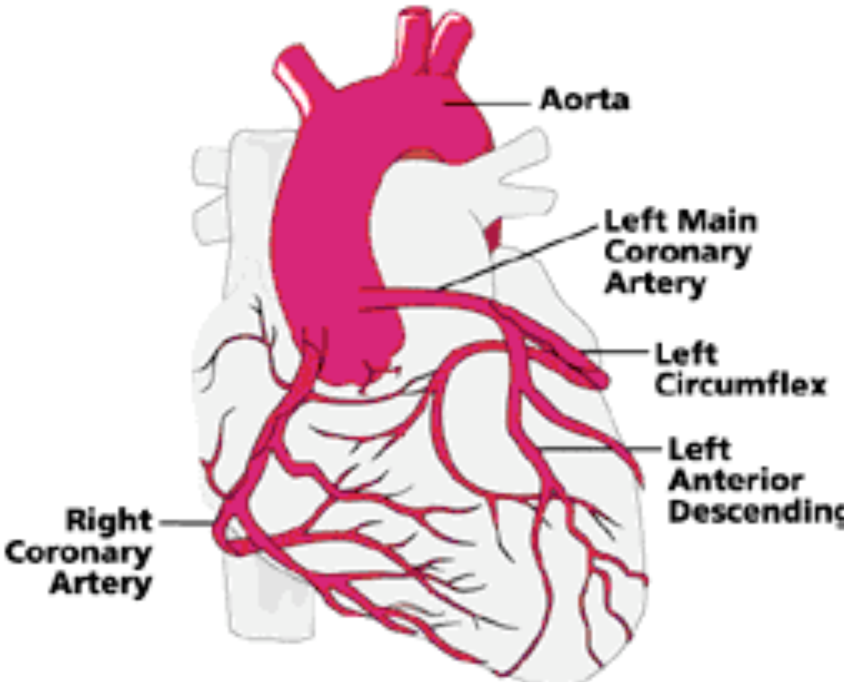
$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.95)^2}} = 3.2$$

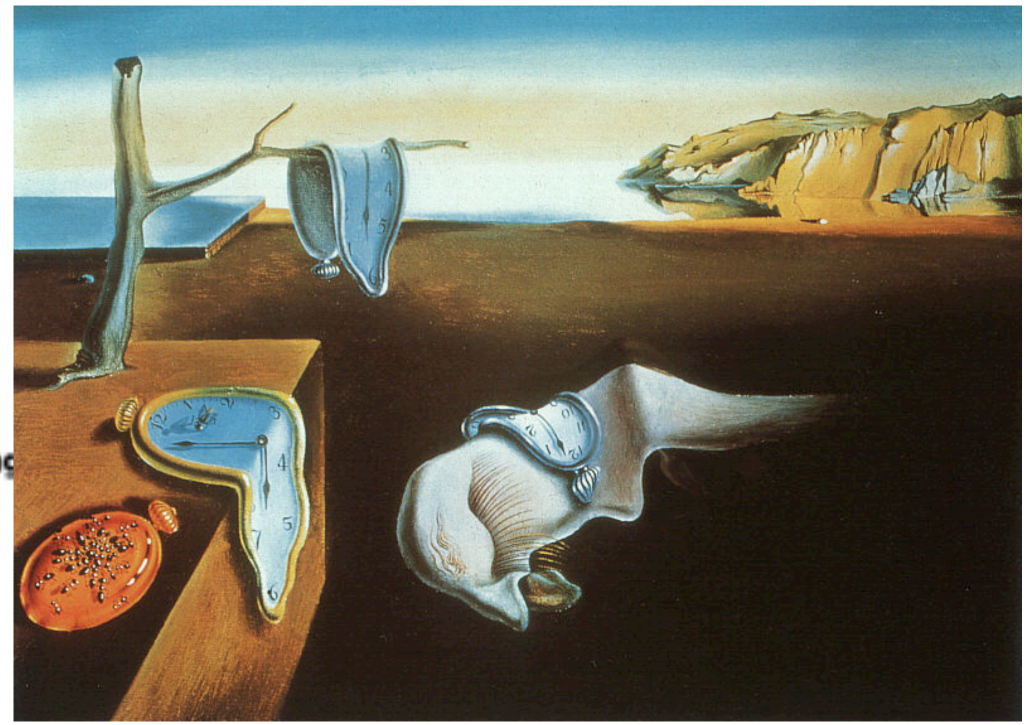
$$\Rightarrow T = \gamma T' = 3.2 \times 3.0s = 9.6s$$

