Physics 2D Lecture Slides
Lecture 14 Feb. 8, 2010

Sunil Sinha
UCSD Physics

Bohr's Explanation of Hydrogen like atoms

- Bohr's Semiclassical theory explained some spectroscopic data → Nobel Prize : 1922
- The "hodge-podge" of classical & quantum attributes left many (Einstein) unconvinced
  - "appeared to me to be a miracle – and appears to me to be a miracle today ...... One ought to be ashamed of the successes of the theory"
- Problems with Bohr's theory:
  - Failed to predict INTENSITY of spectral lines
  - Limited success in predicting spectra of Multi-electron atoms (He)
  - Failed to provide "time evolution " of system from some initial state
  - Overemphasized Particle nature of matter-could not explain the wave-particle duality of light
  - No general scheme applicable to non-periodic motion in subatomic systems
- Without fundamental insight …raised the question : Why was Bohr successful?

Prince Louis de Broglie & Matter Waves

- Key to Bohr atom was Angular momentum quantization
- Why this Quantization: $mvr = |L| = nh/2\pi$?
- Invoking symmetry in nature, Louis de Broglie conjectured:

Because photons have wave and particle like nature → particles may have wave like properties !!

Electrons have accompanying "pilot" wave (not EM) which guide particles thru spacetime

A PhD Thesis Fit For a Prince

- Matter Wave !
  - **"Pilot wave" of $\lambda = h/p = h / (\gamma mv)$**
  - **frequency $f = E/h$**

- **Consequence:**
  - **If matter has wave like properties then there would be interference (destructive & constructive)**
    - **Use analogy of standing waves on a plucked string to explain the quantization condition of Bohr orbits**

Matter Waves : How big, how small

1.Wavelength of baseball, m=140g, v=27m/s

$$\lambda = \frac{\mathrm{h}}{\mathrm{p}} = \frac{h}{mv} = \frac{6.63\times10^{-34}\,J.s}{(.14kg)(27m/s)} = 1.75\times10^{-34}\,m$$

$$\Rightarrow \quad \boxed{\lambda_{baseball} <<< \text{ size of nucleus}}$$

$$\Rightarrow \quad \text{Baseball "looks" like a particle}$$

2. Wavelength of electron K=120eV (assume NR)

$$\mathrm{K} = \frac{\mathrm{p}^2}{2\mathrm{m}} \Rightarrow p = \sqrt{2mK}$$

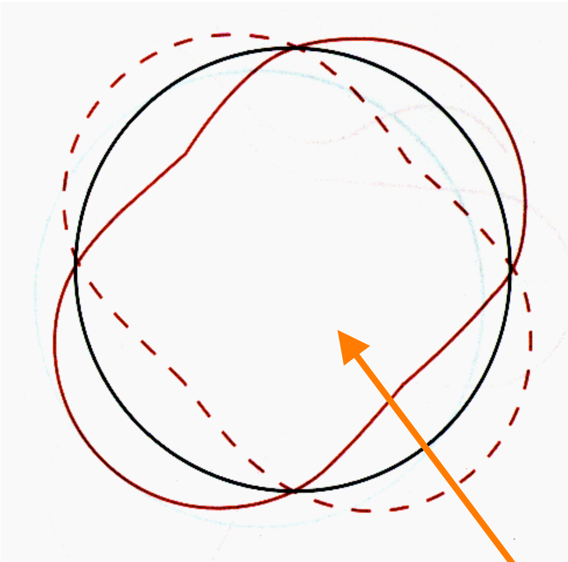
$$= \sqrt{2(9.11\times10^{-31})(120eV)(1.6\times10^{-19})}$$

$$= 5.91\times 10^{-24}\,Kg.m/s$$

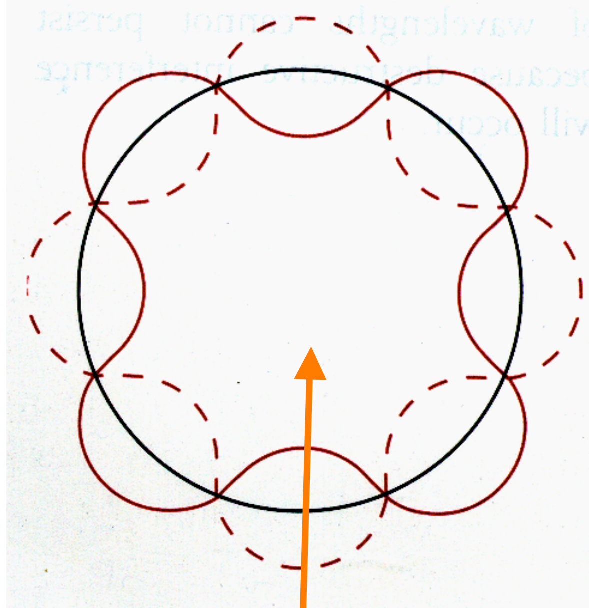
$$\lambda_e = \frac{h}{p} = \frac{6.63\times10^{-34}\,J.s}{5.91\times10^{-24}\,kg.m/s} = 1.12\times10^{-10}\,m$$

$$\Rightarrow \quad \boxed{\lambda_e \simeq \text{ Size of atom}} \text{ !!}$$

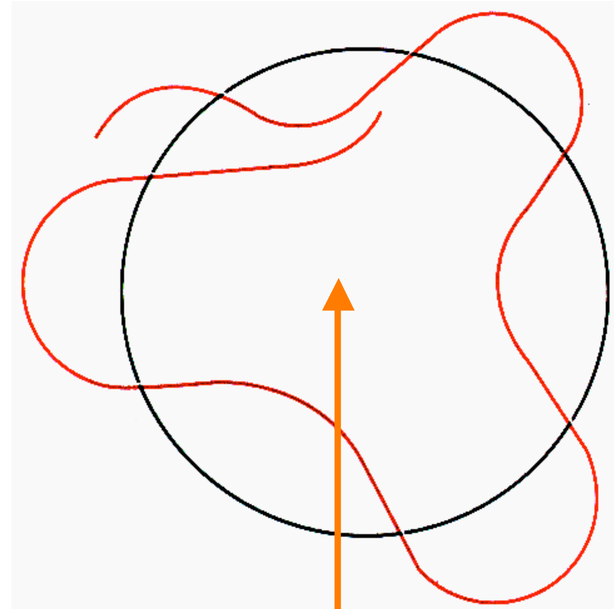
Models of Vibrations on a Loop: Model of e in atom

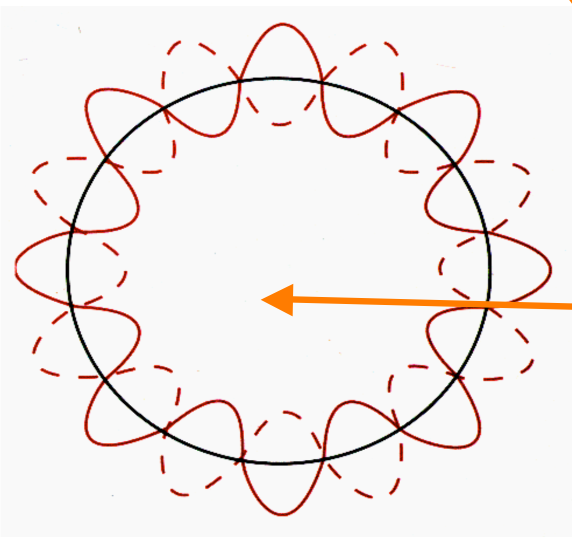


Circumference = 2 wavelengths



Circumference = 4 wavelengths





Circumference = 8 wavelengths

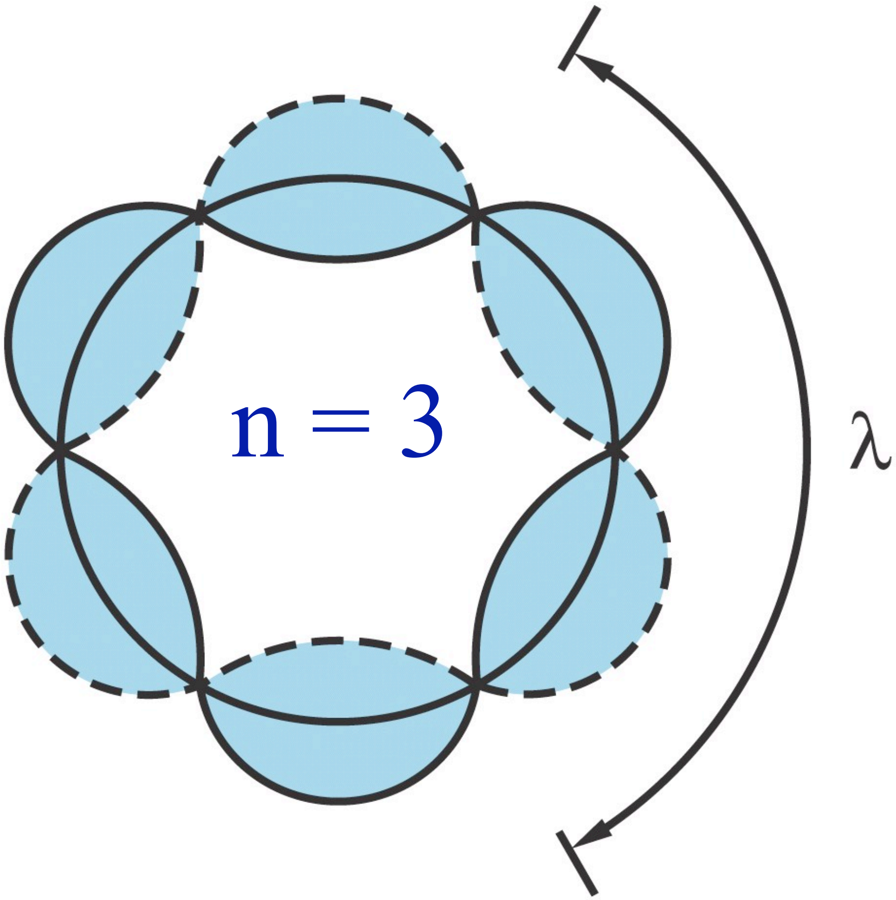
Modes of vibration when a integral # of λ fit into loop

(Standing waves)

vibrations continue Indefinitely

Fractional # of waves in a loop can not persist due to destructive interference

De Broglie's Explanation of Bohr's Quantization



Standing waves in H atom:

Constructive interference when

$n\lambda = 2\pi r$

since $\lambda = \frac{h}{p} = \frac{h}{mv}$ ......(NR)

$\Rightarrow \frac{nh}{mv} = 2\pi r$

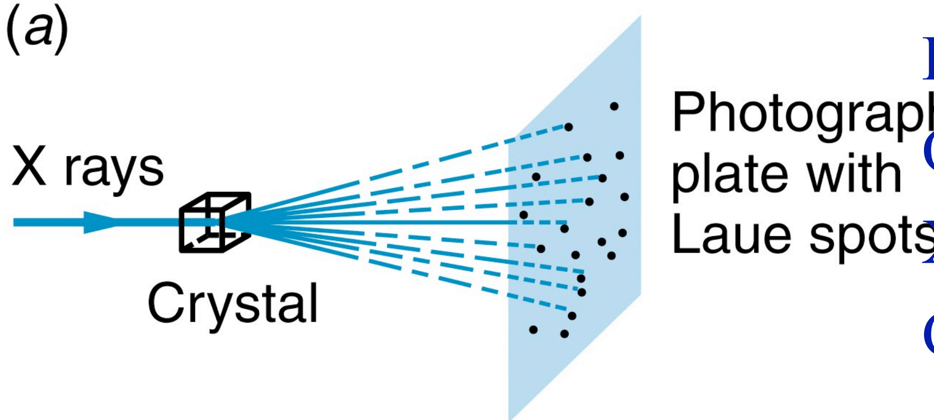
$\Rightarrow \boxed{n\hbar = mvr}$

Angular momentum

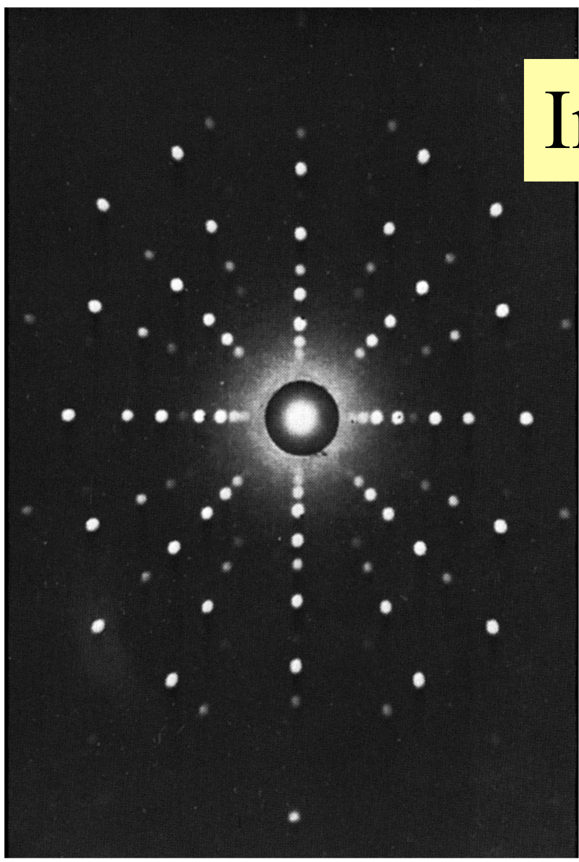
Quantization condition!

