

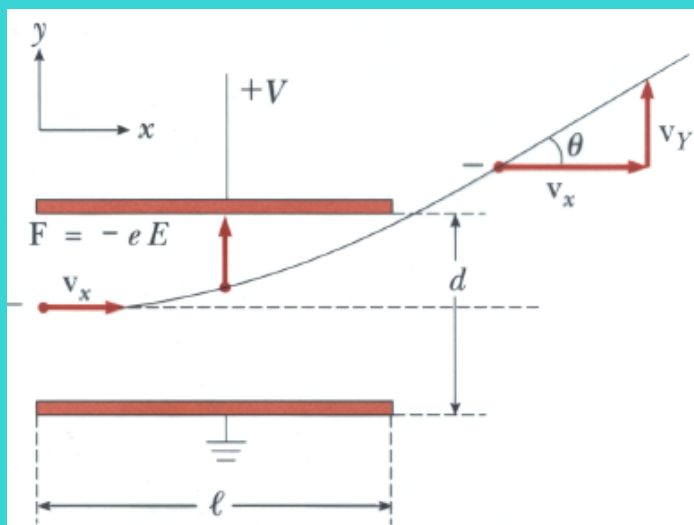
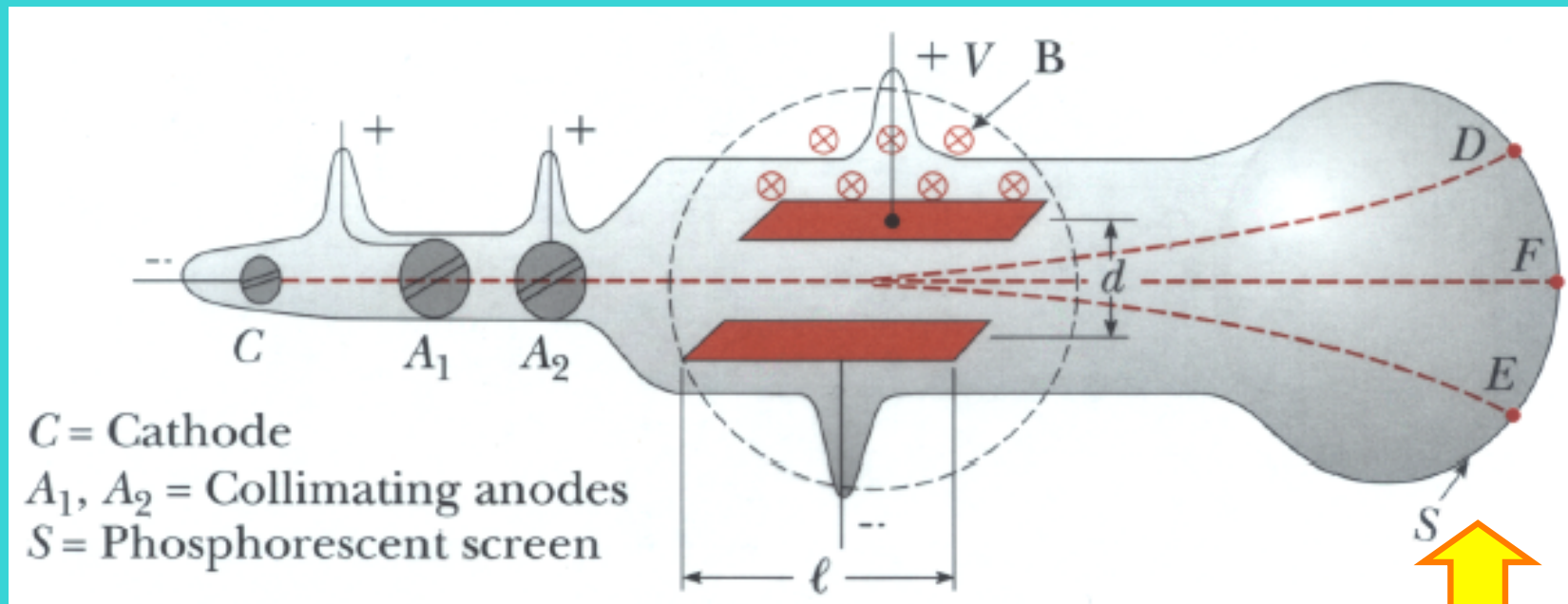


# Physics 2D Lecture Slides

## Feb. 2, 2010

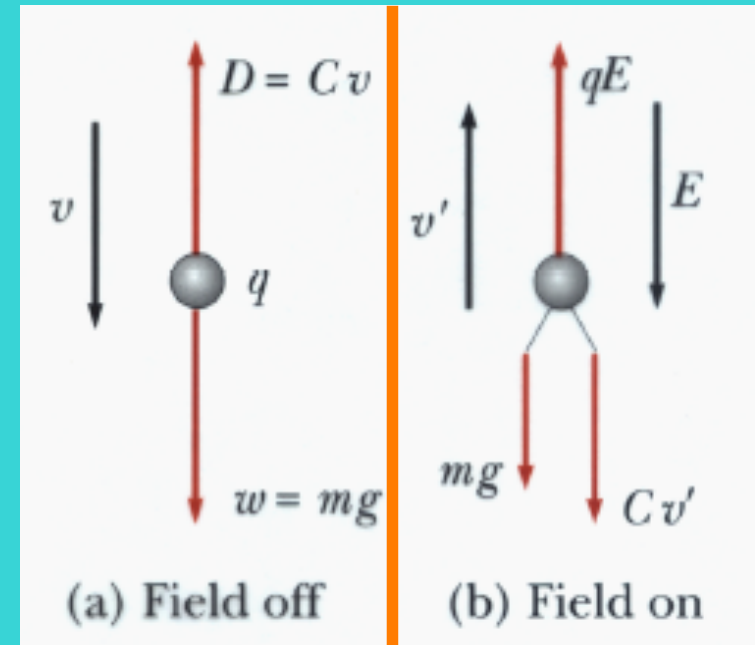
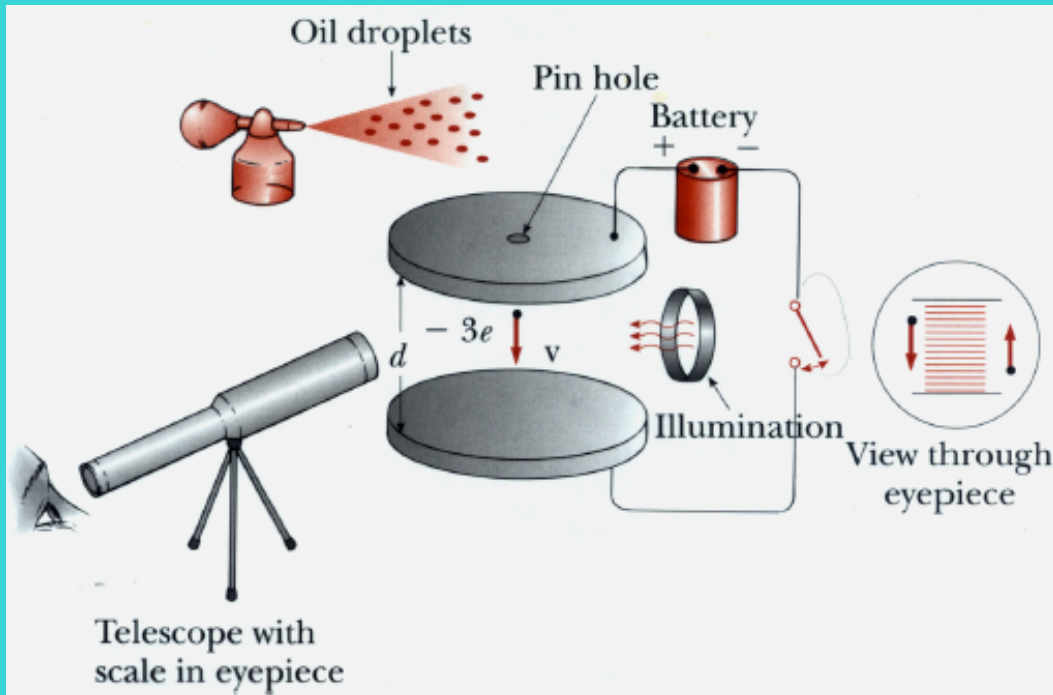
Sunil Sinha  
UCSD Physics