Moving pendulum slows down → takes longer to complete a period

All Measures of Time Slow down from a <u>Moving Observer's</u> Perspective !

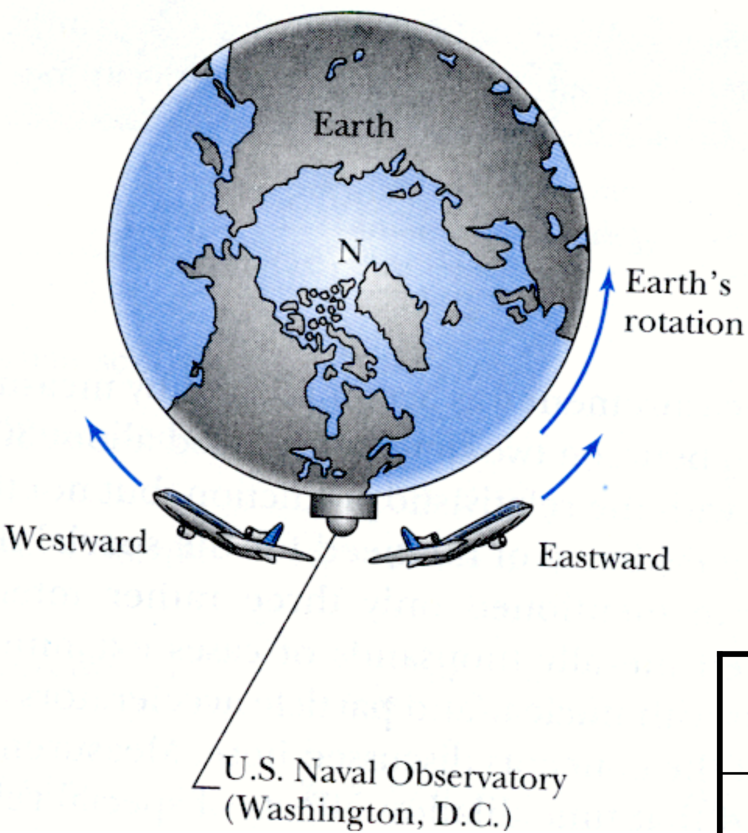
- Your heartbeat or your pulse rate





- …all measures of time interval

Round The World With An Atomic Clock !