This is too intense ! Must verify such "loony tunes" with experiment

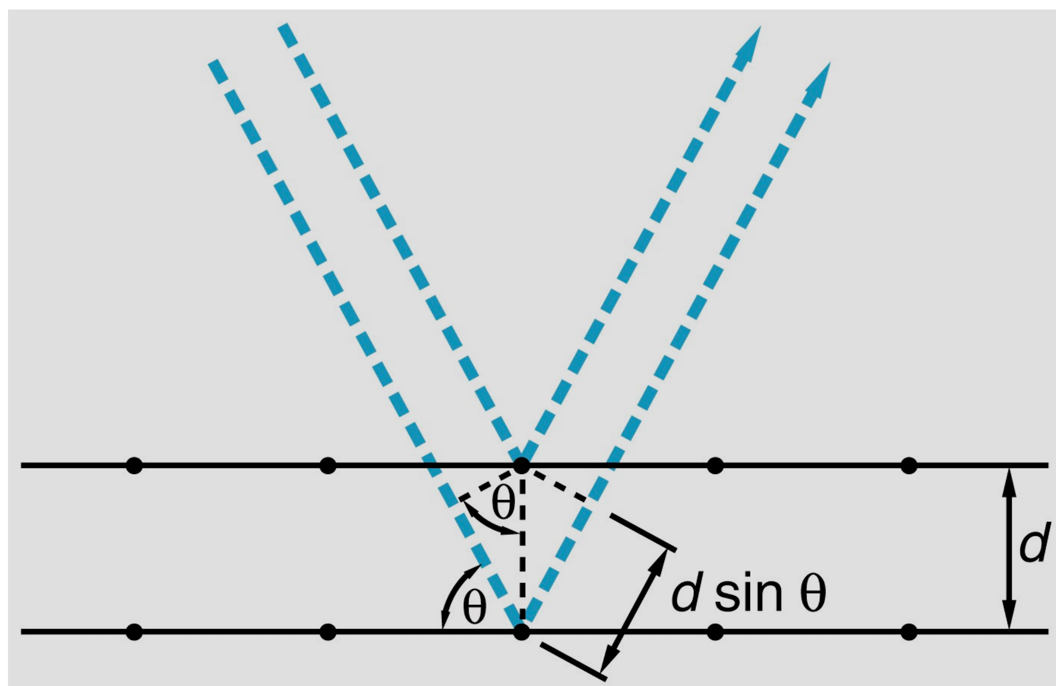
Reminder: Light as a Wave : Bragg Scattering Expt



Range of X-ray wavelengths scatter
Off a crystal sample
X-rays constructively interfere from
Certain planes producing bright spots



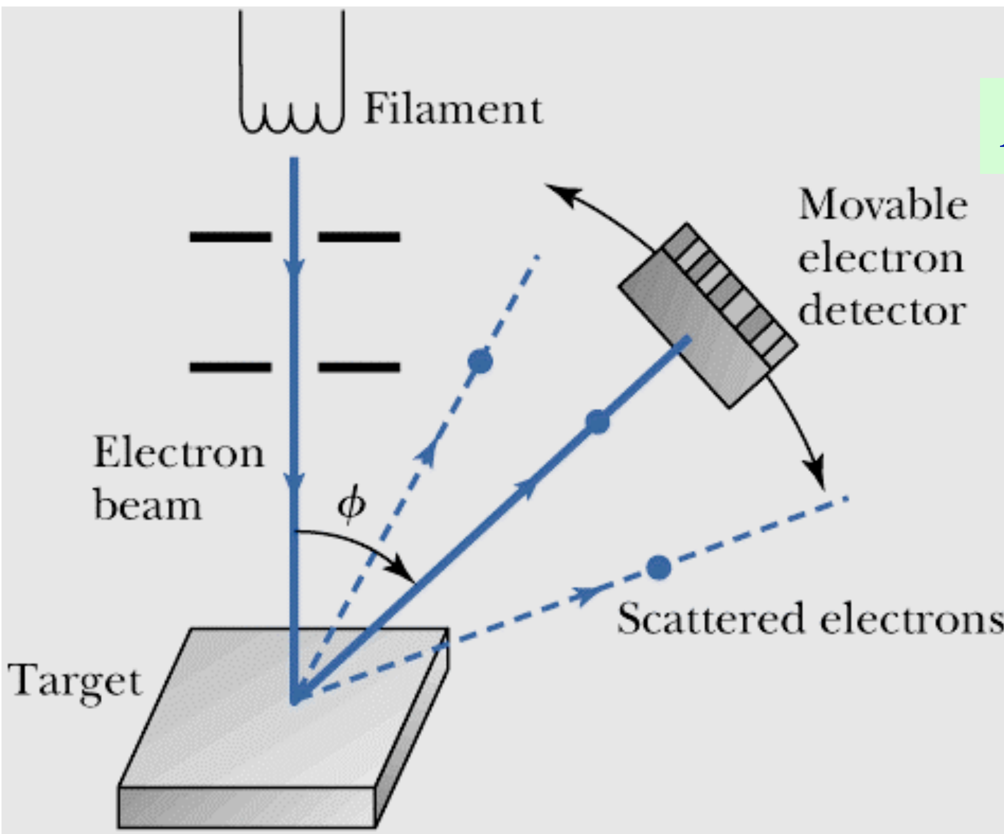
Interference → Path diff $= 2d\sin\vartheta = n\lambda$



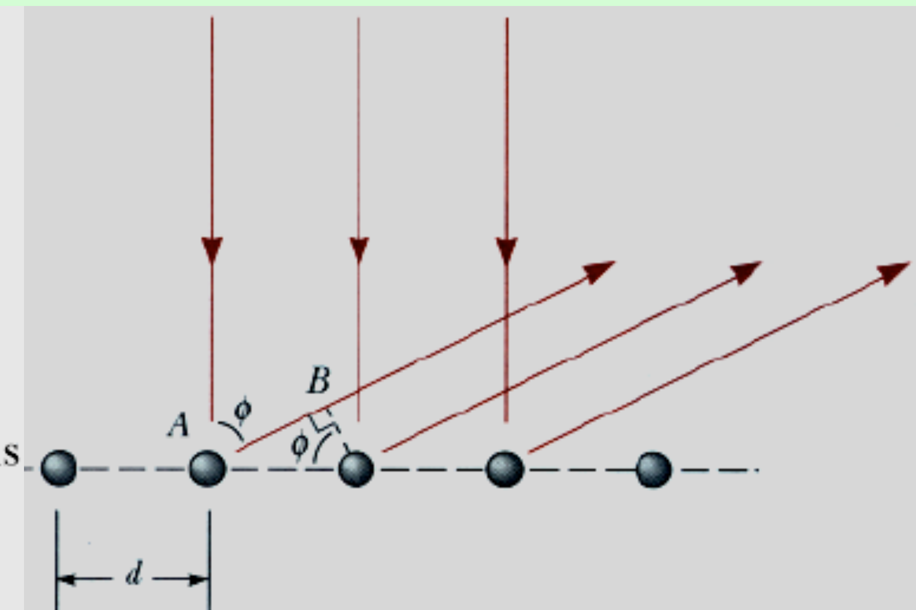
Verification of Matter Waves: Davisson & Germer Expt

If electrons have associated wave like properties → expect interference pattern when incident on a layer of atoms (reflection diffraction grating) with inter-atomic separation d such that

$$\text{path diff AB} = d\sin\varphi = n\lambda$$

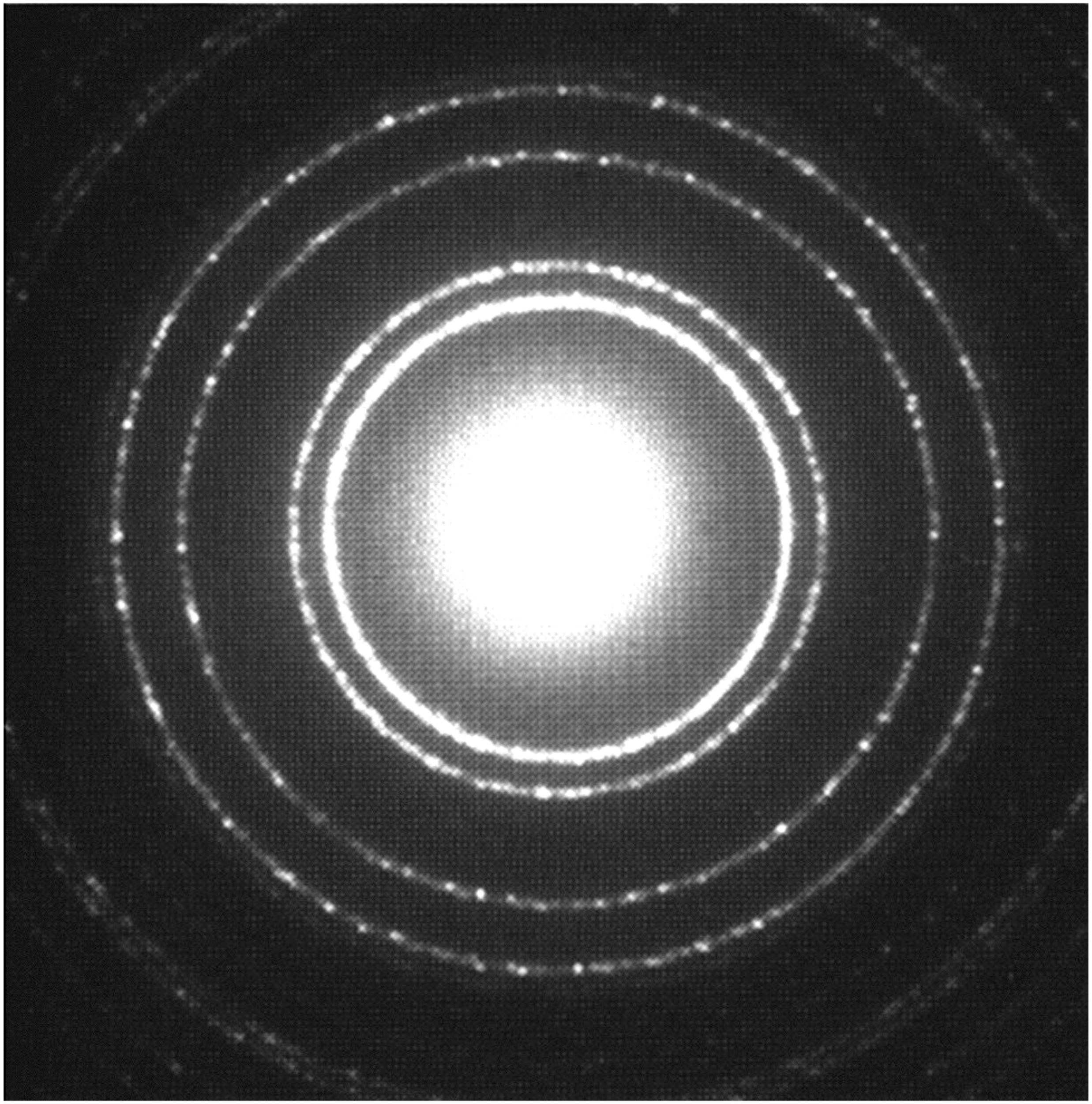


Atomic lattice as diffraction grating



Layer of Nickel atoms

Electrons Diffract in Crystal, just like X-rays

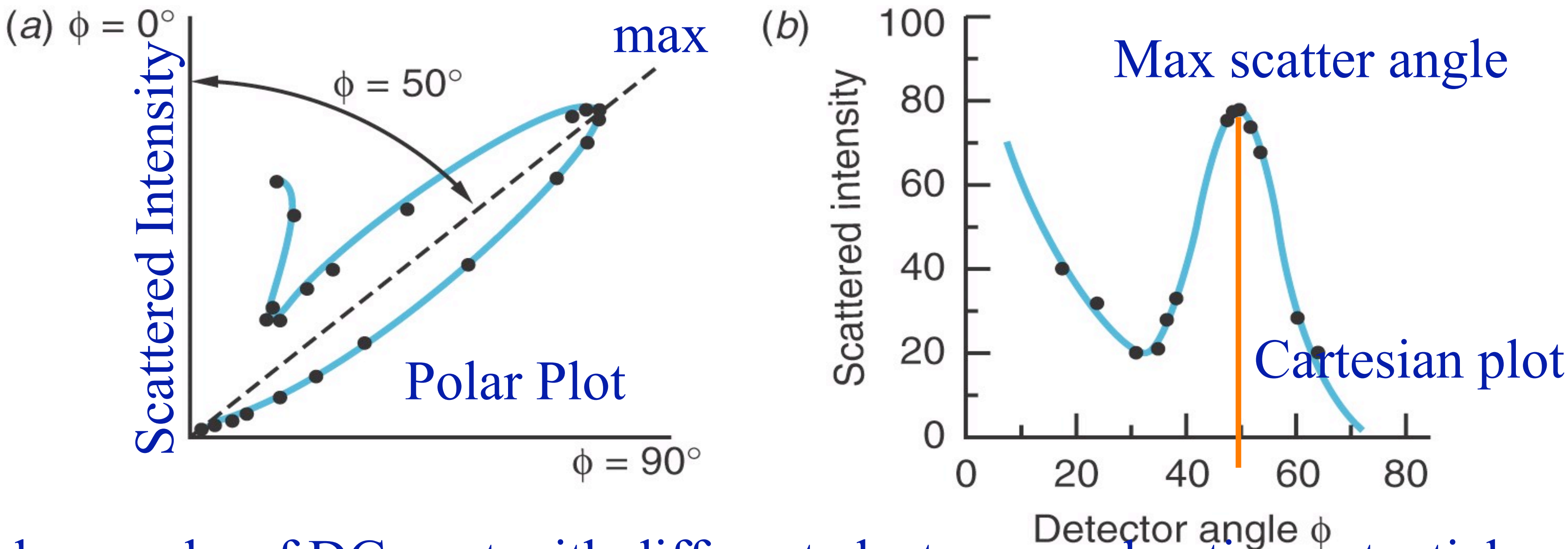


Diffraction pattern produced by 600eV electrons incident on a Al foil target

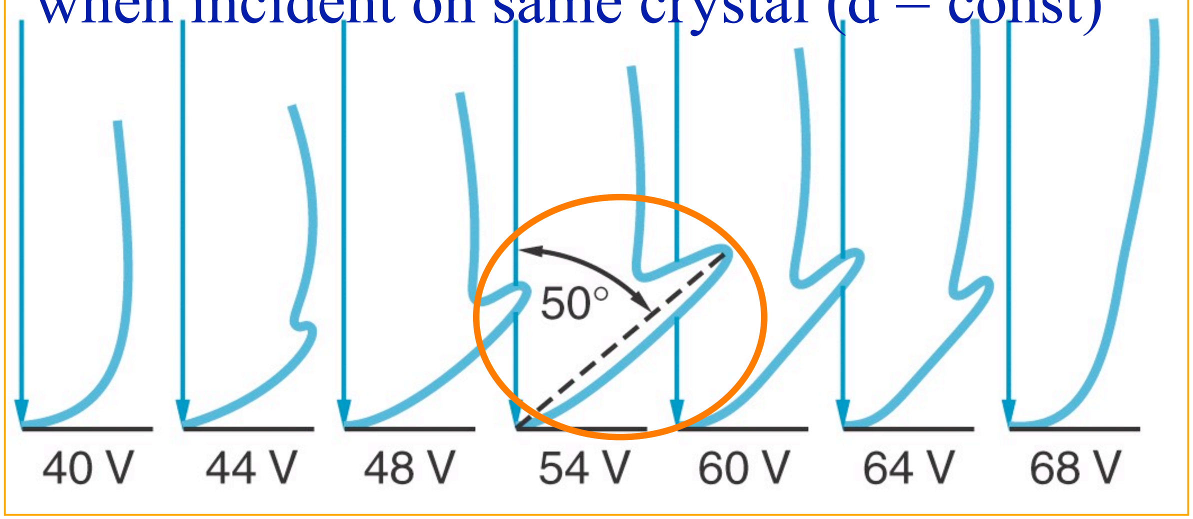
Notice the waxing and waning of scattered electron Intensity.