# Thomson's Determination of $e/m$ of the Electron



- In E Field alone, electron lands at D
- In B field alone, electron lands at E
- When E and B field adjusted to cancel each other's force  $\rightarrow$  electron lands at F  
 $\rightarrow e/m = 1.7588 \times 10^{11} \text{ C/Kg}$

# Millikan's Measurement of Electron Charge



Find charge on oil drop is always in integral multiple of some  $Q$

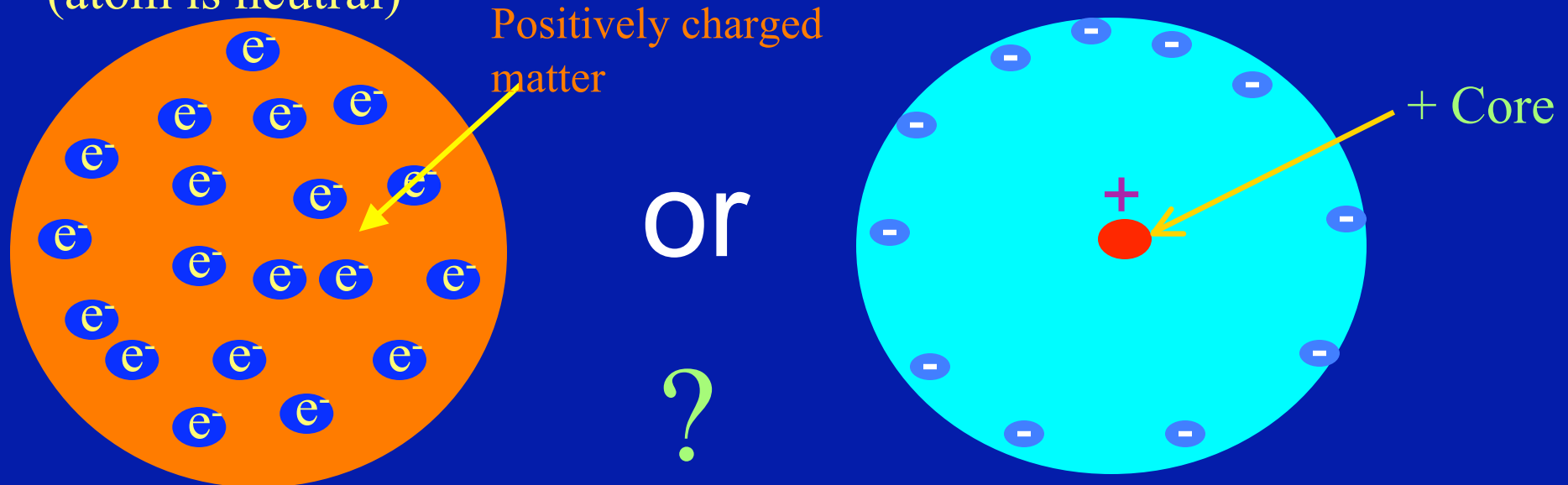
$$q_e = 1.688 \times 10^{-19} \text{ Coulombs}$$

$$\rightarrow m_e = 9.1093 \times 10^{-31} \text{ Kg}$$

$\rightarrow$  Fundamental properties (finger print) of electron  
(similarly can measure proton properties etc)

# Where are the electrons inside the atom?

Early Thought: “Plum pudding” model  $\rightarrow$  Atom has a homogenous distribution of Positive charge with electrons embedded in them (atom is neutral)

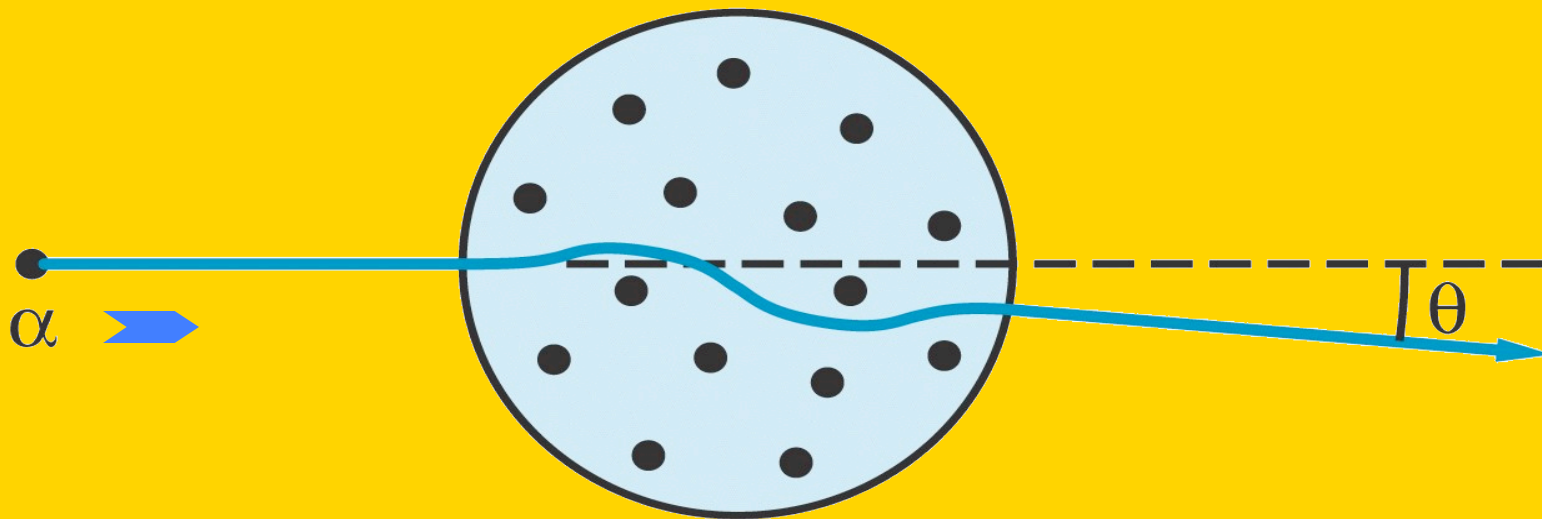


- How to test these hypotheses?  $\rightarrow$  Shoot “bullets” at the atom and watch their trajectory. **What Kind of bullets ?**
  - Indestructible charged bullets  $\rightarrow$  Ionized  $\text{He}^{++}$  atom =  $\alpha^{++}$  particles
  - $Q = +2e$ , Mass  $M_\alpha = 4\text{amu} \gg m_e$ ,  $V_\alpha = 2 \times 10^7 \text{ m/s}$  (non-relativistic)  
[charged to probe charge & mass distribution inside atom]