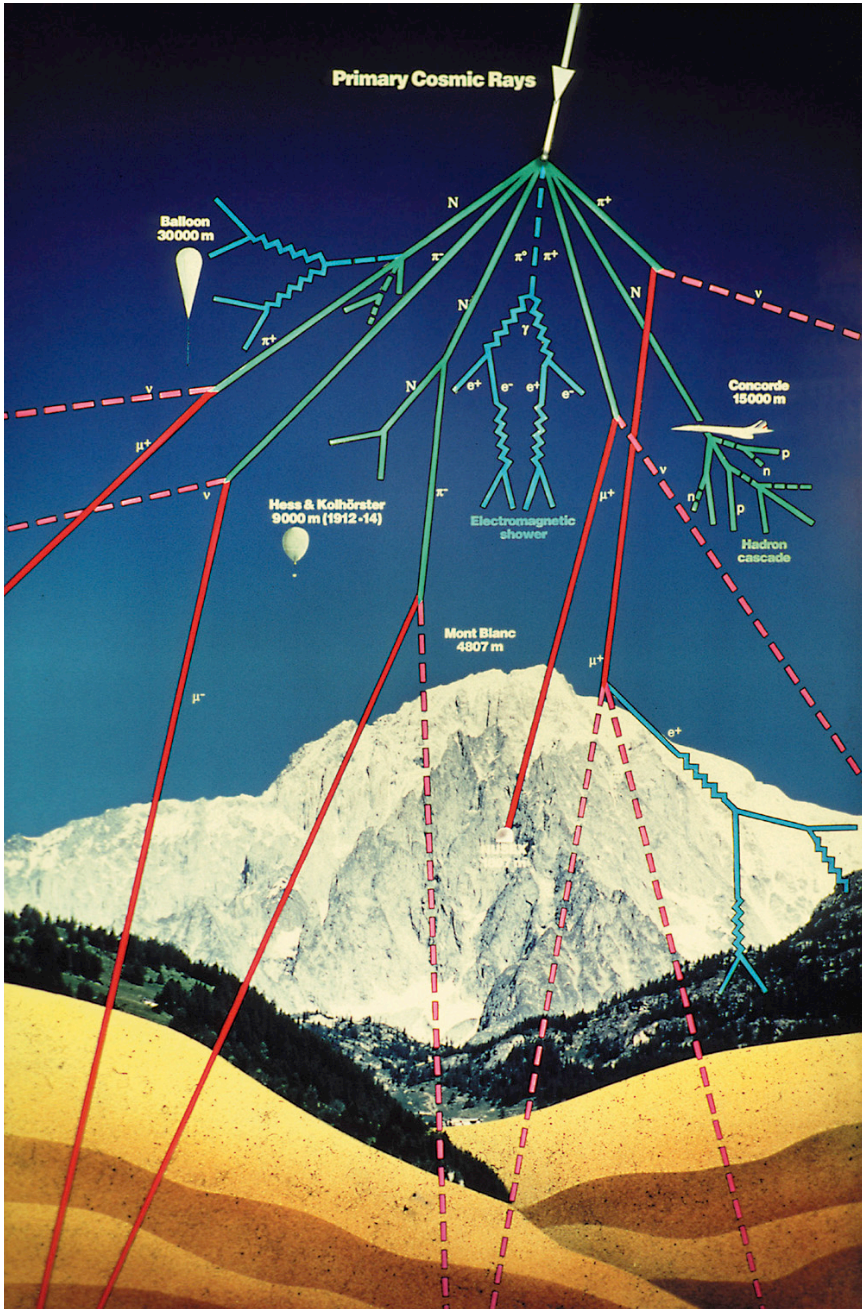


- Atomic Clock : certain atomic level transitions in Cesium atom
- Two planes take off from DC, travel east and west
  - Eastward trip took 41.2 hrs
  - Westward trip took 48.6
- Atomic clocks compared to similar ones kept in DC
- Need to account for Earth's rotation + GR etc

Travel	Predicted	Measured
Eastward	-40 ± 23 ns	-59 ± 10 ns
Westward	275 ± 21 ns	273 ± 7 ns

Flying clock ticked faster or slower than reference clock. Slow or fast is due to Earth's rotation

Cosmic Particles Are Bombarding the Earth



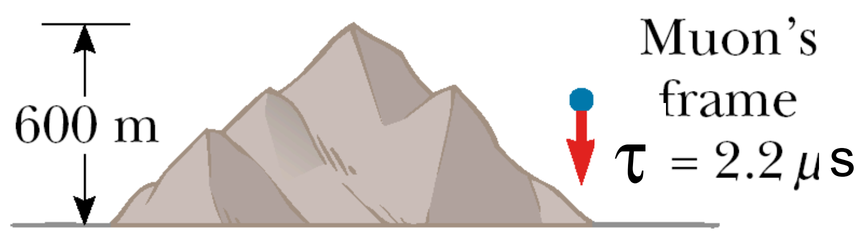
- Cosmic "rays" are messengers from space
- **Produced in violent collisions in the cosmos**
- **Typical Kinetic energy ~ 100 GeV**
- **Smash into Earth's outer atmosphere**
  - **4700 m from sea level**
- **Sometimes produce short lived Muons**