What to expect if electron had no wave like attribute

Davisson-Germer Experiment: 54 eV electron Beam



Polar graphs of DG expt with different electron accelerating potential when incident on same crystal (d = const)



Peak at $\Phi = 50^o$ when $V_{acc} = 54$ V

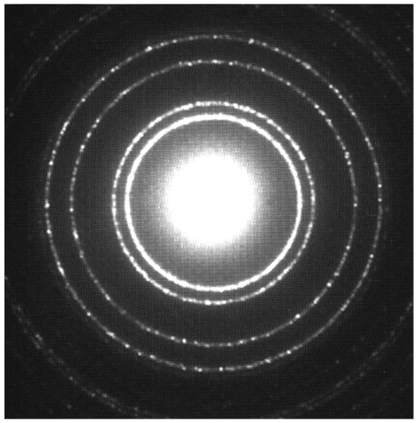
Analyzing Davisson-Germer Expt with de Broglie idea

de Broglie λ for electron accelerated thru V_{acc}=54V

- $\frac{1}{2}mv^2 = K = \frac{p^2}{2m} = eV \Rightarrow v = \sqrt{\frac{2eV}{m}}$; $p = mv = m\sqrt{\frac{2eV}{m}}$

If you believe de Broglie

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \boxed{\frac{h}{\sqrt{2meV}} = \lambda^{predict}}$$



For V_{acc} = 54 Volts $\Rightarrow \lambda = 1.67\times10^{-10}\,m$ (de Broglie)

Exptal data from Davisson-Germer Observation:

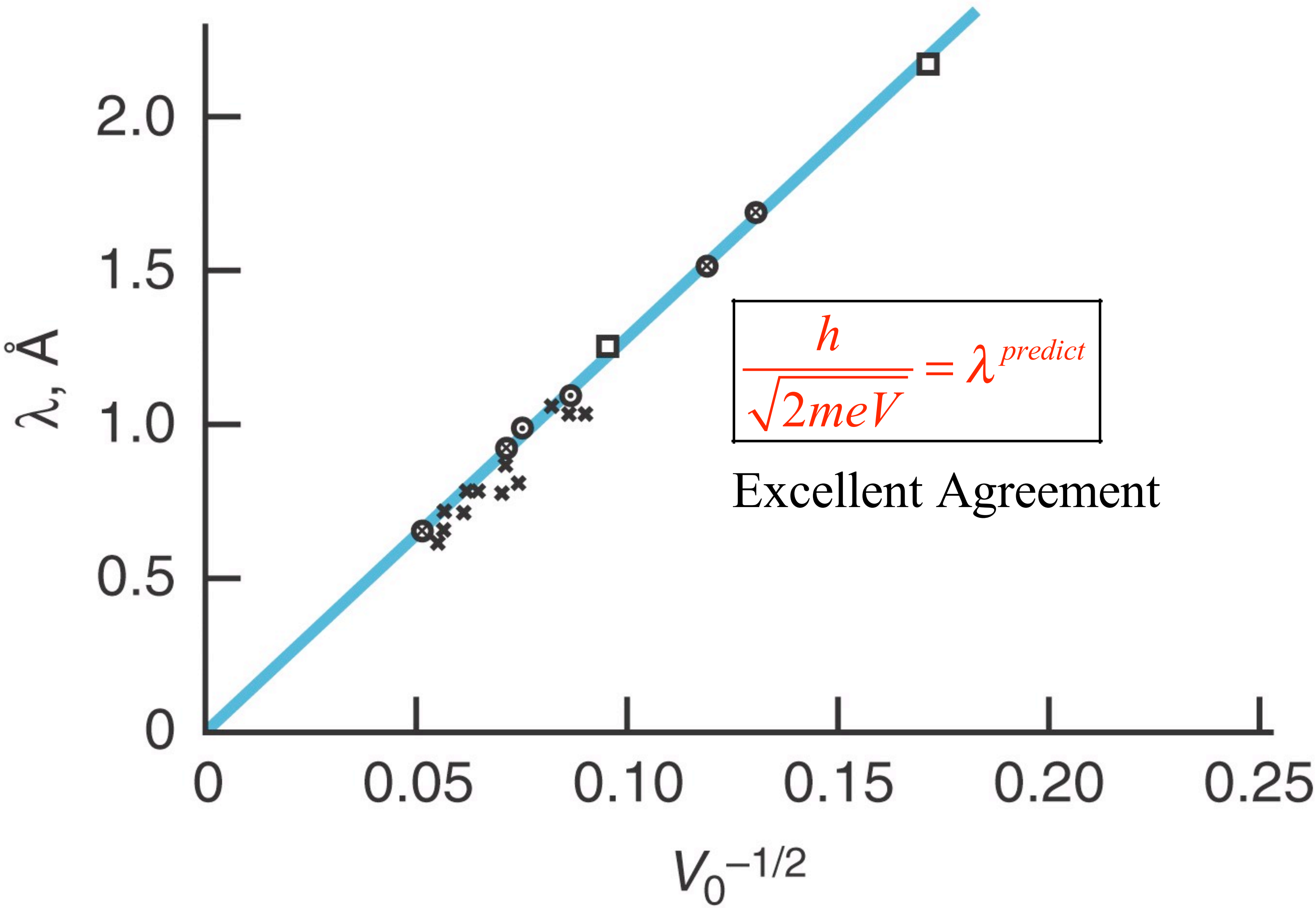
d_{nickel}=2.15 Å =$2.15\times10^{-10}\,m$ (from Bragg Scattering)

$\theta_{diff}^{max} = 50^o$ (observation from scattering intensity plot)

$$\boxed{\text{Diffraction Rule : d } \sin\phi = n\lambda}$$

$\Rightarrow$ For Principal Maxima (n=1); λ^{meas}=(2.15 Å)(sin 50°) $\boxed{\begin{array}{l}\lambda^{predict} = 1.67\,\dot{A} \\ \lambda^{observ} = 1.65\,\dot{A}\end{array}}$ *Excellent* agreement

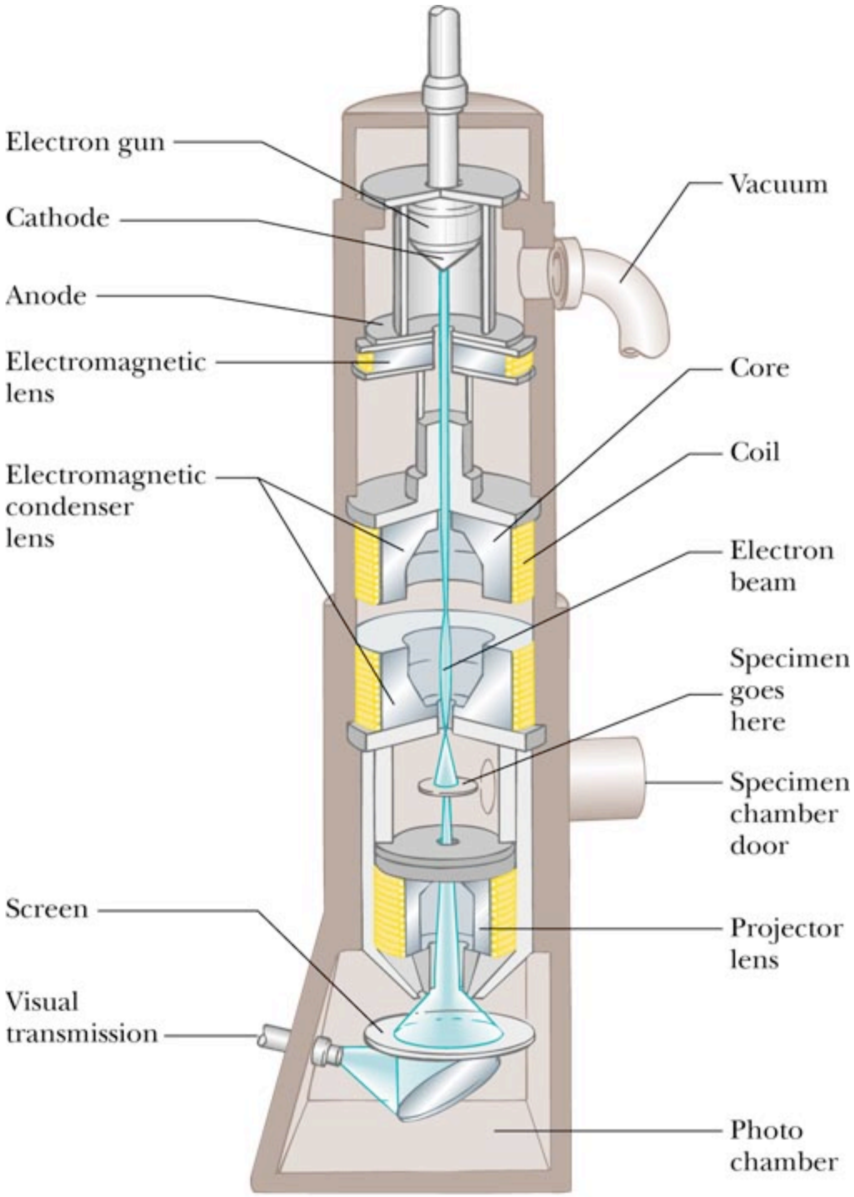
Davisson Germer Experiment: Matter Waves !