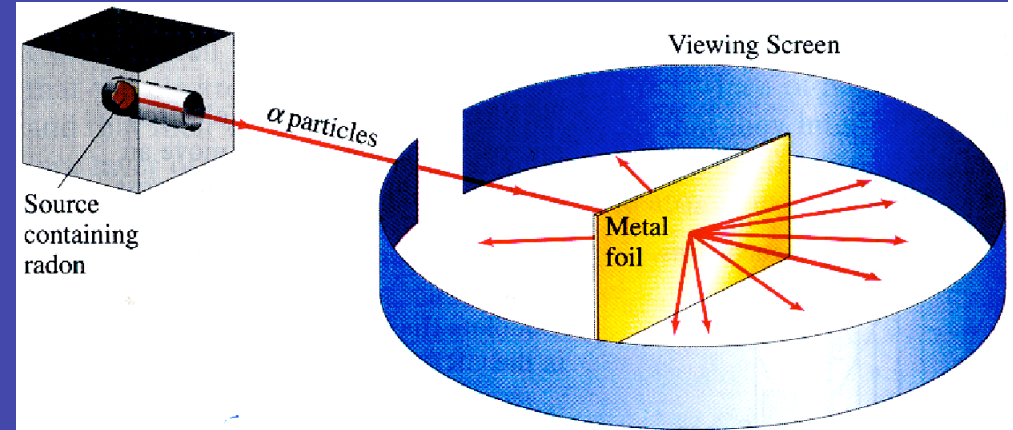
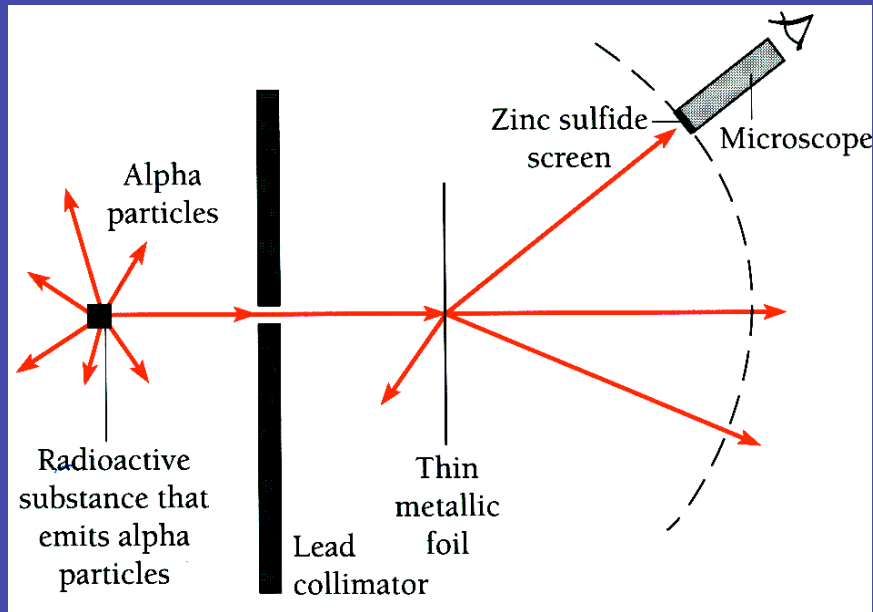
# Plum Pudding Model of Atom

- Non-relativistic mechanics ( $V_{\alpha}/c = 0.1$ )
- In Plum-pudding model,  $\alpha$ -rays hardly scatter because
  - Positive charge distributed over size of atom ( $10^{-10}\text{m}$ )
  - $M_{\alpha} \gg M_e$  (like moving truck hits a bicycle)
  - $\rightarrow$  predict  $\alpha$ -rays will pass thru array of atoms with little scatter ( $\sim 1^{\circ}$ )

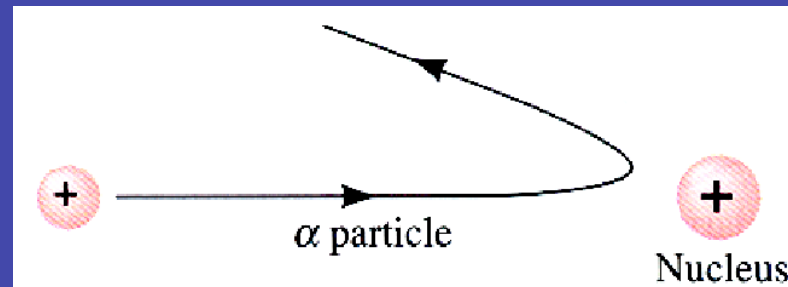


Need to test this hypothesis  $\rightarrow$  Ernest Rutherford

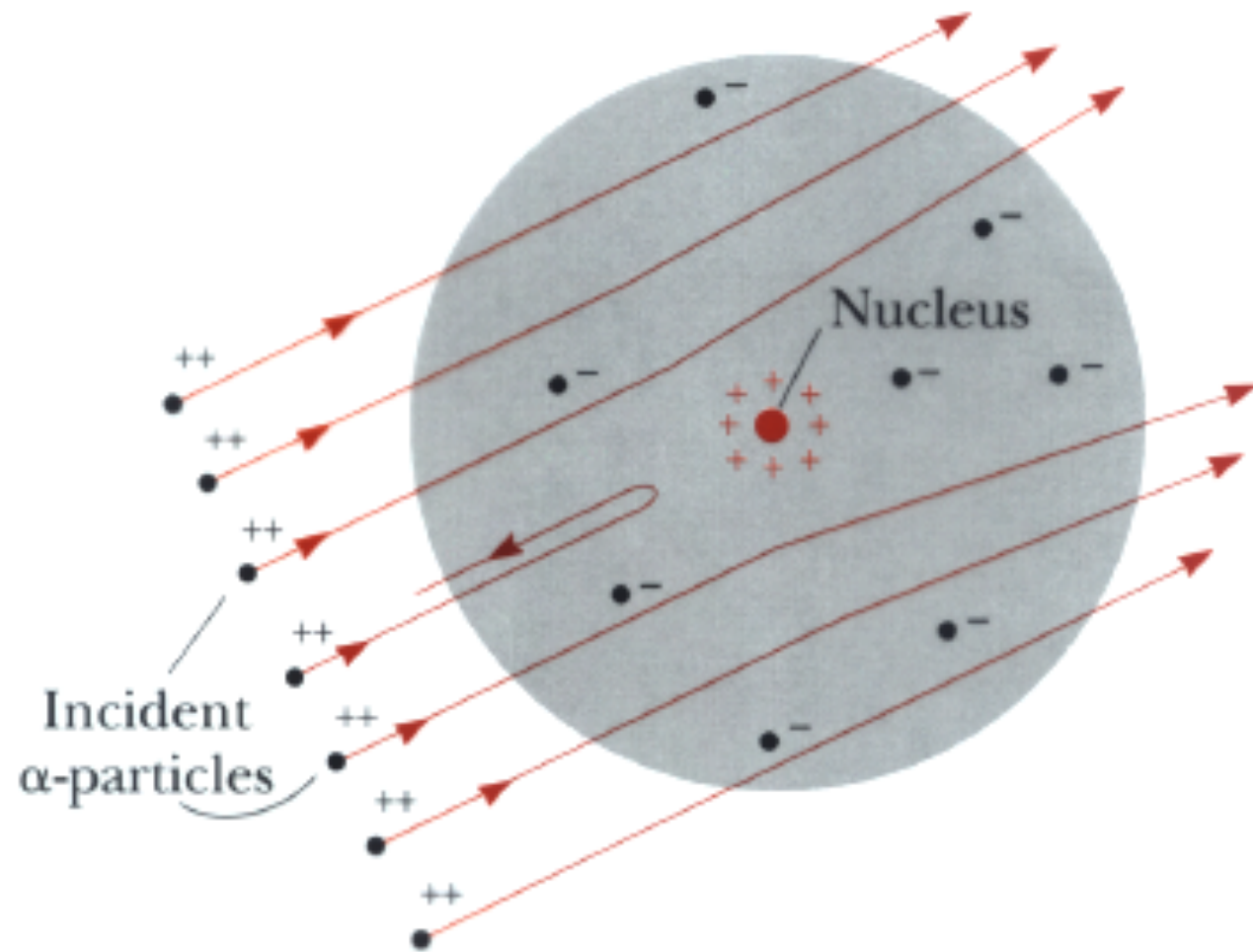
# Probing Within an Atom with $\alpha$ Particles



- Most  $\alpha$  particles pass thru gold foil with no deflection
- SOME ( $\cong 10^{-4}$ ) scatter at LARGE angles  $\Phi$
- Even fewer scatter almost backwards  $\rightarrow$  Why