- **Muon is electron like charged particle**
  - **~ 200 times heavier , same charge**
  - **Lifetime $\tau = 2.2\mu s = 2.2 \times 10^{-6}$ s**
  - **Produced with speed $v \equiv c$**
  - **Distance traveled in its lifetime**

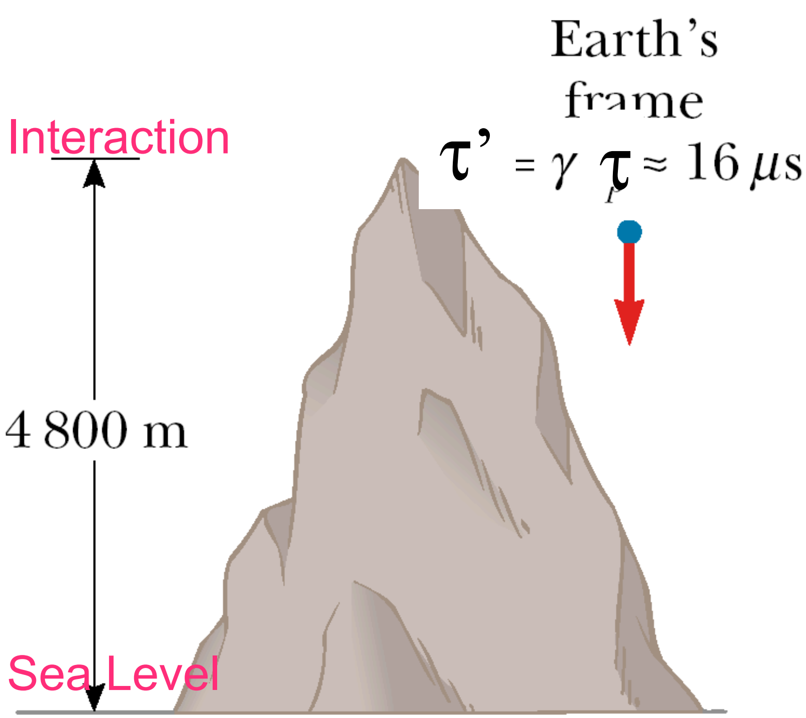
$$d = c\tau = 650m$$

  - **Yet they seem to reach the surface!!**
    - **Why => Time Dilation**
    - **Must pay attention to frames of references involved**

Cosmic Rays Are Falling On Earth : Example of Time Dilation



(a)



- Consider Two frames of references
  1. You Riding on the Muon Particle
  2. Your twin watching On surface of earth

– Muon Rider has "Proper Time"
  – Time measured by observer moving along with clock
  – $\Delta t' = \tau = 2.2\,\mu S$
  – $D' = v\,\Delta t' = 650m$