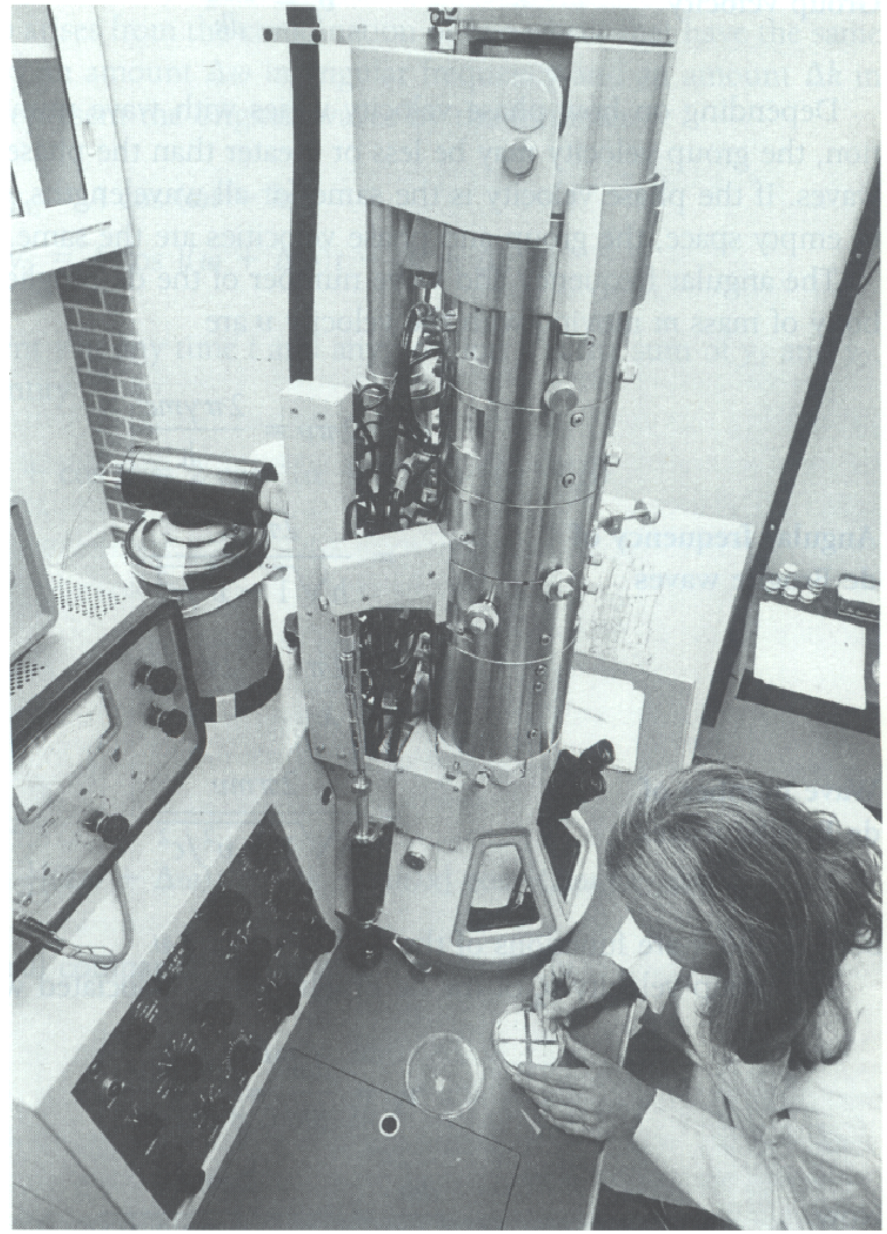


$$\frac{h}{\sqrt{2meV}} = \lambda^{predict}$$

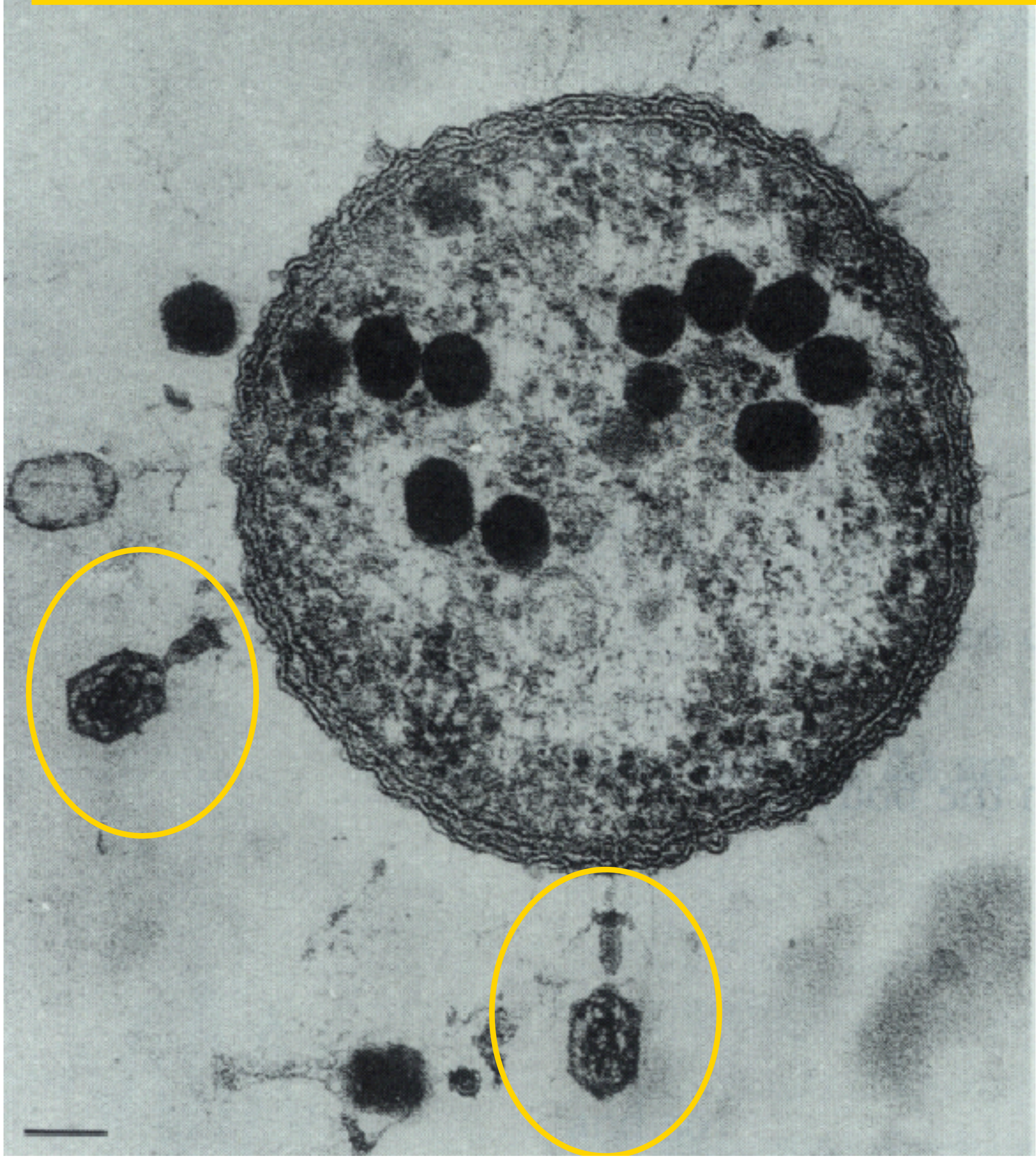
Excellent Agreement

Practical Application : Electron Microscope





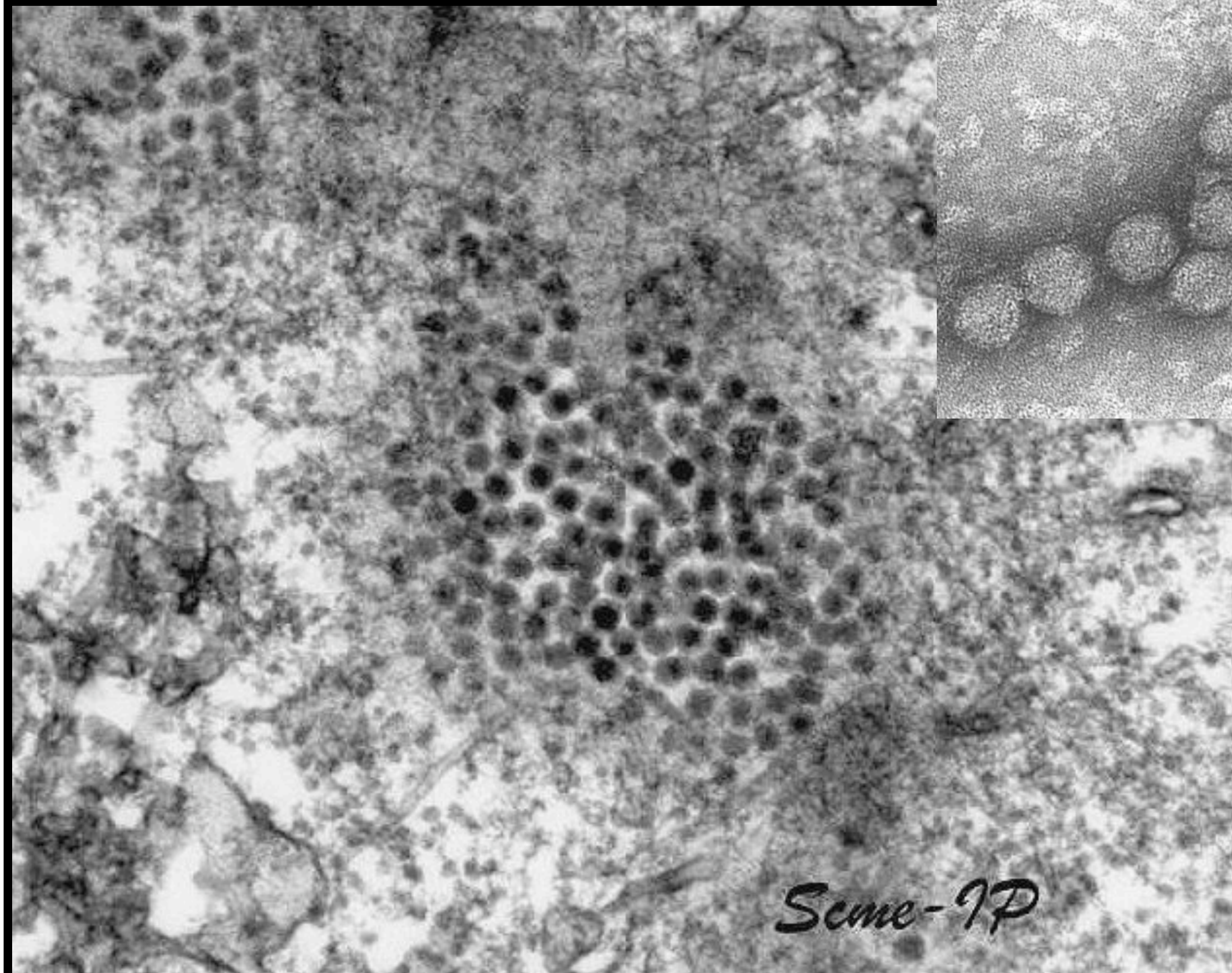
Electron Microscope : Excellent Resolving Power

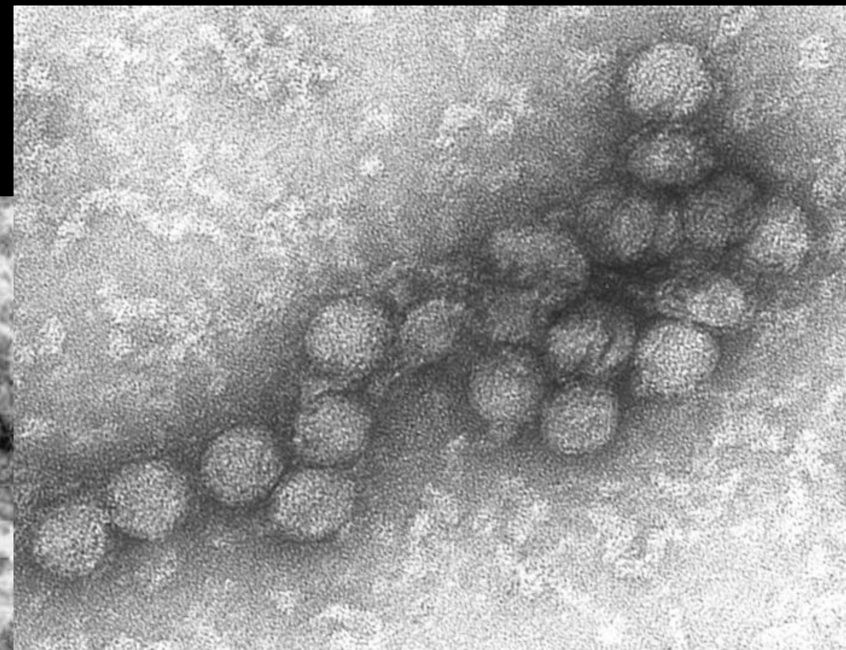


Electron Micrograph
Showing Bacteriophage
Viruses in E. Coli bacterium

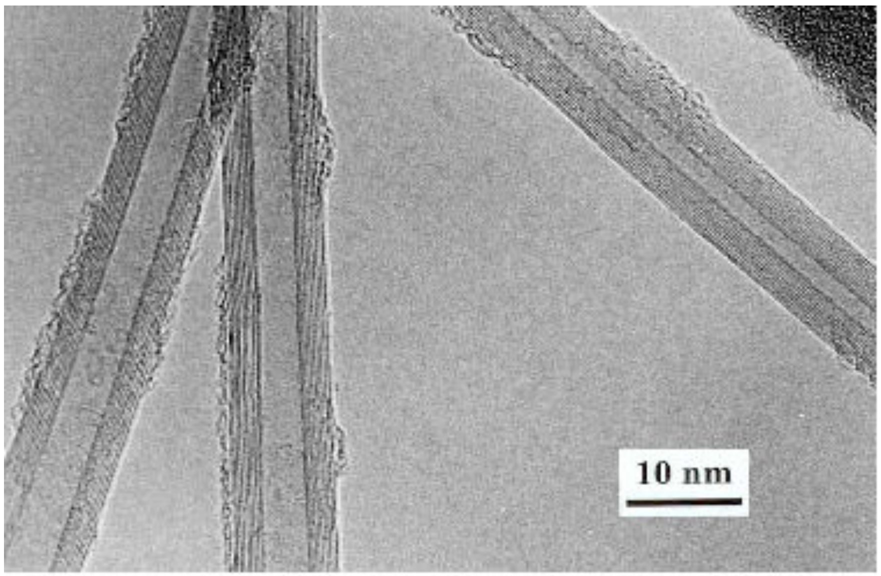
The bacterium is ≅ 1μ size

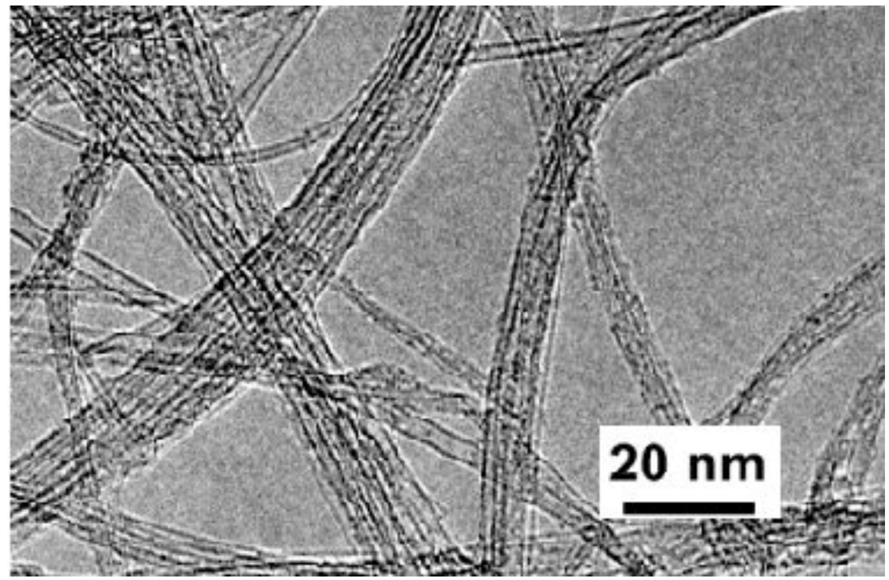
West Nile Virus extracted from a crow brain





STM pictures of Carbon Nanotubes





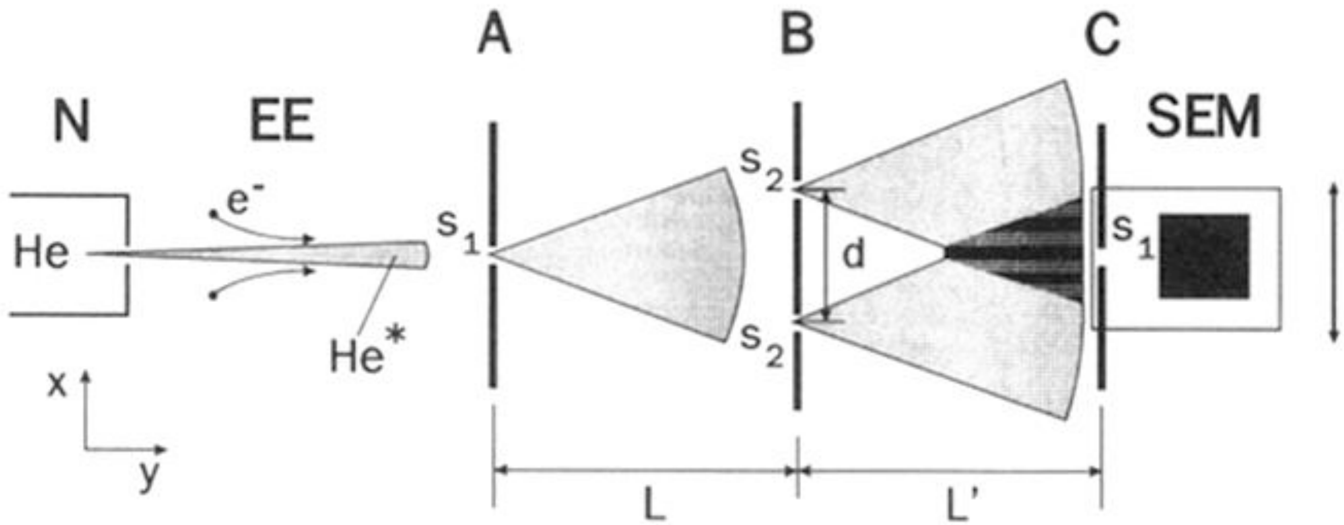
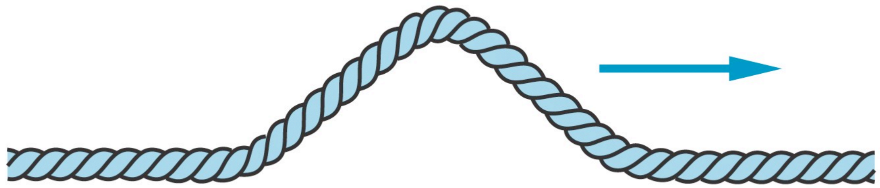


FIG. 2. Schematic representation of the experimental setup: nozzle system and gas reservoir N; electron impact excitation EE; entrance slit A, double slit B, and detector screen C; secondary electron multiplier SEM (mounted together with C on a translation stage). Dimensions: $d=8\ \mu$m, $L=L'=64$ cm; slit widths: $s_1=2\ \mu$m, $s_2=1\ \mu$m.

Just What is Waving in Matter Waves ?