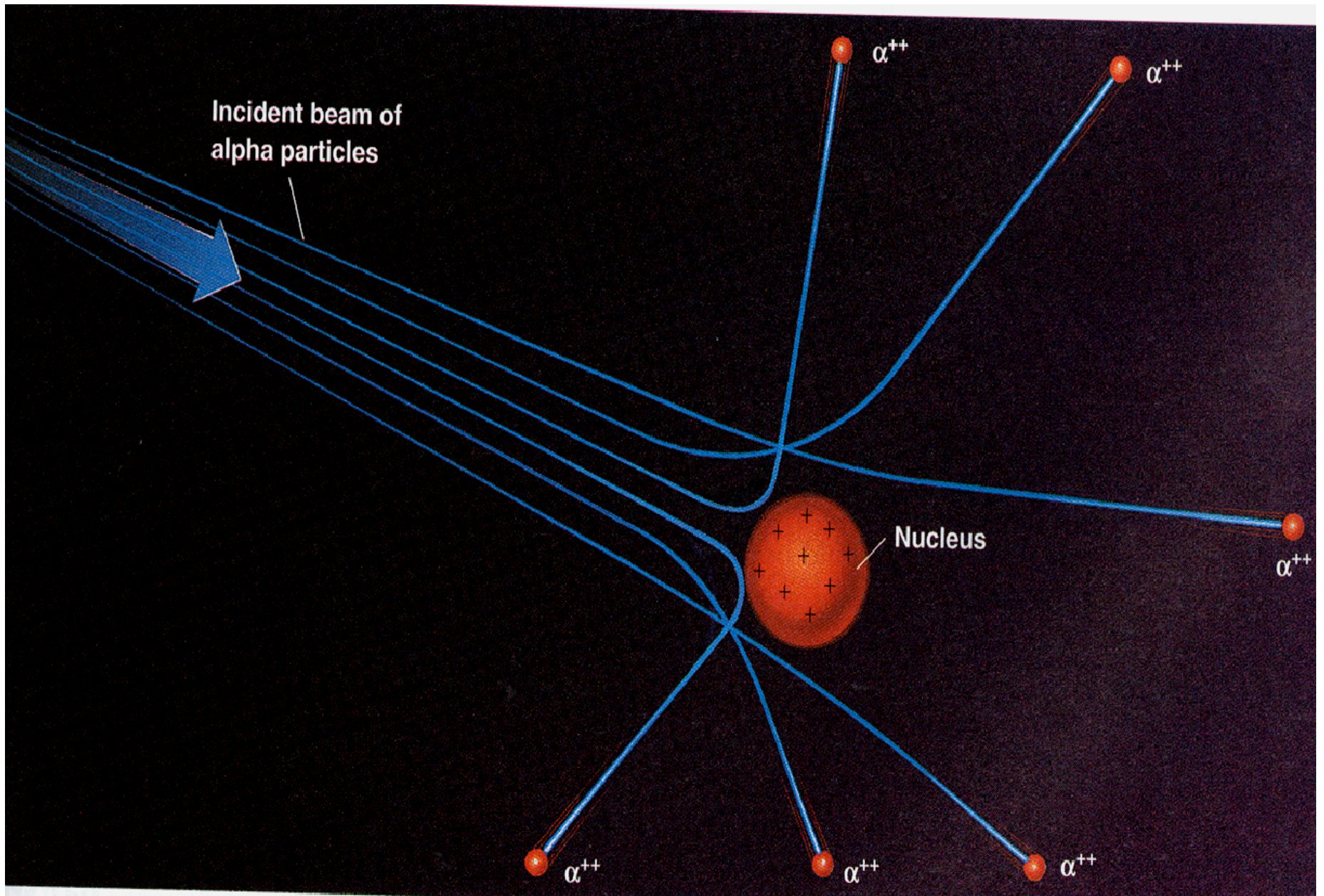


“Rutherford Scattering” discovered by his PhD Student (Marsden)

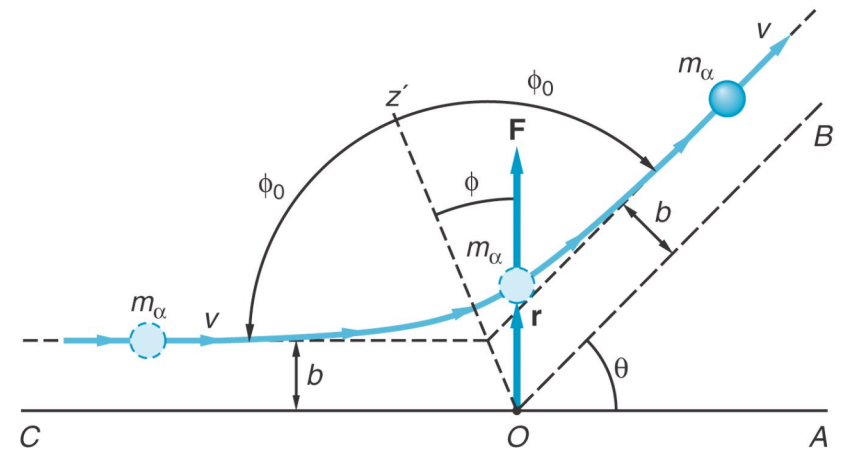
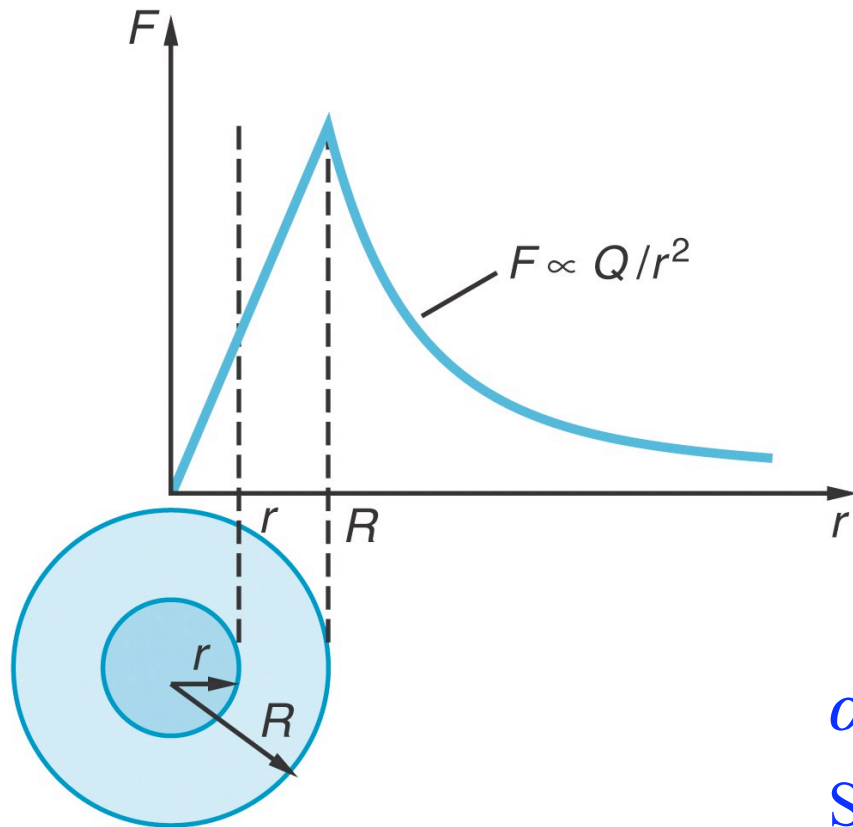