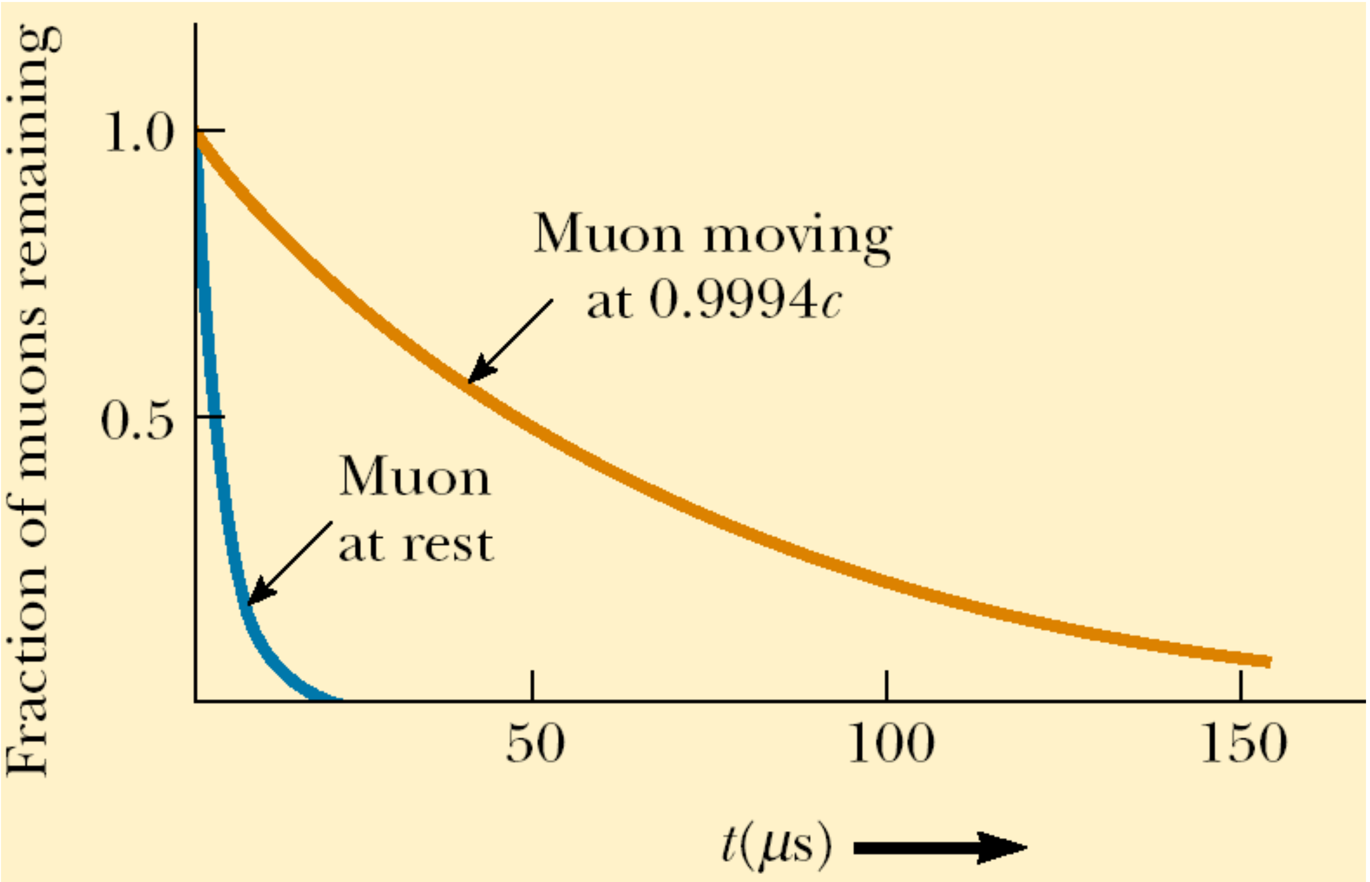
– Earthling watches a moving clock (muon's) run slower
  – $\Delta t' = \gamma\tau$
    – $v = 0.99c$, => $\gamma = 7.1$
    – $D = v\,\Delta t = 4700m$

Muon Decay Distance Distribution

Relative to Observer on Earth Muons have a lifetime

$$t = \gamma\tau = 7.1\ \tau$$

Exponential Decay time Distribution : As in Radioactivity



Pop Quiz !

- What happens when I reverse the clocks being watched ?
  - Sally now watches Sam's clock
  - Sally is moving w.r.t. Sam's clock. Sam is at rest w.r.t the clock.
  - What does she make of time intervals as measured by his clock ?

Synchronizing Clocks

After coincidence of their origins at t=0, t′ = 0
Sam and Sally agree to send light signals to each other after time t on their respective clocks. Let t_1 = time on Sam's clock at which he would receive signal if clocks ran at same time and t_2 be actual time he received it.
Then,

$$t_1 = t(1 + v / c)$$

$$t_2 = \gamma t(1 + v / c) = t\sqrt{\frac{1 + v / c}{1 - v / c}}$$

Let t'_1 = time on Sally's clock at which she would receive signal if clocks ran at same time and t'_2 be actual time she received it. According to Sam she should get it at $(t + t_0)$ where

$c\, t_0 = vt + vt_0$ or

$t_0 = [v/(c-v)]\, t$ so

$t'_1 = t\,[1 + v/(c-v)\,] = t/(1 - v/c)$

But Sally's clock shows $t'_2 = t'_1/\gamma = \quad t\sqrt{\dfrac{1+v/c}{1-v/c}}$

So each of them sees the others clock slow down!