- For waves in an ocean, it's the water that "waves"
- For sound waves, it's the molecules in medium
- For light it's the **E** & **B** vectors
- What's waving for matter waves ?
  - **It's the PROBABLILITY OF FINDING THE PARTICLE that waves !**
  - **Particle can be represented by a wave packet in**
    - **Space**
    - **Time**
    - **Made by superposition of many sinusoidal waves of different λ**
    - **It's a "pulse" of probability**

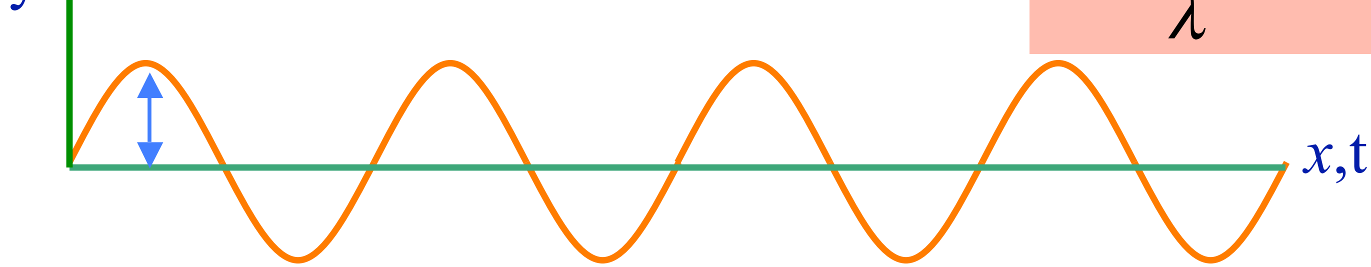
Imagine Wave pulse moving along a string: its localized in time and space (unlike a pure harmonic wave)



What Wave Does Not Describe a Particle

$$y = A\ \cos\ (kx - \omega t + \Phi)$$

$$k = \frac{2\pi}{\lambda}, w = 2\pi f$$



- What wave form can be associated with particle's pilot wave?
- A traveling sinusoidal wave? $y = A\ \cos\ (kx - \omega t + \Phi)$
- Since de Broglie "pilot wave" represents particle, it must travel with same speed as particle ……(like me and my shadow)

Phase velocity (v_p) of sinusoidal wave: $v_p = \lambda f$

In Matter:

$$(a)\ \ \lambda = \frac{h}{p} = \frac{h}{\gamma m v}$$

$$(b)\ \ f = \frac{E}{h} = \frac{\gamma m c^2}{h}$$

$$\Rightarrow v_p = \lambda f = \frac{E}{p} = \frac{\gamma m c^2}{\gamma m v} = \frac{c^2}{v} > c!$$

Conflicts with Relativity → Unphysical

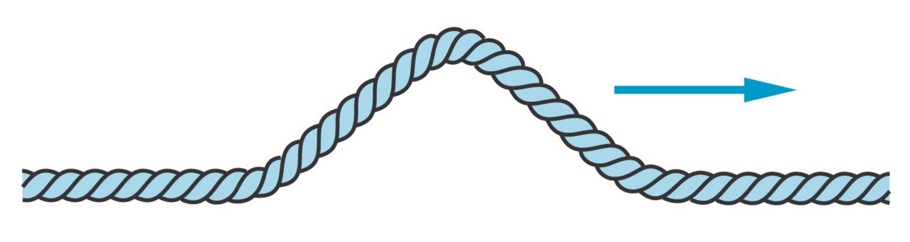
Single sinusoidal wave of infinite extent does not represent particle localized in space

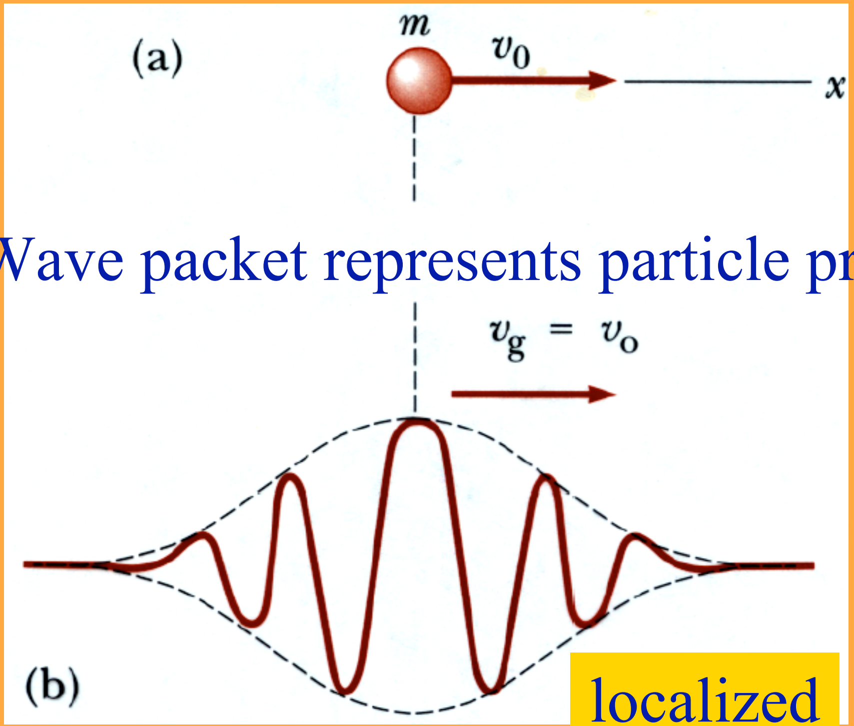
Need "wave packets" localized Spatially (x) and Temporally (t)

Wave Group or Wave Pulse

- Wave Group/packet:
  - Superposition of many sinusoidal waves with different wavelengths and frequencies
  - Localized in space, time
  - Size designated by
    - Δx or Δt
  - Wave groups travel with the speed $v_g = v_0$ of particle
- Constructing Wave Packets
  - Add waves of diff λ,
  - For each wave, pick
    - Amplitude
    - Phase
  - Constructive interference over the space-time of particle
  - Destructive interference elsewhere !

Imagine Wave pulse moving along a string: its localized in time and space (unlike a pure harmonic wave)





Wave packet represents particle prob

localized

Resulting wave's "displacement" $y = y_1 + y_2$:

$$y = A\left[\cos(k_1 x - \omega_1 t) + \cos(k_2 x - \omega_2 t)\right]$$

Trignometry : $\cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$

$$\therefore\ y = 2A\left[\left(\cos(\frac{k_2 - k_1}{2}x - \frac{\omega_2 - \omega_1}{2}t)\right)\left(\cos(\frac{k_2 + k_1}{2}x - \frac{\omega_2 + \omega_1}{2}t)\right)\right]$$

since $k_2 \cong k_1 \cong k_{ave}$, $\omega_2 \cong \omega_1 \cong \omega_{ave}$, $\Delta k \ll k, \Delta\omega \ll \omega$

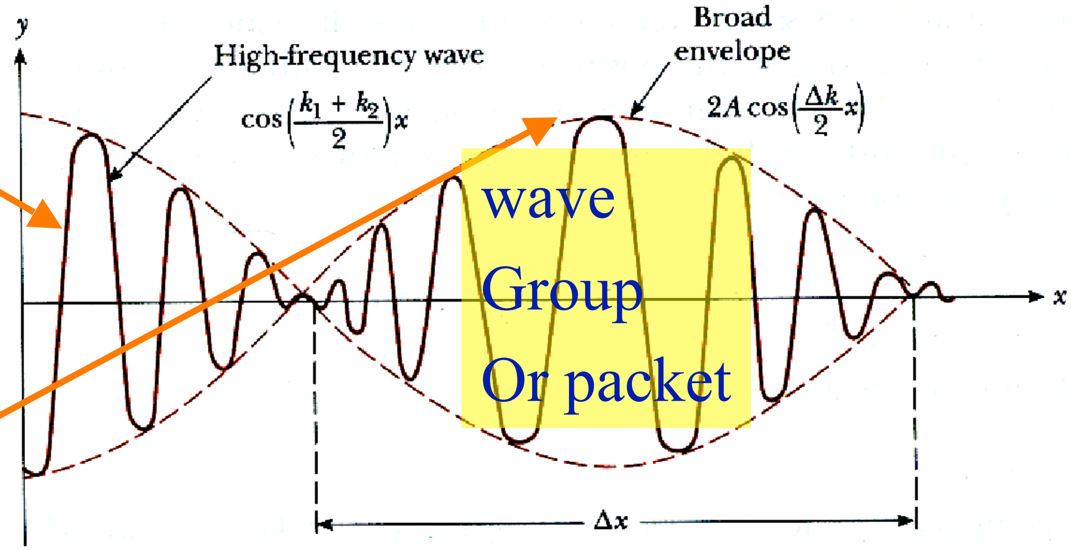
$$\therefore \boxed{y = 2A\left[\left(\cos(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t)\right)\cos(kx - \omega t)\right]} \equiv\ y = A' \cos(kx - \omega t),\ A' \text{ oscillates in x,t}$$

$$A' = 2A\left(\cos(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t)\right) = \text{modulated amplitude}$$

Phase Vel $\quad V_p = \frac{w_{ave}}{k_{ave}}$

Group Vel $\quad V_g = \frac{\Delta w}{\Delta k}$

V_g : Vel of envelope$= \frac{dw}{dk}$



wave
Group
Or packet

Wave Packet : Localization

•Finite # of diff. Monochromatic waves always produce INFINTE sequence of repeating wave groups → can't describe (localized) particle

•To make localized wave packet, add " infinite" # of waves with Well chosen Ampl A, Wave# k, ang.

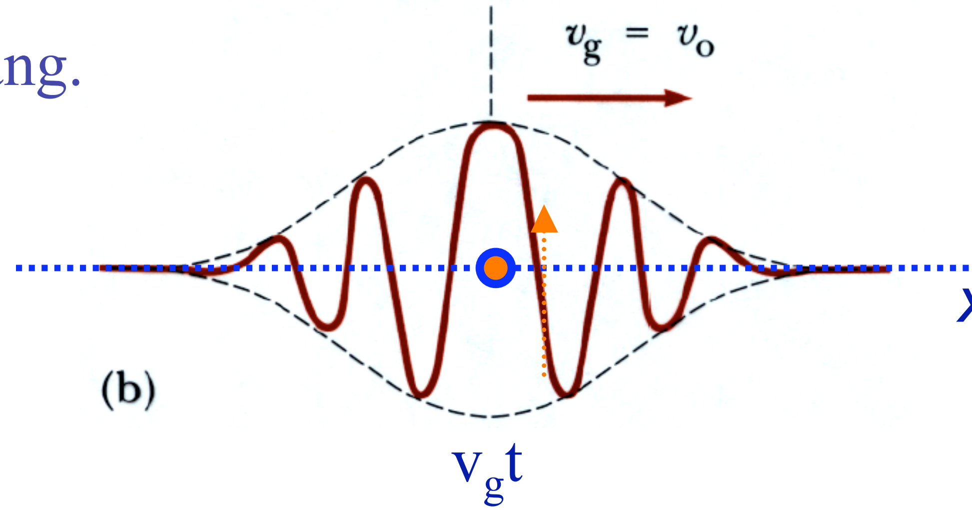
$$\psi(x,t) = \int_{-\infty}^{\infty} A(k)\, e^{i(kx-\omega t)} dk$$

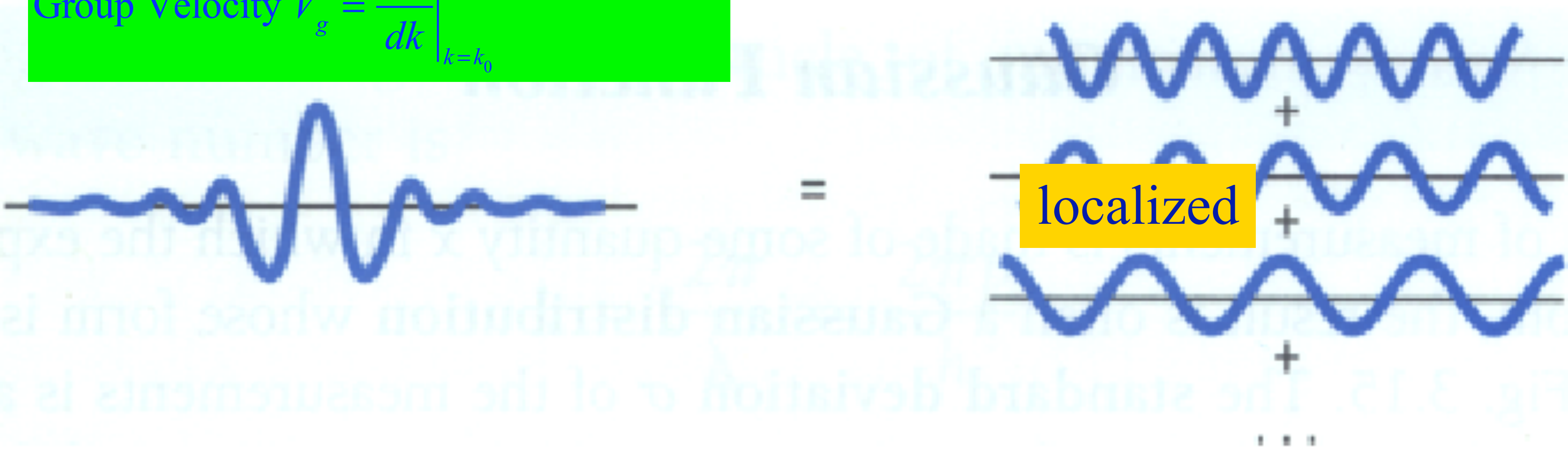
$A(k)$ = Amplitude Fn

$\Rightarrow$ diff waves of diff k have different amplitudes A(k)

$\omega = \omega$(k), depends on type of wave, media

Group Velocity $V_g = \left.\frac{d\omega}{dk}\right|_{k=k_0}$





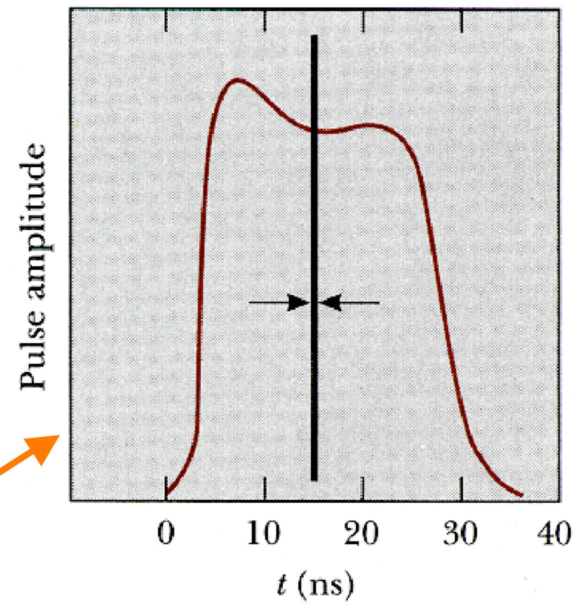
Group, Velocity, Phase Velocity and Dispersion

In a Wave Packet: $\omega = \omega(k)$

Group Velocity $V_g = \left.\dfrac{d\omega}{dk}\right|_{k=k_0}$

Since $\mathrm{V_p} = \omega / k \quad (def) \Rightarrow \omega = kV_p$

$$\therefore \boxed{V_g = \left.\frac{d\omega}{dk}\right| = V_p\Big|_{k=k_0} + k\left.\frac{dV_p}{dk}\right|_{k=k_0}}$$



1ns laser pulse disperse
By x30 after travelling
1km in optical fiber

usually $\mathrm{V_p} = V_p(k \ or \lambda)$

Material in which $\mathrm{V_p}$ varies with λ are said to be Dispersive

Individual harmonic waves making a wave pulse travel at different $\mathrm{V_p}$ thus changing shape of pulse and become spread out

In non-dispersive media, $\mathrm{V_g} = V_p$

In dispersive media $\mathrm{V_g} \neq V_p$, depends on $\dfrac{dV_p}{dk}$

Matter Wave Packets

Consider An Electron:

mass = m velocity = v, momentum = p

Energy $E = hf = \gamma mc^2$; $\omega = 2\pi f = \frac{2\pi}{h}\gamma mc^2$

Wavelength $\lambda = \frac{h}{p}$; $k = \frac{2\pi}{\lambda} \Rightarrow k = \frac{2\pi}{h}\gamma mv$