# Rutherford Discovers Nucleus (Nobel Prize)



# Force on $\alpha$ -particle due to heavy Nucleus



$\alpha$  particle trajectory is hyperbolic

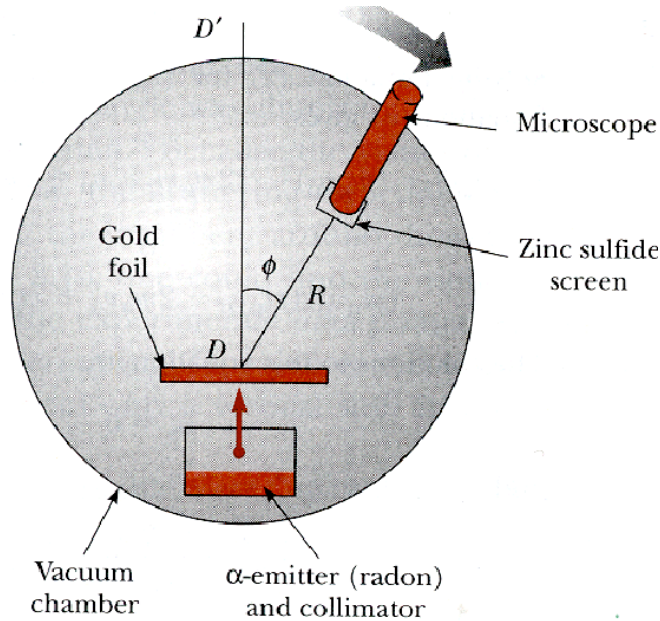
Scattering angle is related to impact par.

$$\text{Impact Parameter } b = \left( \frac{kq_\alpha Q}{m_\alpha v_\alpha^2} \right) \left( \cot \frac{\theta}{2} \right)$$

- Outside radius  $r = R$ ,  $F \propto Q/r^2$
- Inside radius  $r < R$ ,  $F \propto q/r^2 = Qr/R^2$
- Maximum force at radius  $r = R$

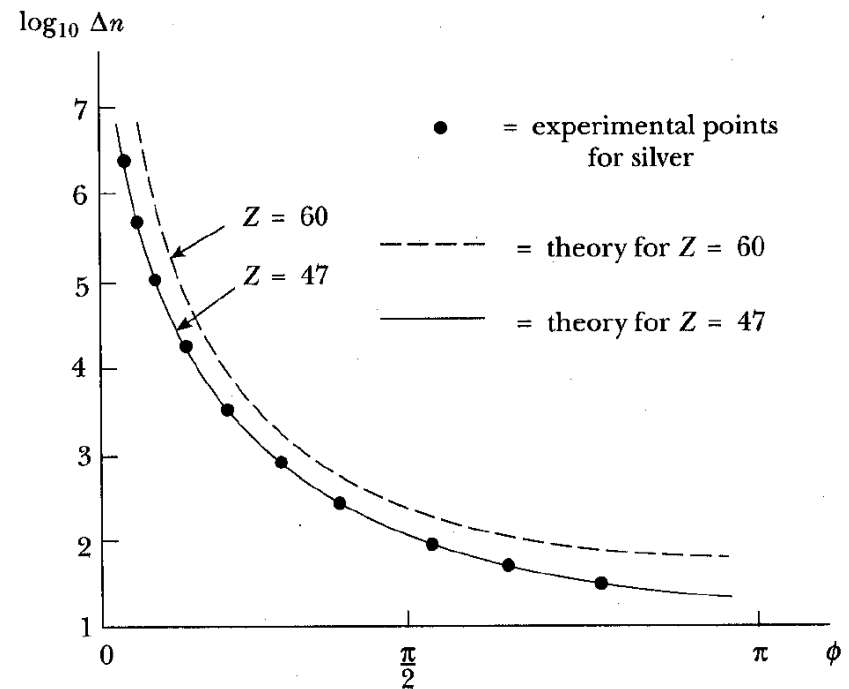


# Rutherford Scattering: Prediction and Experimental Result



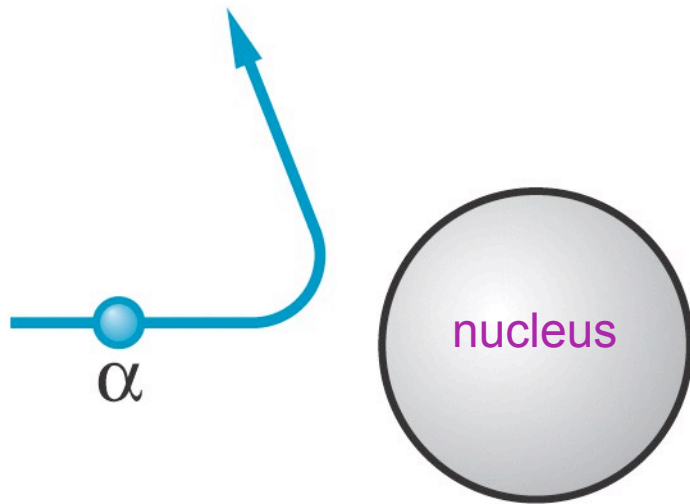
$$\Delta n = \frac{k^2 Z^2 e^4 N n A}{4R^2 \left( \frac{1}{2} m_\alpha v_\alpha^2 \right)^2 \text{Sin}^4(\phi / 2)}$$

- # scattered Vs  $\phi$  depends on :
  - $n$  = # of incident alpha particles
  - $N$  = # of nuclei/area of foil
  - $Ze$  = Nuclear charge
  - $K_\alpha$  of incident alpha beam
  - $A$  = detector area



# Rutherford Scattering & Size of Nucleus

(a)



distance of closest approach  $\propto$  r size of nucleus

$$\text{Kinetic energy of } \alpha = K_{\alpha} = \frac{1}{2} m_{\alpha} v_{\beta}^2$$

$\alpha$  particle will penetrate thru a radius r until all its kinetic energy is used up to do work AGAINST the Coulomb potential of the Nucleus:

$$K_{\alpha} = \frac{1}{2} m_{\alpha} v_{\beta}^2 = 8 \text{ MeV} = k \frac{(Ze)(2e)}{r}$$

$$\Rightarrow \boxed{r = \frac{2kZe^2}{K_{\alpha}}}$$

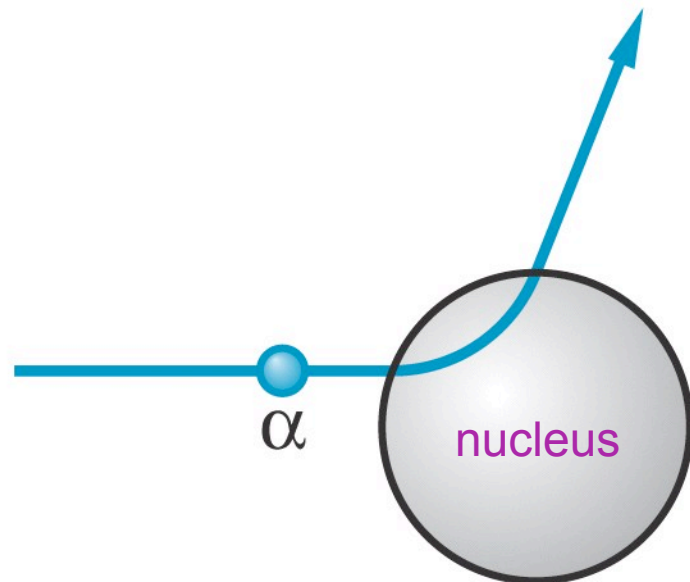
$$\text{For } K_{\alpha} = 7.7 \text{ MeV}, Z_{\text{Al}} = 13$$

$$\Rightarrow \boxed{r = \frac{2kZe^2}{K_{\alpha}} = 4.9 \times 10^{-15} \text{ m}}$$

Size of Nucleus =  $10^{-15} \text{ m}$

Size of Atom =  $10^{-10} \text{ m}$

(b)