Jack and Jill's Excellent Adventure: Twin Paradox

Jack & Jill are <u>20 yr old twins</u>, with same heartbeat
Jack takes off with V = 0.8c = (4/5)c to a star
20 light years Away.

Jill stays behind, watches Jack by telescope. They
Eventually compare notes

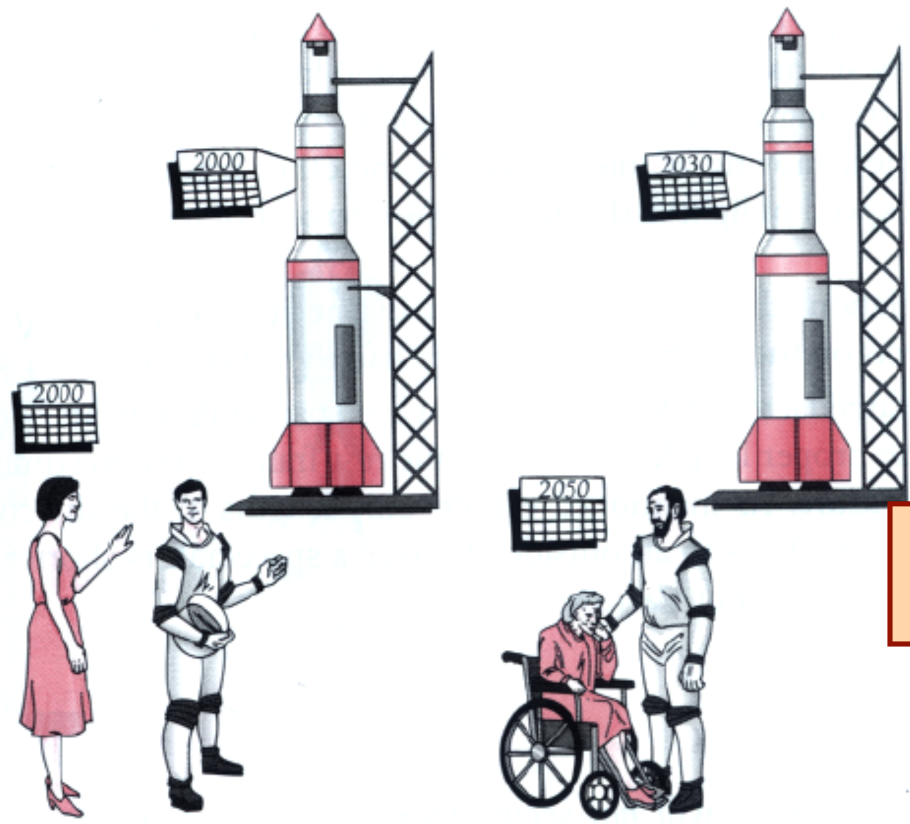
Jill sees <u>Jack's heart</u> slow down
Compared to her by the factor :

$$\sqrt{1-(v/c)^2} = \sqrt{1-(0.8c/c)^2} = 0.6$$

For every 5 beats of her heart
She sees Jack's beat only 3 !

Jack has only 3 thoughts for 5 that
Jill has ! …..Every things slows!