Group Velocity : $V_g = \frac{d\omega}{dk} = \frac{d\omega / dv}{dk / dv}$

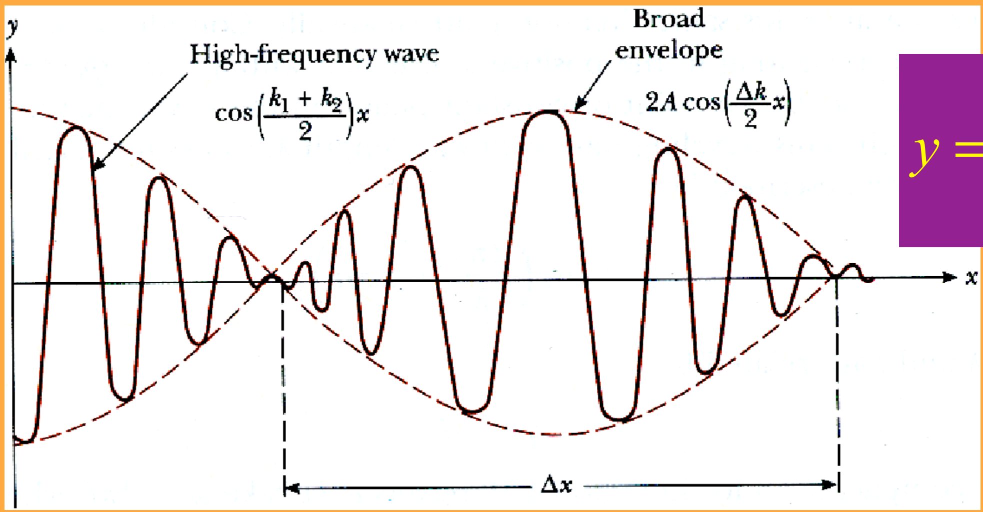
$$\frac{d\omega}{dv} = \frac{d}{dv}\left[\frac{\frac{2\pi}{h}mc^2}{[1-(\frac{v}{c})^2]^{1/2}}\right] = \frac{2\pi mv}{h[1-(\frac{v}{c})^2]^{3/2}} \quad \& \quad \frac{dk}{dv} = \frac{d}{dv}\left[\frac{2\pi}{h[1-(\frac{v}{c})^2]^{1/2}} mv\right] = \frac{2\pi m}{h[1-(\frac{v}{c})^2]^{3/2}}$$

$V_g = \frac{d\omega}{dk} = \frac{d\omega / dv}{dk / dv} = v \Rightarrow$ Group velocity of electron Wave packet "pilot wave"

is same as electron's physical velocity

But velocity of individual waves making up the wave packet $V_p = \frac{\omega}{k} = \frac{c^2}{v} > c!$ (not physical)

Wave Packets & Uncertainty Principle



$$y = 2A\left[\left(\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\right)\cos(kx-\omega t)\right]$$

Amplitude Modulation

- Distance ΔX between adjacent minima = $(X_2)_{node}$ - $(X_1)_{node}$
- Define $X_1=0$ then phase diff from $X_1 \rightarrow X_2 = \pi$

Node at y = 0 = 2A cos $\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)$

$\Rightarrow$ $\boxed{\Delta k.\Delta x = 2\pi}$ $\Rightarrow$ Need to combine more k to make small Δx packet

also implies $\Rightarrow$ $\boxed{\Delta p.\Delta x = h}$

and

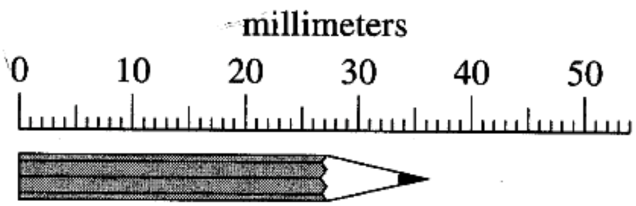
$\boxed{\Delta\omega.\Delta t = 2\pi}$ $\Rightarrow$ Need to combine more ω to make small Δt packet

also $\Rightarrow$ $\boxed{\Delta E.\Delta t = h}$

What does This mean?

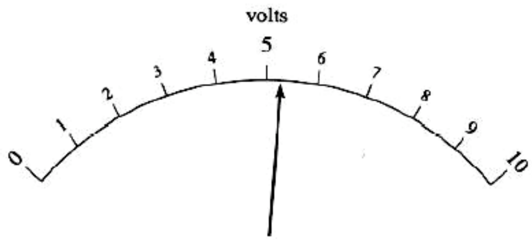
Know the Error of Thy Ways: Measurement Error → Δ

- Measurements are made by observing something : length, time, momentum, energy
- All measurements have some (limited) precision`…no matter the instrument used
- Examples:
  - How long is a desk ? $L = (5 \pm 0.1)$ m = $L \pm \Delta L$ (depends on ruler used)
  - How long was this lecture ? $T = (50 \pm 1)$minutes = $T \pm \Delta T$ (depends on the accuracy of your watch)
  - How much does Prof. Sinha weigh ? $M = (1000 \pm 950)$ kg = $m \pm \Delta m$
    - Is this a correct measure of my weight ?
      - Correct (because of large error reported) but imprecise
      - My correct weight is covered by the (large) error in observation



Best Estimate Length: 36 mm
Probable Range: 35.5 to 36.5 mm

Length Measure

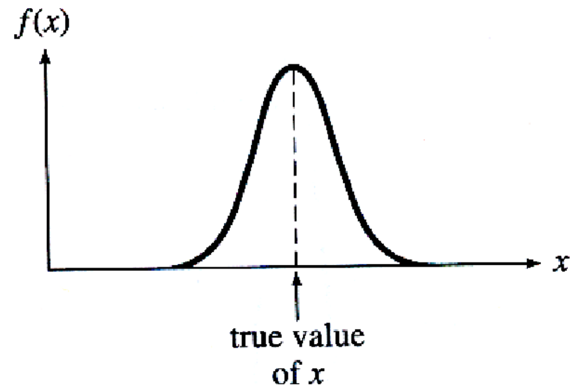


Best Estimate of Voltage: 5.3 V
Estimated Range: 5.2 to 5.4 mm

Voltage (or time) Measure

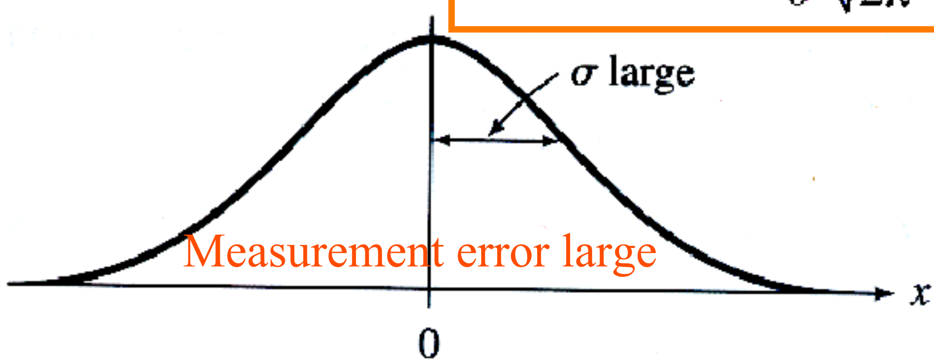
Measurement Error : x ± Δx

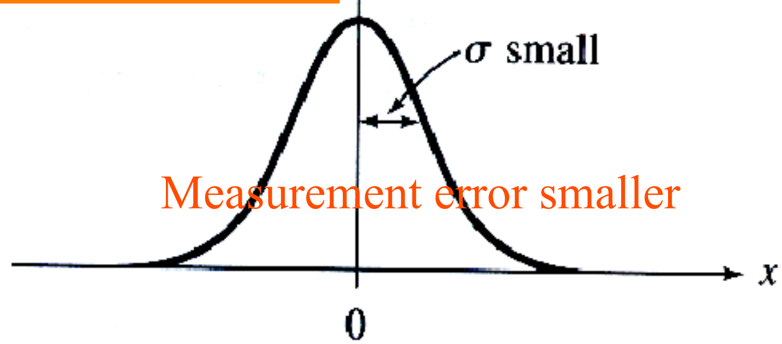
- Measurement errors are unavoidable since the measurement procedure is an experimental one
- True value of an measurable quantity is an abstract concept
- In a set of repeated <u>measurements with random errors</u>, the distribution of measurements resembles a Gaussian distribution characterized by the parameter σ or Δ characterizing the width of the distribution



The Gauss, or Normal, Distribution

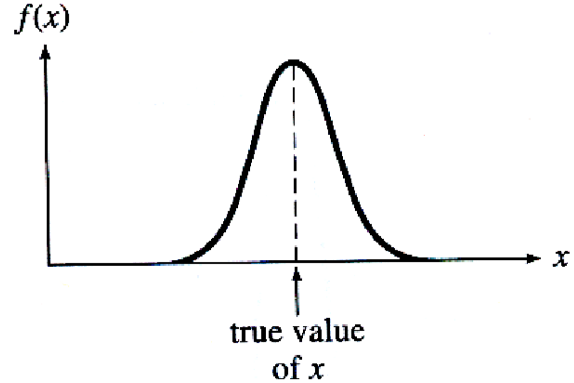
$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{(x-X)^2/2\sigma^2}.$$





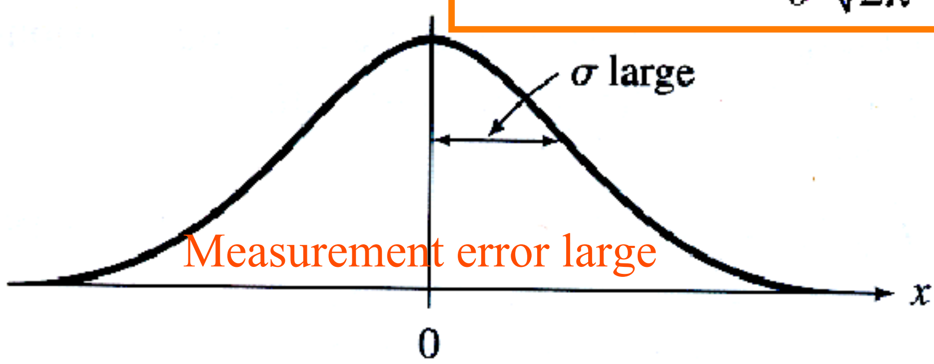
Measurement Error : x ± Δx

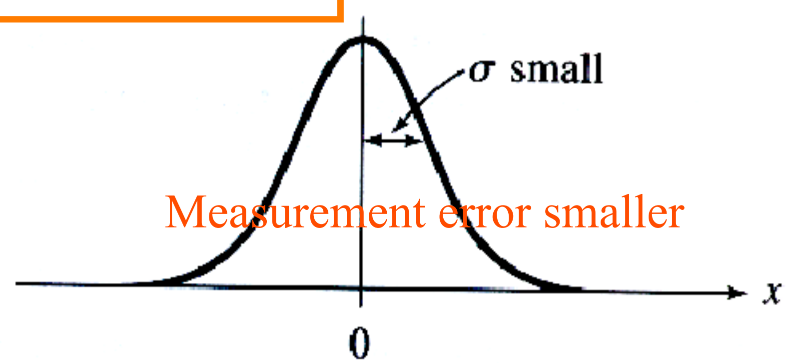
- Measurement errors are unavoidable since the measurement procedure is an experimental one
- True value of an measurable quantity is an abstract concept
- In a set of repeated measurements with random errors, the distribution of measurements resembles a Gaussian distribution characterized by the parameter σ or Δ characterizing the width of the distribution



The Gauss, or Normal, Distribution

$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{(x-X)^2/2\sigma^2}.$$





Interpreting Measurements with random Error : Δ

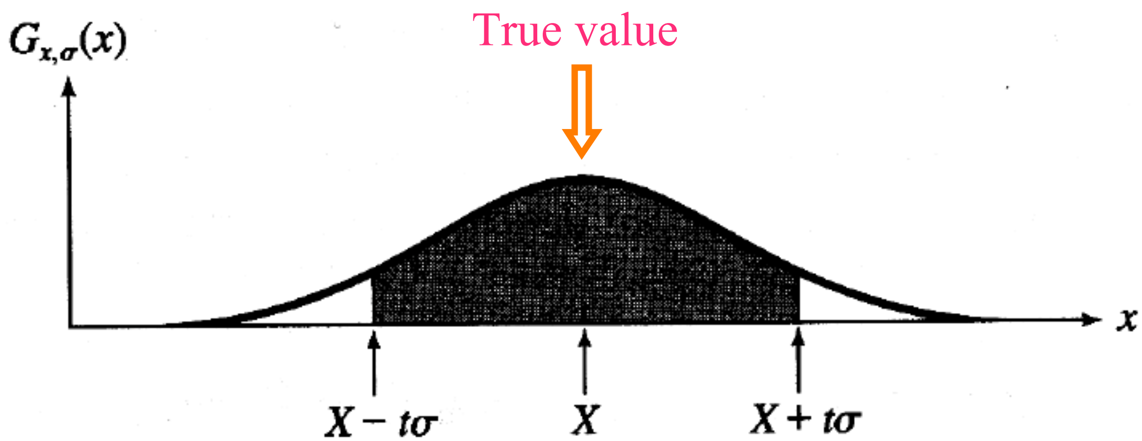
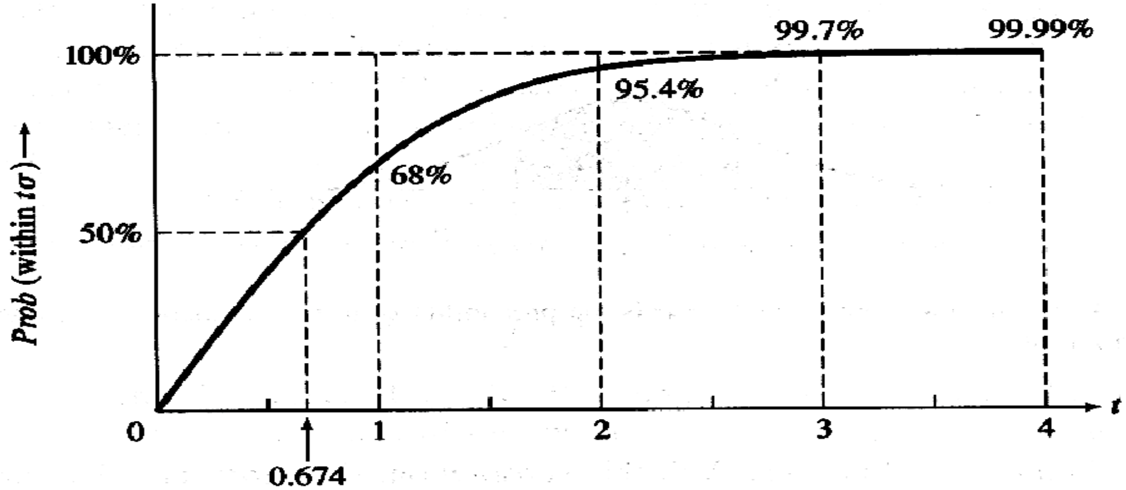


Figure 5.12. The shaded area between $X \pm t\sigma$ is the probability of a measurement within t standard deviations of X.



t	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0	2.5	3.0	3.5	4.0
Prob (%)	0	20	38	55	68	79	87	92	95.4	98.8	99.7	99.95	99.99

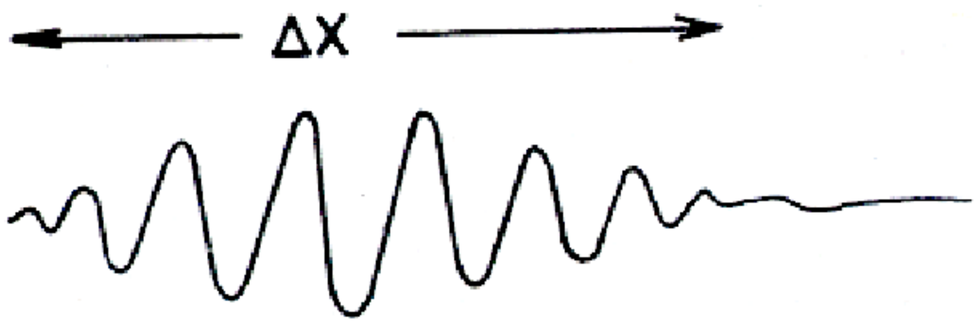
Where in the World is Carmen San Diego?