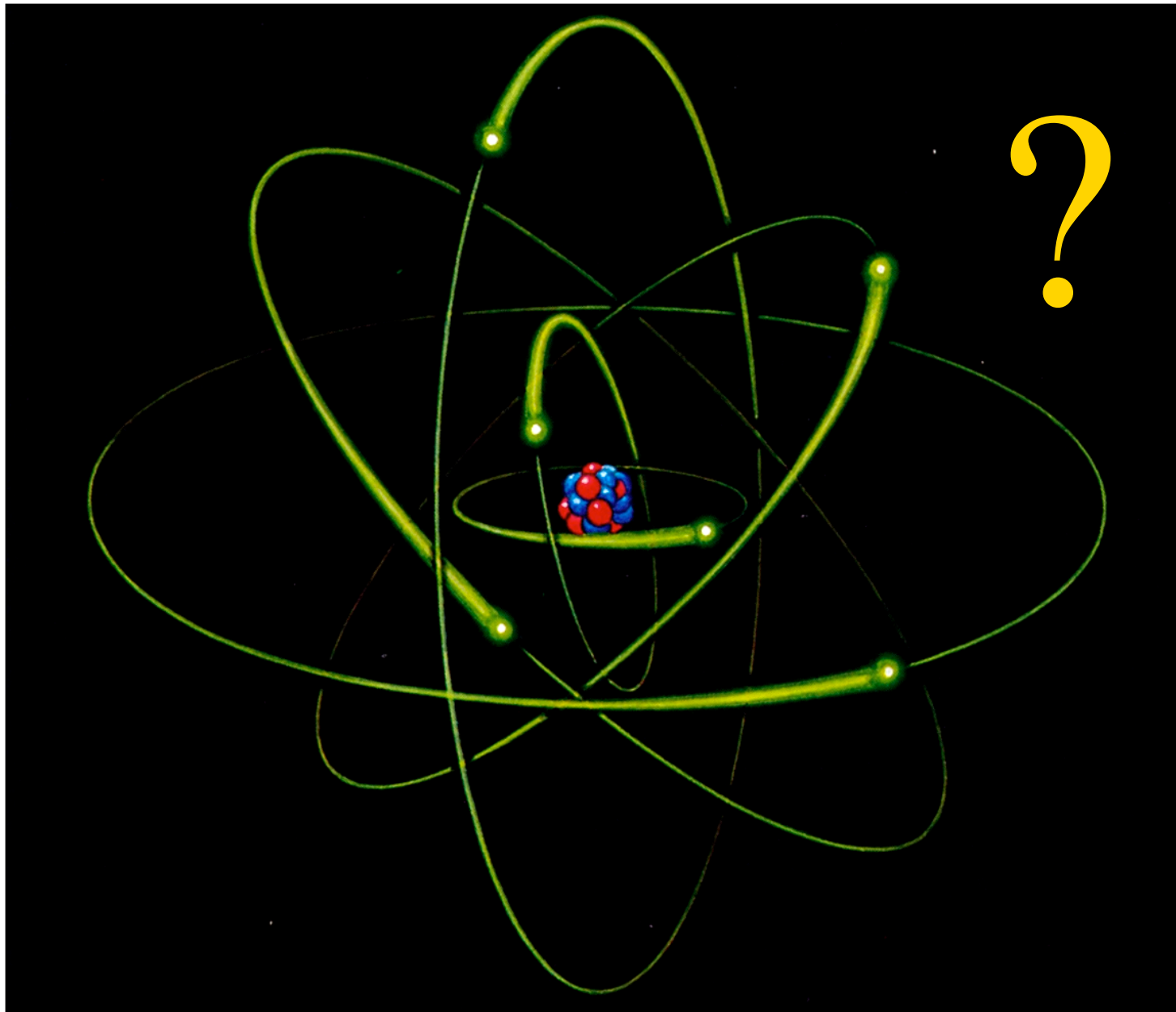
## Dimension Matters !

Size of Nucleus =  $10^{-15} m$

Size of Atom =  $10^{-10} m$

- how are the electrons located inside an atom?
- How are they held in a stable fashion?  
(necessary condition for us to exist )
  - What makes up the rest of the mass of the nucleus?
  - Why doesn't Coulomb repulsion blow the nucleus apart?
  - All these discoveries will require new experiments and observations

# Rutherford Atom & Classical Physics



# Continuous & Discrete spectra of Elements

Hot blackbody



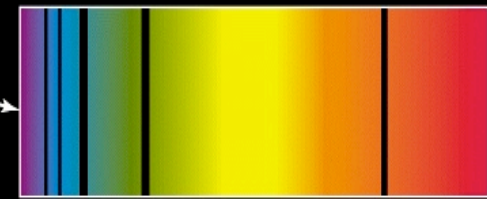
Prism



a Continuous spectrum

Cloud of cooler gas

Prism



b Absorption line spectrum

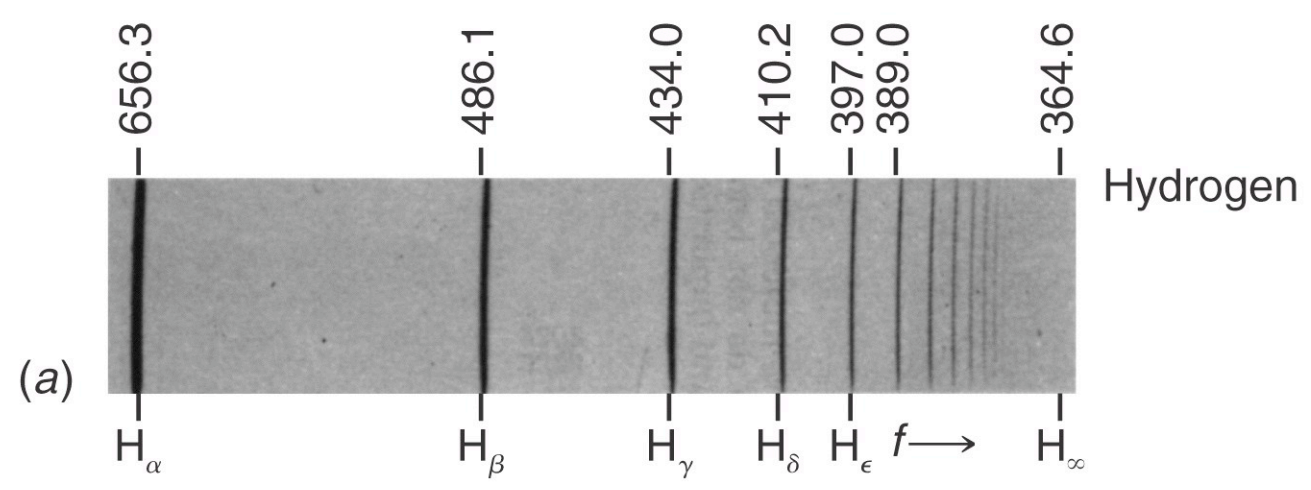
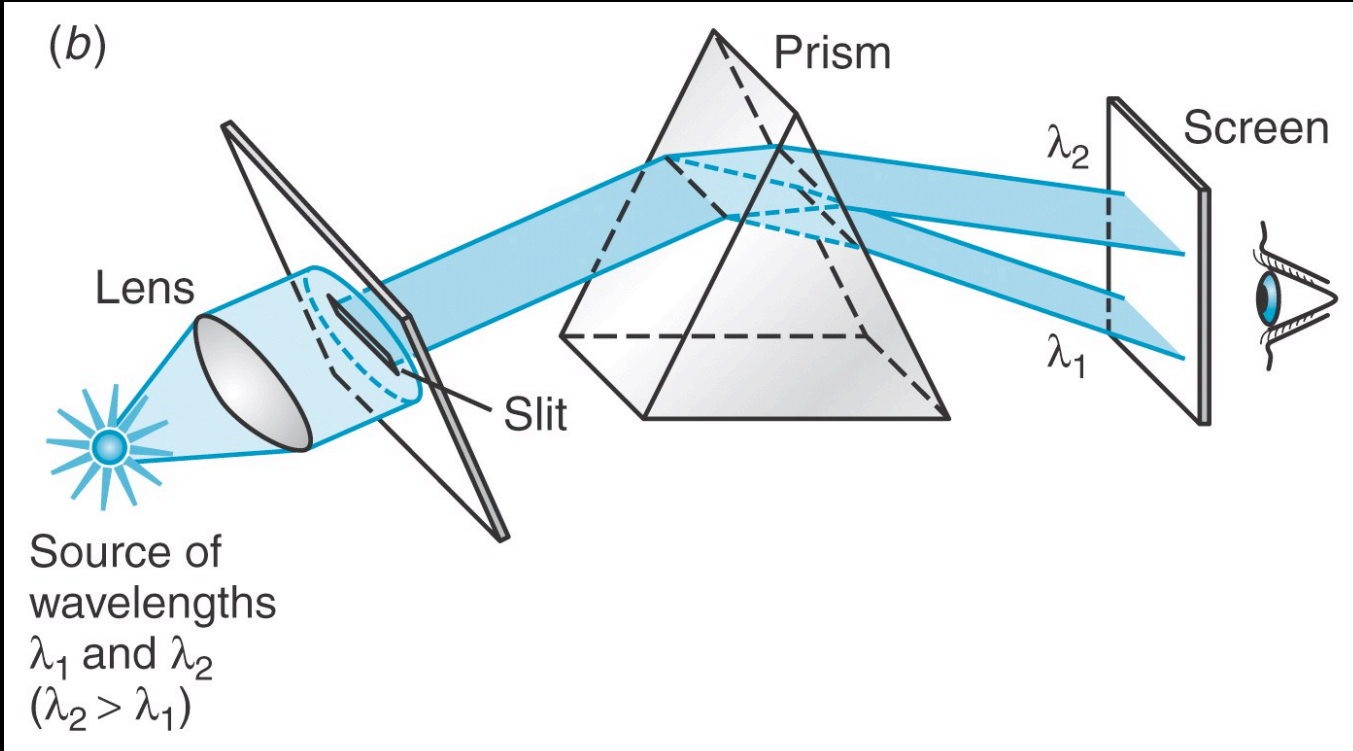