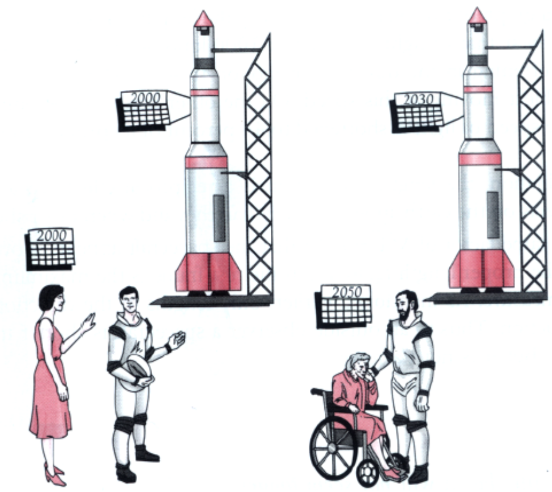
Finally Jack returns after 50 yrs
gone by according to Jill's calendar



Only 30 years have gone by Jack's calendar
SO Jack is 50 years old but Jill is 70 !

Where is the paradox ??

Twin Paradox ?



- Paradox : Turn argument around, motion is relative. Look at Jack's point of view !
- Jack claims he at rest, Jill is moving v=0.8c
- Should not Jill be 50 years old when 70 year old Jack returns from space Odyssey?

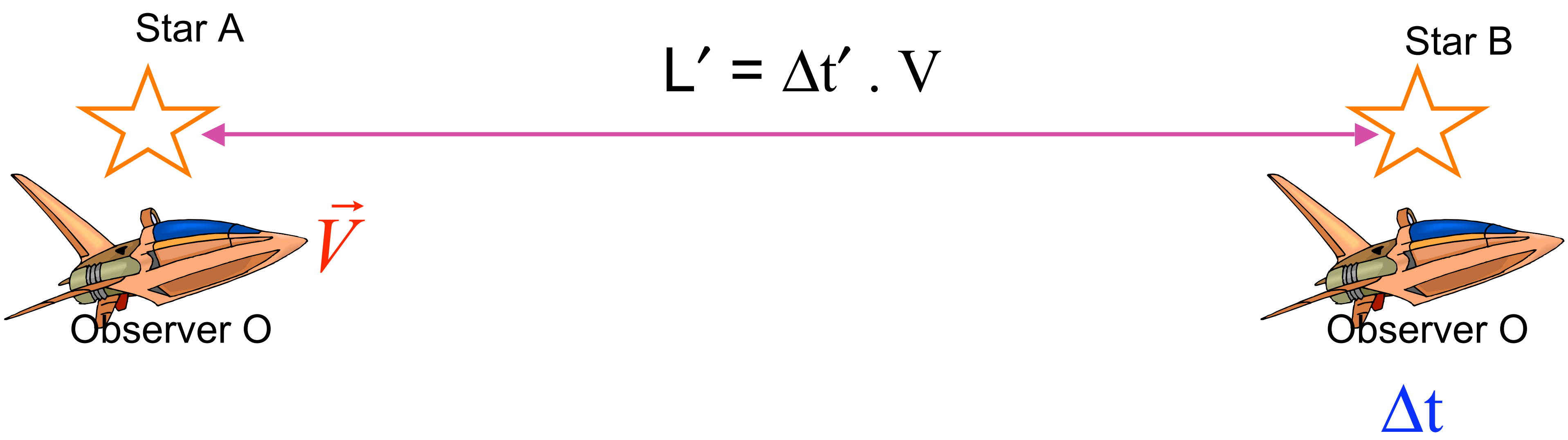
No ! …because Jack is not always traveling in a inertial frame of reference

TO GET BACK TO EARTH HE HAS TO TURN AROUND => decelerate/accelerate

But Jill always remained in Inertial frame
Time dilation formula valid for Jill's observation of Jack but not to Jack's observation of Jill !!….remember this always

Non-symmetric aging verified with atomic clocks taken on airplane trip around world and compared with identical clock left behind. Observer who departs from an inertial system will always find its clock slow compared with clocks that stayed in the system

Offsetting Penalty : Length Contraction



$$\Delta t = L/V$$

Observer O'