- Carmen San Diego hidden inside a big box of length L
- Suppose you can't see thru the (blue) box, what is you best estimate of her location inside box (she could be anywhere inside the box)

Your best unbiased measure would be $x = L/2 \pm L/2$

There is no perfect measurement, there are always measurement error

Wave Packets & Matter Waves



What is the Wave Length of this wave packet?

$\lambda - \Delta\lambda < \lambda < \lambda + \Delta\lambda$

De Broglie wavelength $\lambda = h/p$

→Momentum Uncertainty: $p - \Delta p < p < p + \Delta p$

Similarly for frequency ω or f

$\omega - \Delta\omega < \omega < \omega + \Delta\omega$

Planck's condition $E = hf = h\omega/2$

→ $E - \Delta E < E < E + \Delta E$

Back to Heisenberg's Uncertainty Principle & Δ

- $\Delta x.\ \Delta p \geq h/4\pi \Rightarrow$
  - If the measurement of the position of a particle is made with a precision Δx and a SIMULTANEOUS measurement of its momentum p_x in the X direction , then the product of the two uncertainties (measurement errors) can never be smaller than $\cong h/4\pi$ irrespective of how precise the measurement tools

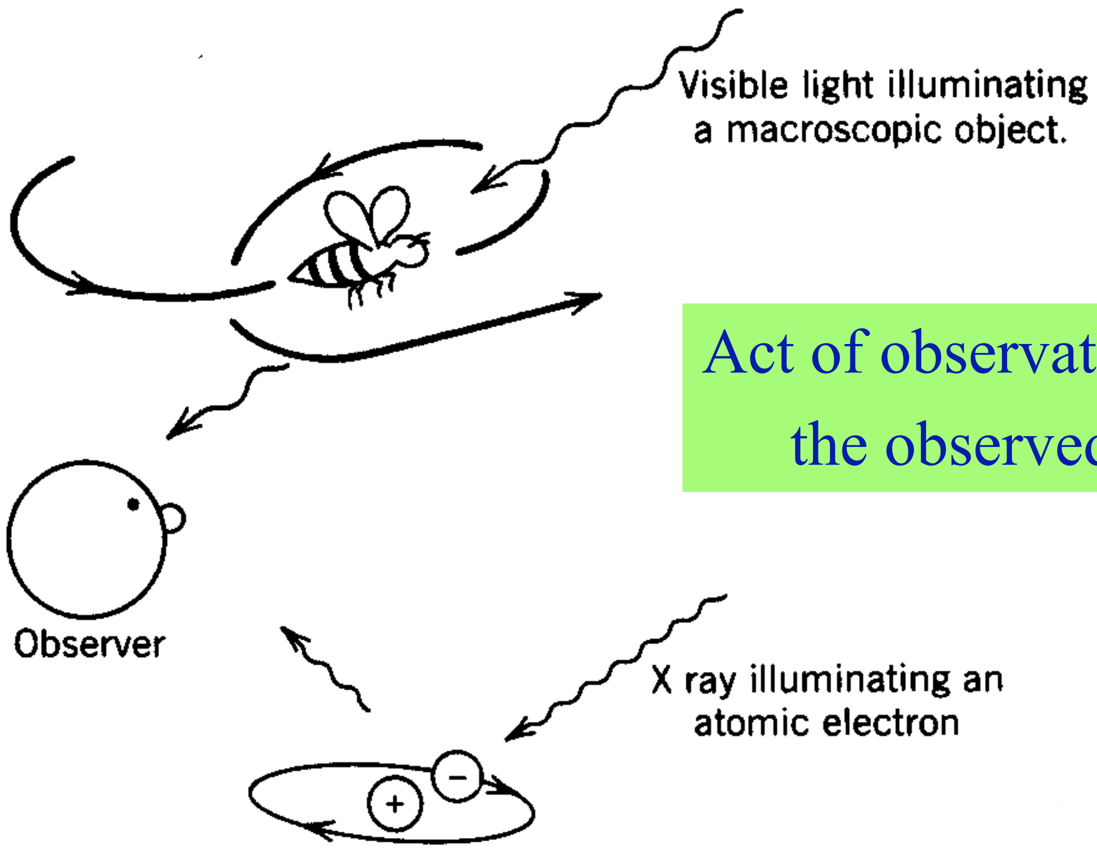
- $\Delta E.\ \Delta t \geq h/4\pi \Rightarrow$
  - If the measurement of the energy E of a particle is made with a precision ΔE and it took time Δt to make that measurement, then the product of the two uncertainties (measurement errors) can never be smaller than $\cong h/4\pi$ irrespective of how precise the measurement tools

These rules arise from the way we constructed the Wave packets describing Matter "pilot" waves

Perhaps these rules Are bogus, can we verify this with some physical picture ??

The Act of Observation (Compton Scattering)

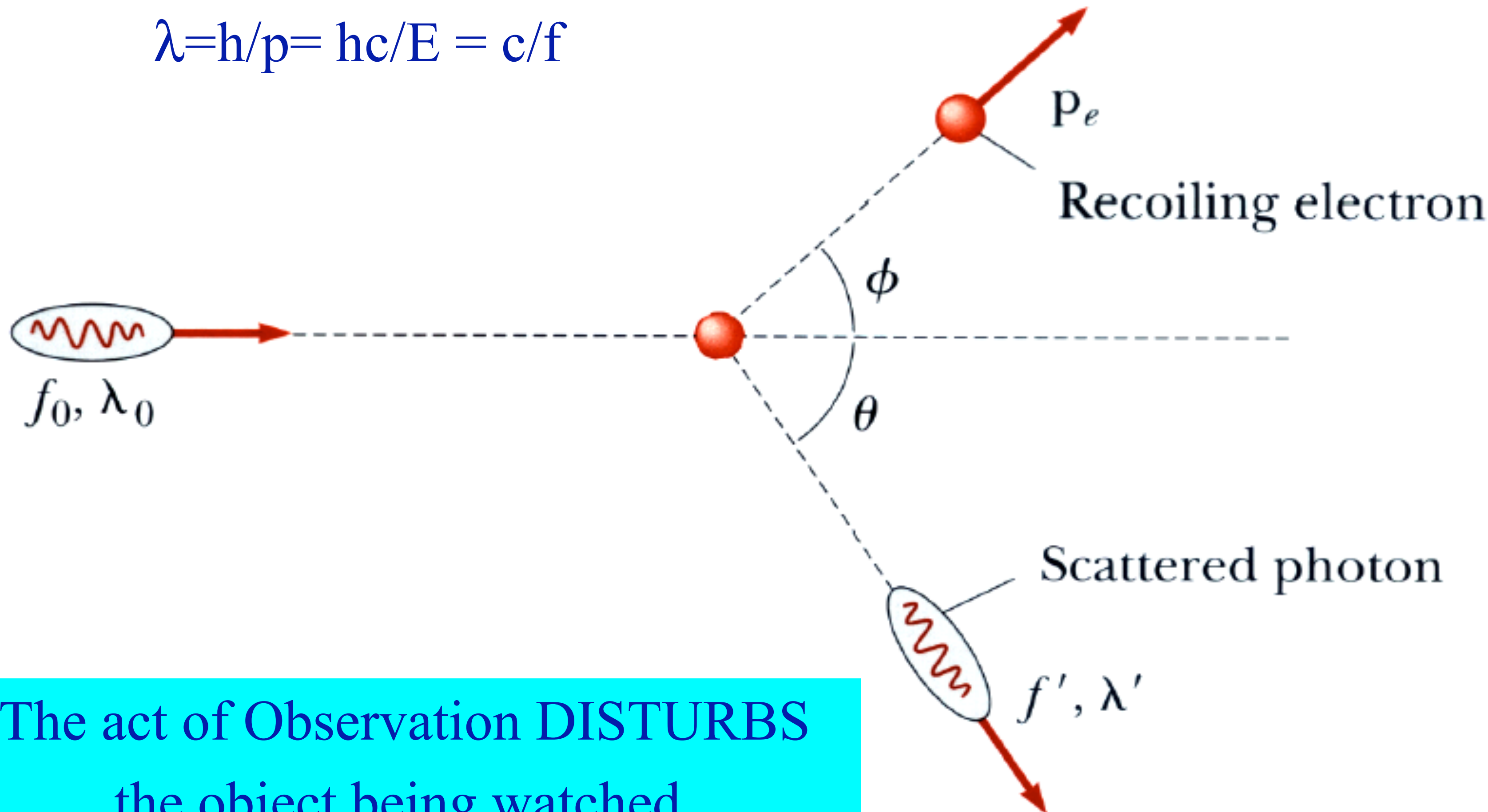
Observations of particle motion by means of scattered illumination. When the incident wavelength is reduced to accommodate the size of the particle, the momentum transferred by the photon becomes large enough to disturb the observed motion.

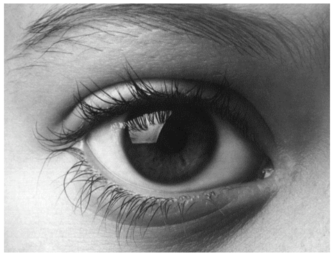


Act of observation disturbs the observed system

Compton Scattering: Shining light to observe electron

$\lambda = h/p = hc/E = c/f$





The act of Observation DISTURBS the object being watched, here the electron moves away from where it was originally

Diffraction By a Circular Aperture (Lens)

See Resnick, Halliday Walker 6th Ed (on S.Reserve), Ch 37, pages 898-900

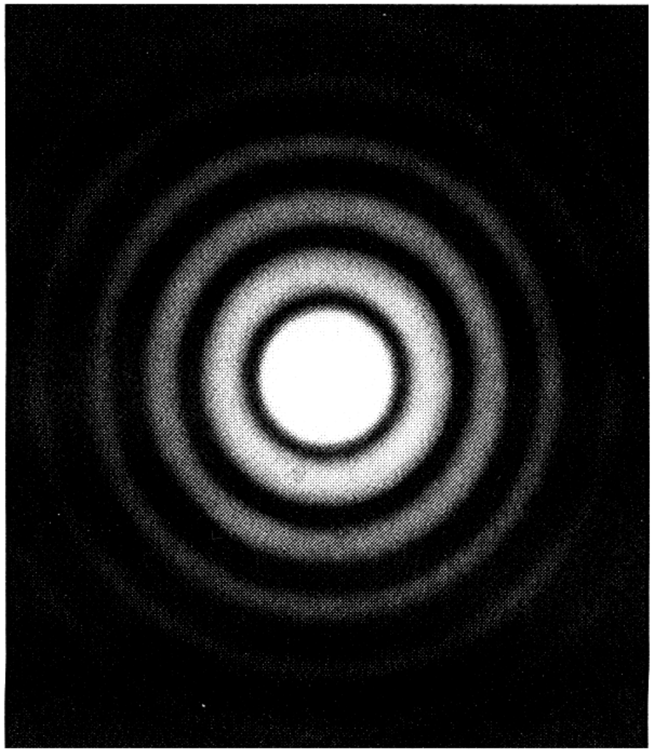


Fig. 37-9 The diffraction pattern of a circular aperture. Note the central maximum and the circular secondary maxima. The figure has been overexposed to bring out these secondary maxima, which are much less intense than the central maximum.