c Emission line spectrum

# Visible Spectrum of Sun Through a Prism

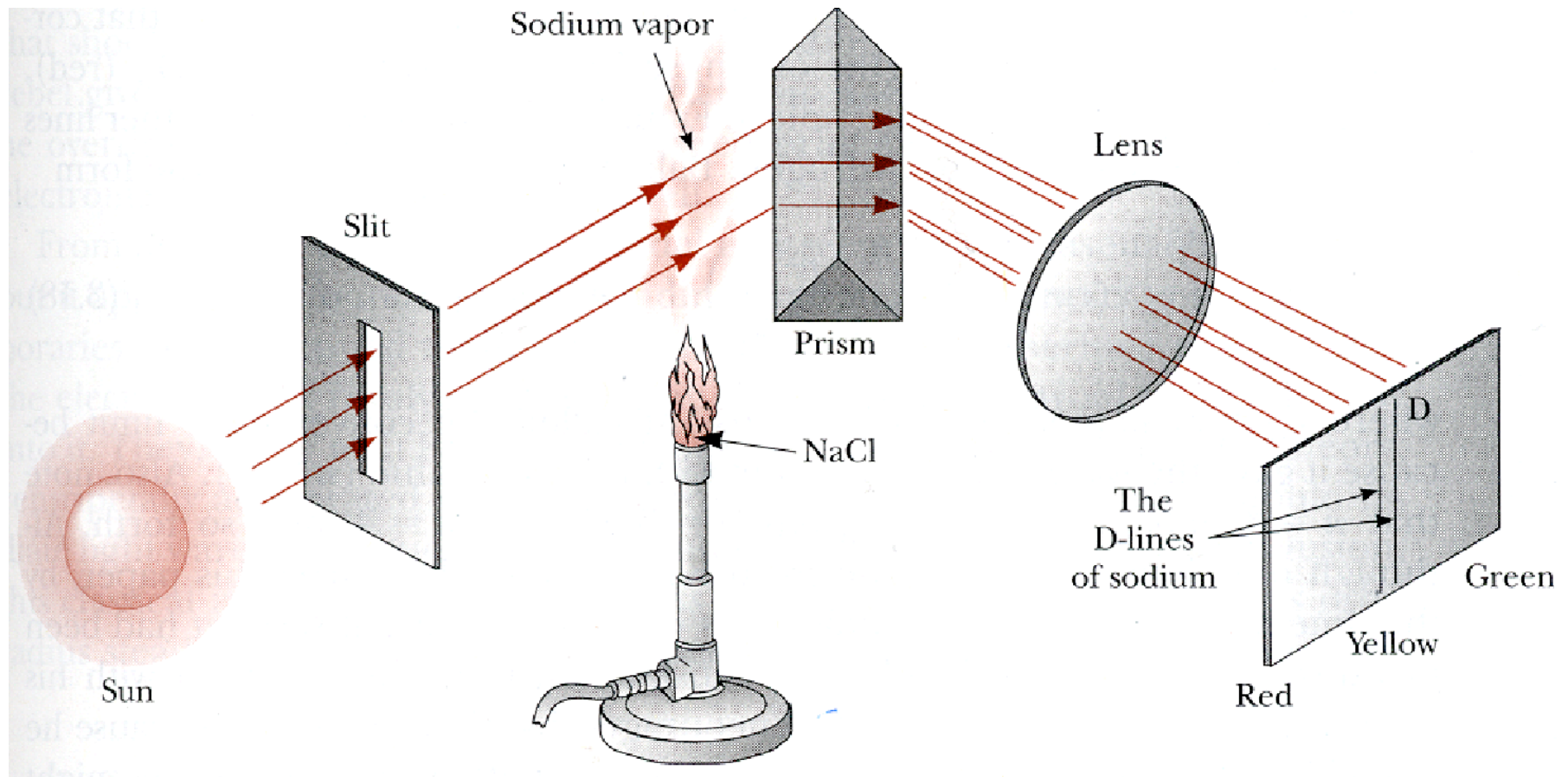


# Emission & Absorption Line Spectra of Elements



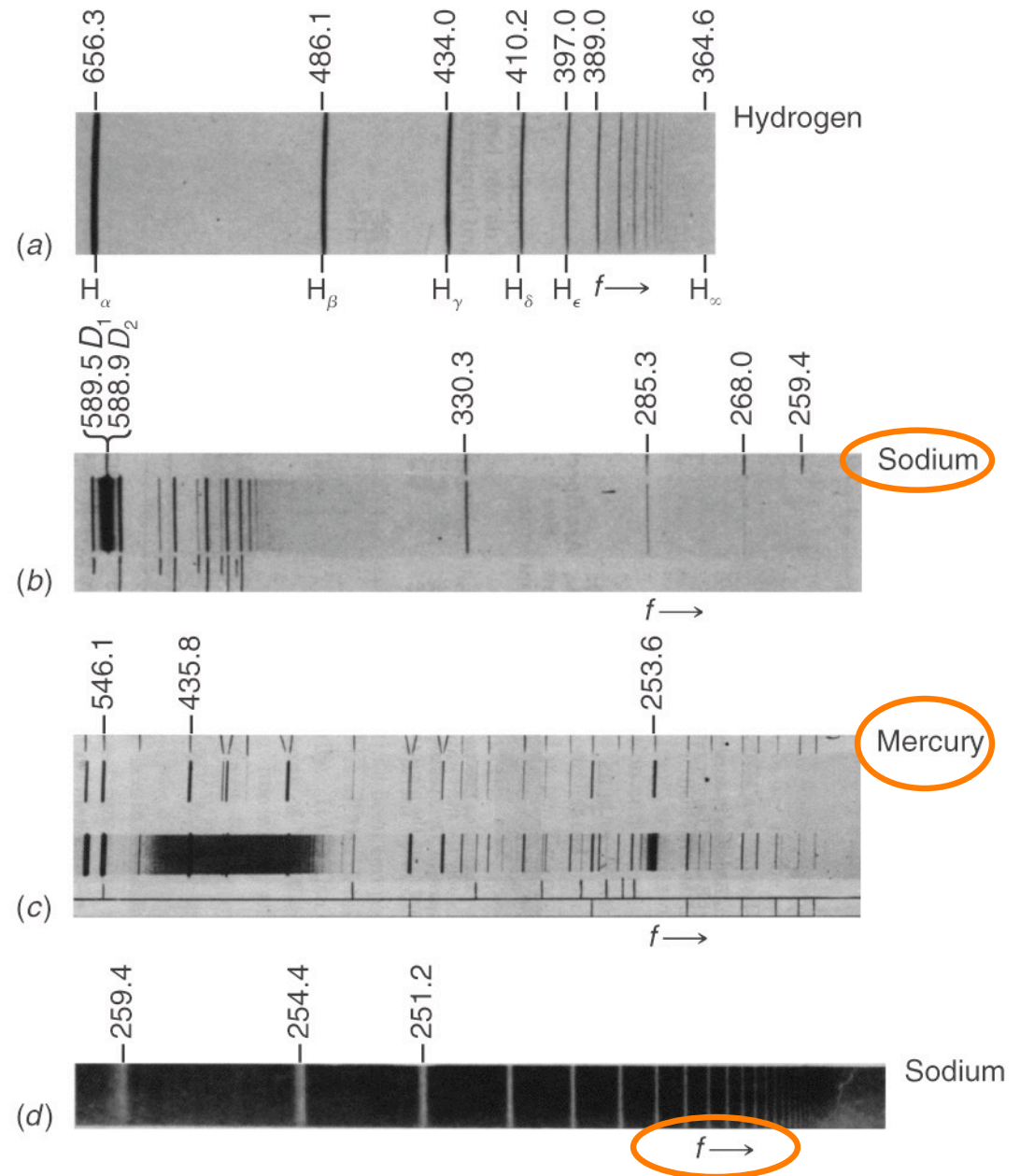


# Kirchhoff' Experiment : "D" Lines in Na



D lines **darken** noticeably when Sodium vapor introduced  
Between slit and prism

# Emission & Absorption Line Spectrum of Elements



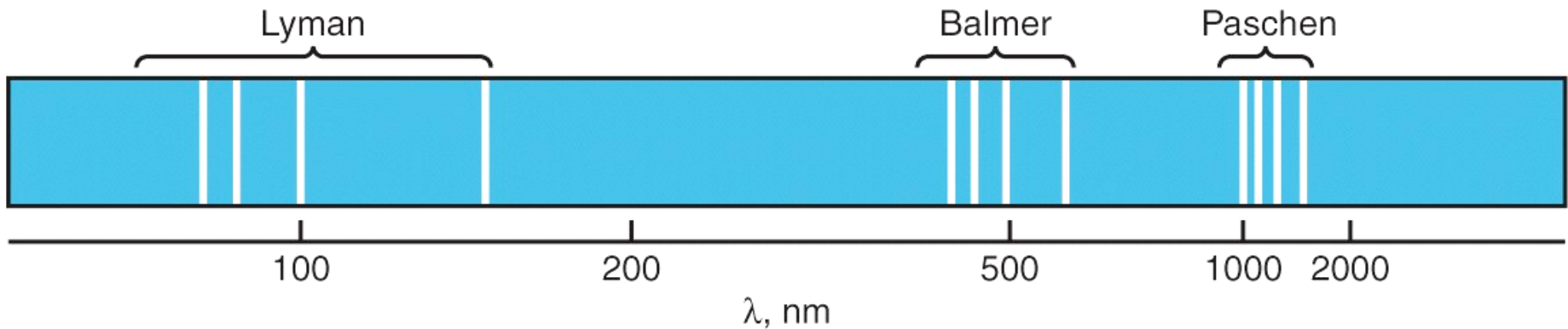
• Emission lines appear dark because of photographic exposure

Absorption spectrum of Na

While light passed thru Na vapor is absorbed at specific  $\lambda$