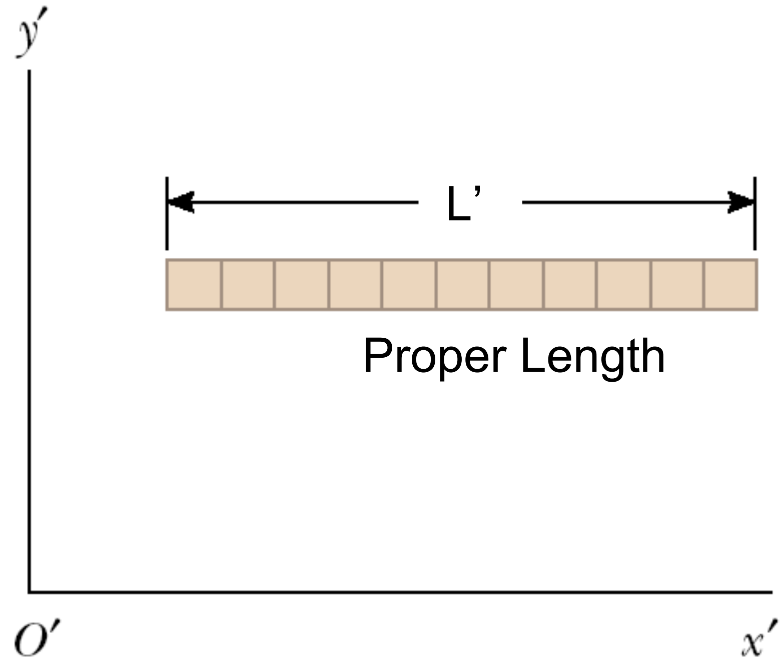
At rest w.r.t stars A & B

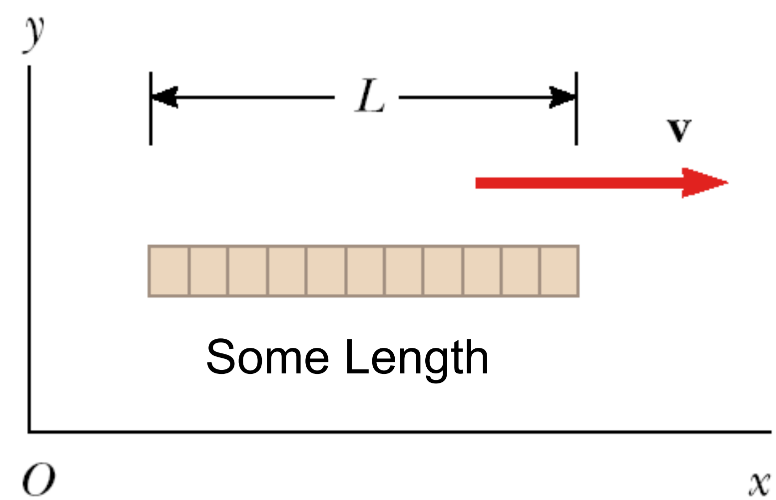
Watches rocketship cross from Star A to Star B in time Δt

Rocketman Vs The Earthling

- Earth Observer saw rocketman take time $\Delta t = (L/V)$
- Rocketman says he is at rest, Star B moving towards him with speed V from right passed him by in time $\Delta t'$, so
  - $L' = \Delta t'. V$
  - But $\Delta t' = \Delta t/\gamma$ (time dilation)
  - $\Rightarrow L' = V\Delta t/\gamma$

    $= L/\gamma$





$$L' = L.\sqrt{1-\frac{V^2}{c^2}}$$

$$L' \leq L$$

Moving Rods Contract in direction Of relative motion

Immediate Consequences of Einstein's Postulates: Recap

- Events that are simultaneous for one Observer are not simultaneous for another Observer in relative motion
- Time Dilation : Clocks in motion relative to an Observer appear to slow down by factor γ
- Length Contraction : Lengths of Objects in motion appear to be contracted in the direction of motion by factor γ^{-1}
- New Definitions :
  - Proper Time (who measures this ?)
  - Proper Length (who measures this ?)
  - Different clocks for different folks !