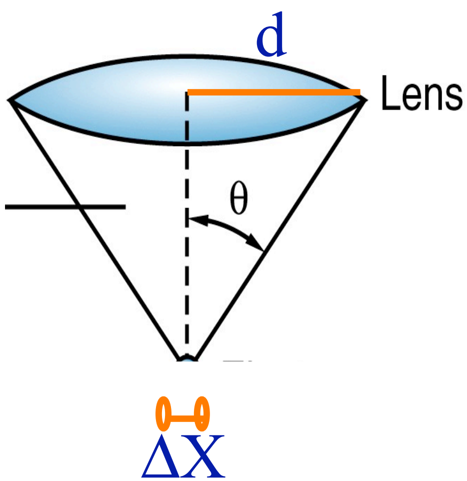
Diffracted image of a point source of light thru a lens (circular aperture of size d)

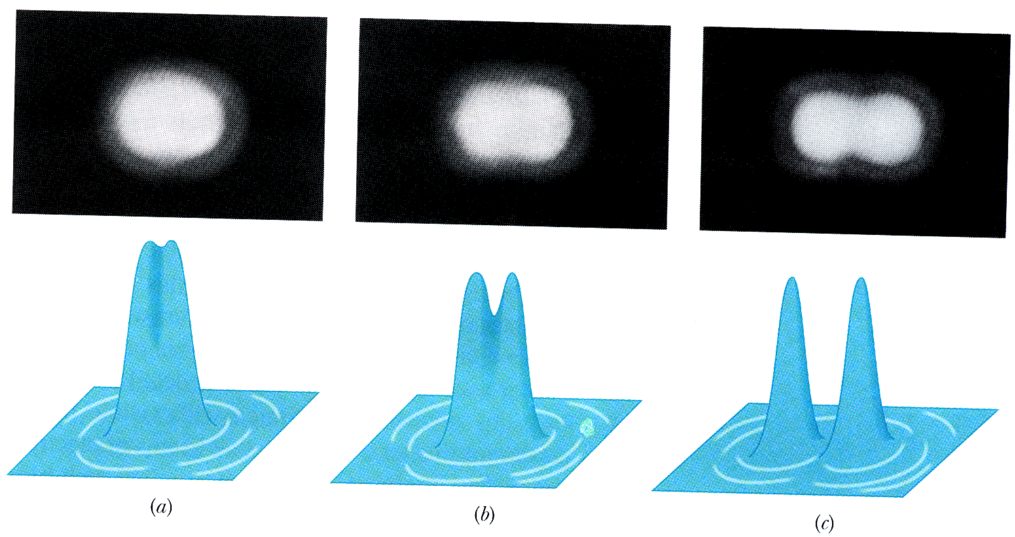
First minimum of diffraction pattern is located by

$$\sin\theta = 1.22\frac{\lambda}{d}$$

Resolving Power of Light Thru a Lens

Image of 2 separate point sources formed by a converging lens of diameter d, ability to resolve them depends on λ & d because of the Inherent diffraction in image formation



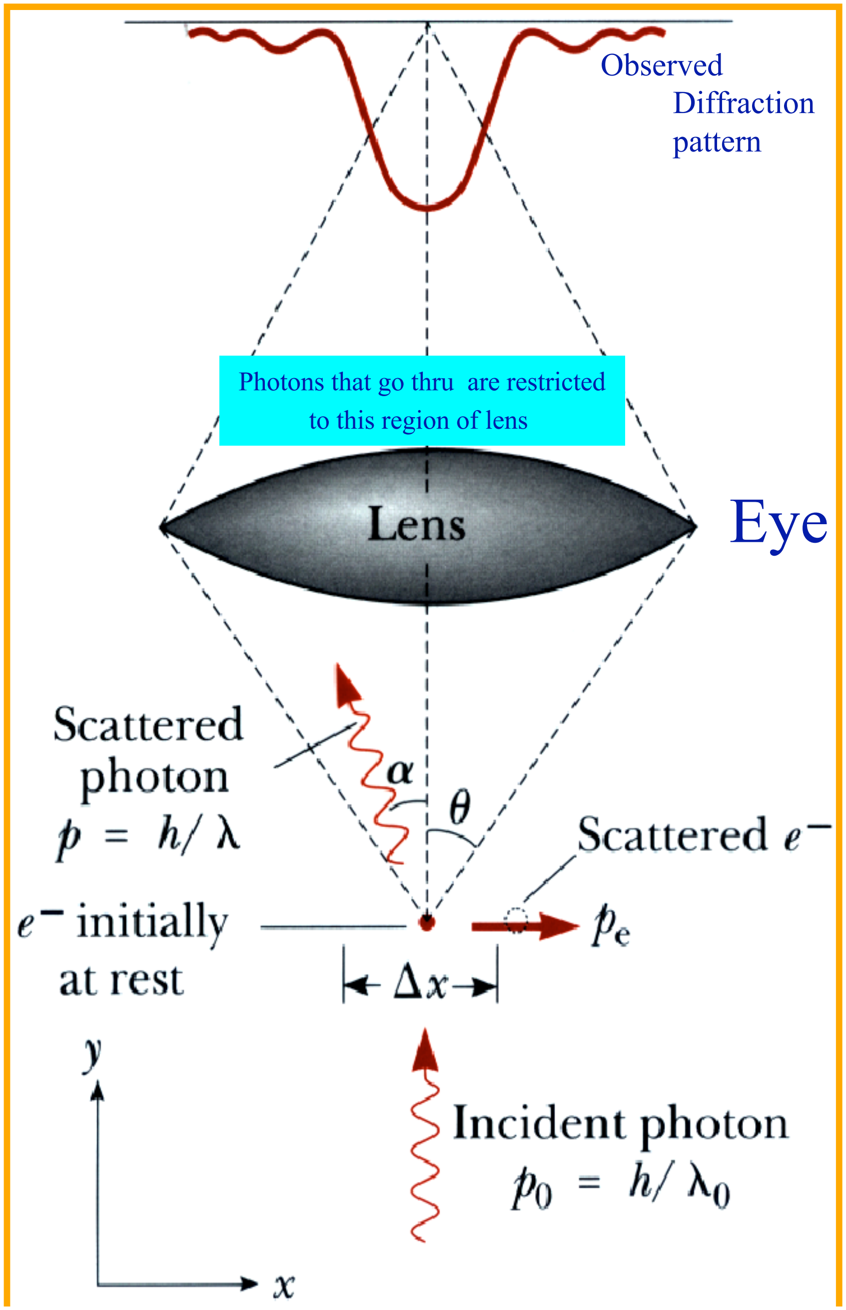


Not resolved　　barely resolved　　resolved

$$\text{Resolving power } \Delta x \simeq \frac{\lambda}{2\sin\theta}$$

ϑ Depends on d

Putting it all together: act of Observing an electron



- Incident light (p,λ) scatters off electron
- To be collected by lens → γ must scatter thru angle α
  - $-\vartheta \leq \alpha \leq \vartheta$
- Due to Compton scatter, electron picks up momentum
  - P_X , P_Y

$$-\frac{h}{\lambda}\sin\theta \leq P_x \leq \frac{h}{\lambda}\sin\theta$$

electron momentum uncertainty is

$$\Delta p \cong \frac{\sim 2h}{\lambda}\sin\theta$$

- After passing thru lens, photon diffracts, lands somewhere on screen, image (of electron) is fuzzy
- How fuzzy ? Optics says shortest distance between two resolvable points is :

$$\Delta x = \frac{\lambda}{2\sin\theta}$$

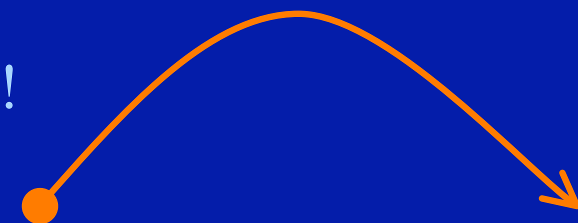
- Larger the lens radius, larger the $\vartheta \Rightarrow$ better resolution

$$\Rightarrow \Delta p.\Delta x \simeq \left(\frac{2h\sin\theta}{\lambda}\right)\left(\frac{\lambda}{2\sin\theta}\right) = h$$

$$\Rightarrow \Delta p.\Delta x \geq \hbar/2$$

Pseudo-Philosophical Aftermath of Uncertainty Principle

- Newtonian Physics & Deterministic physics topples over
  - Newton's laws told you all you needed to know about trajectory of a particle
    - Apply a force, watch the particle go !
      - Know every thing ! X, v, p , F, a
      - Can predict exact trajectory of particle if you had perfect device



- No so in the subatomic world !
  - Of small momenta, forces, energies
  - Cant predict anything exactly
    - Can only predict probabilities
      - There is so much chance that the particle landed here or there
      - Cant be sure !....cognizant of the errors of thy observations

Philosophers went nuts !...what has happened to nature

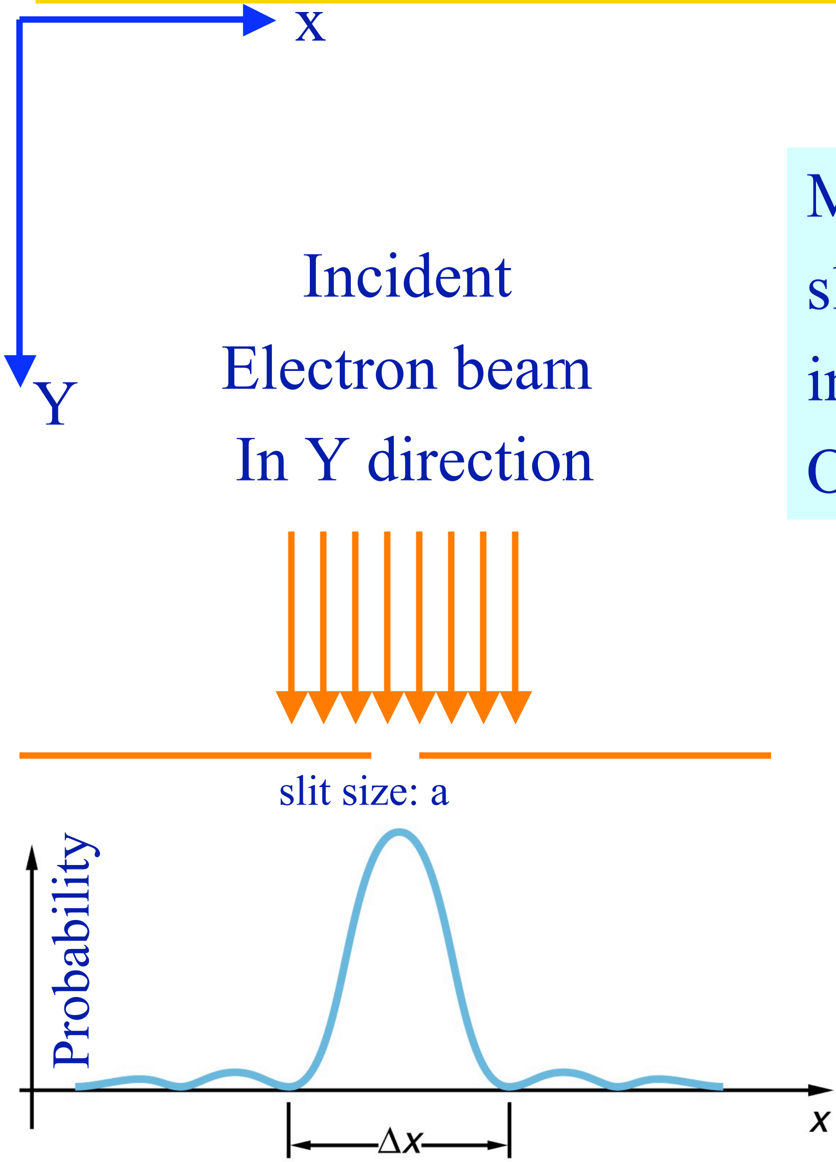
Philosophers just talk, don't do real life experiments!

All Measurements Have Associated Errors

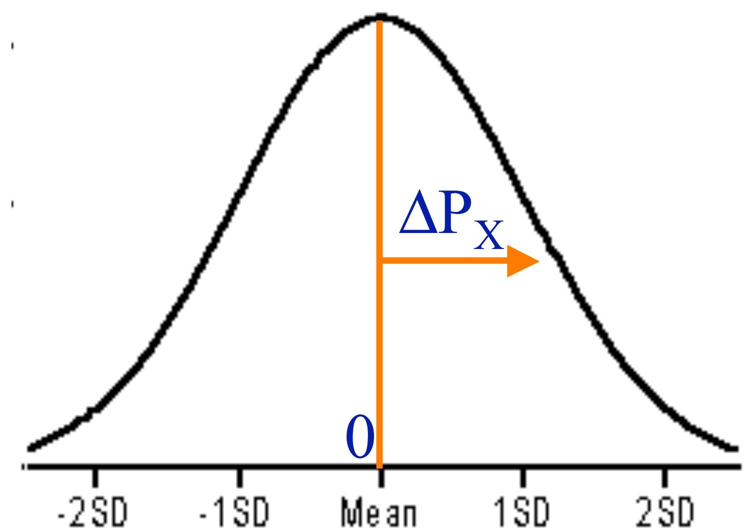
- If your measuring apparatus has an intrinsic inaccuracy (error) of amount Δp
- Then results of measurement of momentum p of an object at rest can easily yield a range of values accommodated by the measurement imprecision :
  - $-\Delta p \leq p \leq \Delta p$
- Similarly for all measurable quantities like x, t, Energy !

Matter Diffraction & Uncertainty Principle

Momentum measurement beyond slit show particle not moving exactly in Y direction, develops a X component Of motion $\Delta P_X = h/(2\pi\ a)$



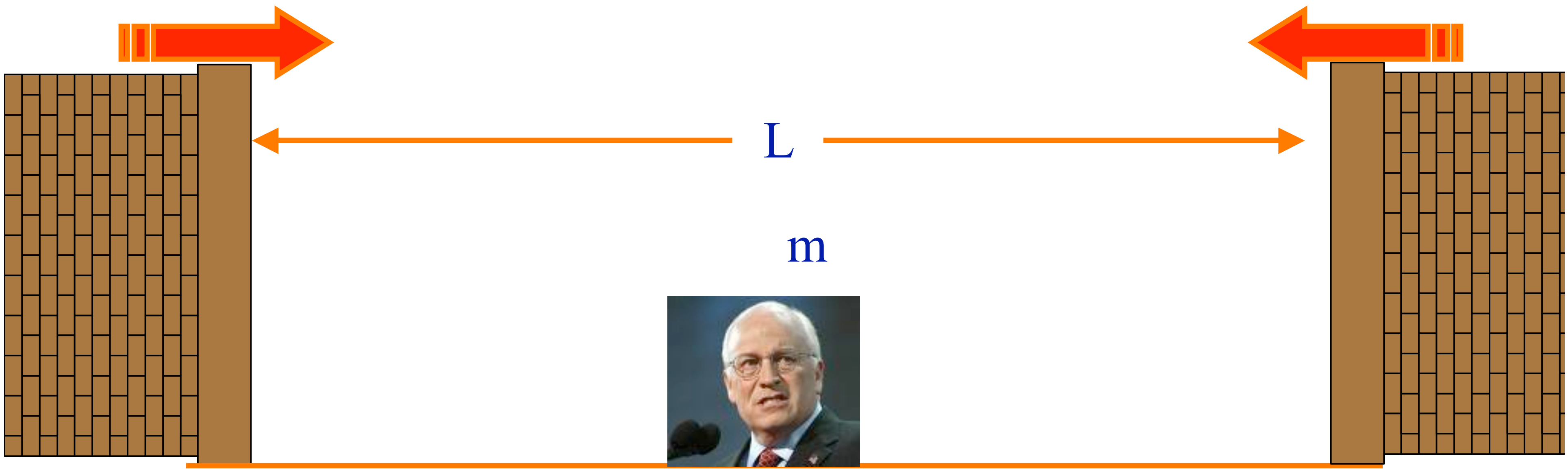
Diffraction pattern
seen on screen

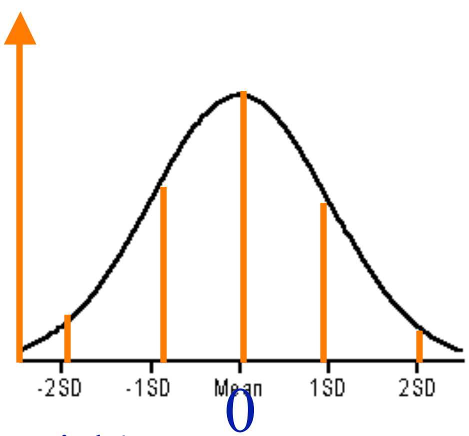


X component P_X of momentum

Particle at Rest Between Two Walls

- Object of mass M at rest between two walls originally at infinity
- What happens to our perception of Dick as the walls are brought in ?





Dick’s Momentum p

On average, measure <p> = 0

but there are quite large fluctuations!

Width of Distribution $= \Delta P$

$$\Delta P = \sqrt{(P^2)_{ave} - (P_{ave})^2}; \quad \Delta P \sim \frac{\hbar}{L}$$