## Spectral Observations : series of lines with a pattern



- Empirical observation (by trial & error)
- All these series can be summarized in a simple formula

$$\frac{1}{\lambda} = R \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right), n_i > n_f, n_i = 1, 2, 3, 4..$$

Fitting to spectral line series data

$$R = 1.09737 \times 10^7 \text{ m}^{-1}$$

How does one explain this ?

# The Rapidly Vanishing Atom: A Classical Disaster !

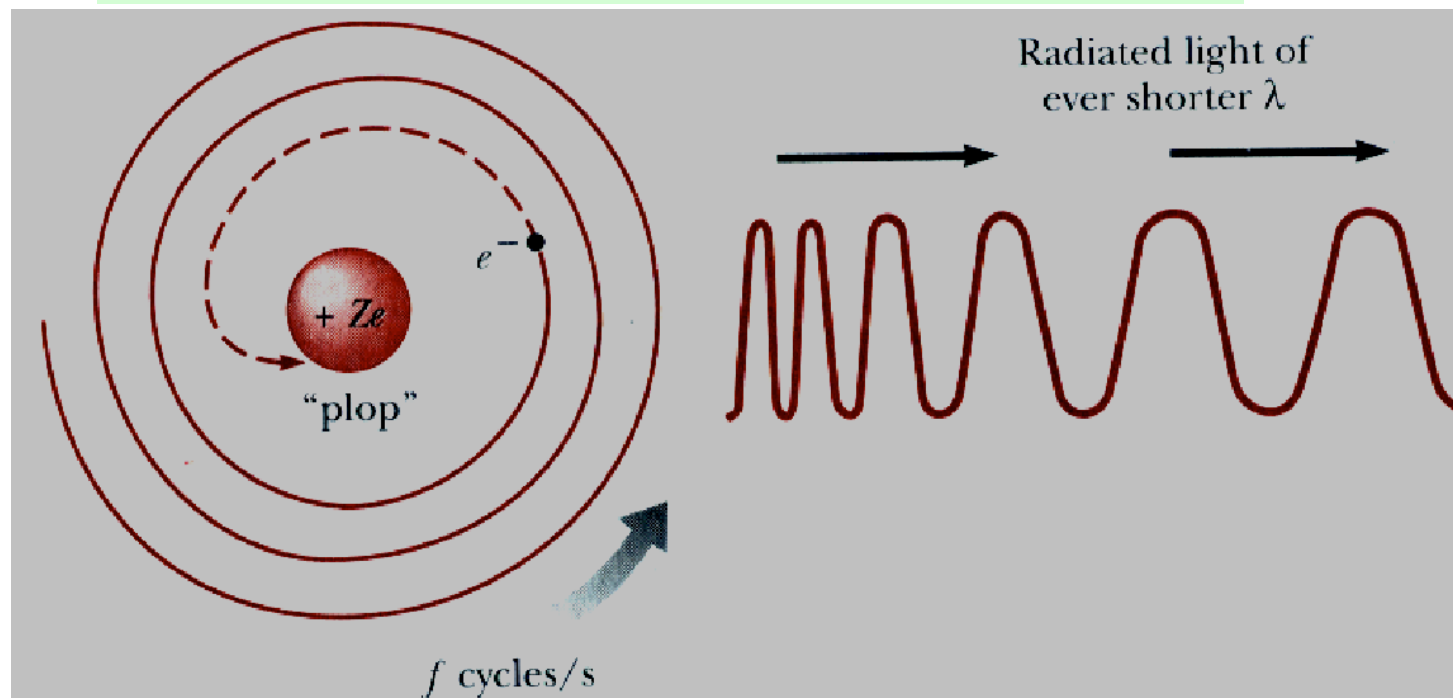
Not too hard to draw analogy with dynamics under another Central Force

Think of the Gravitational Force between two objects and their circular orbits.

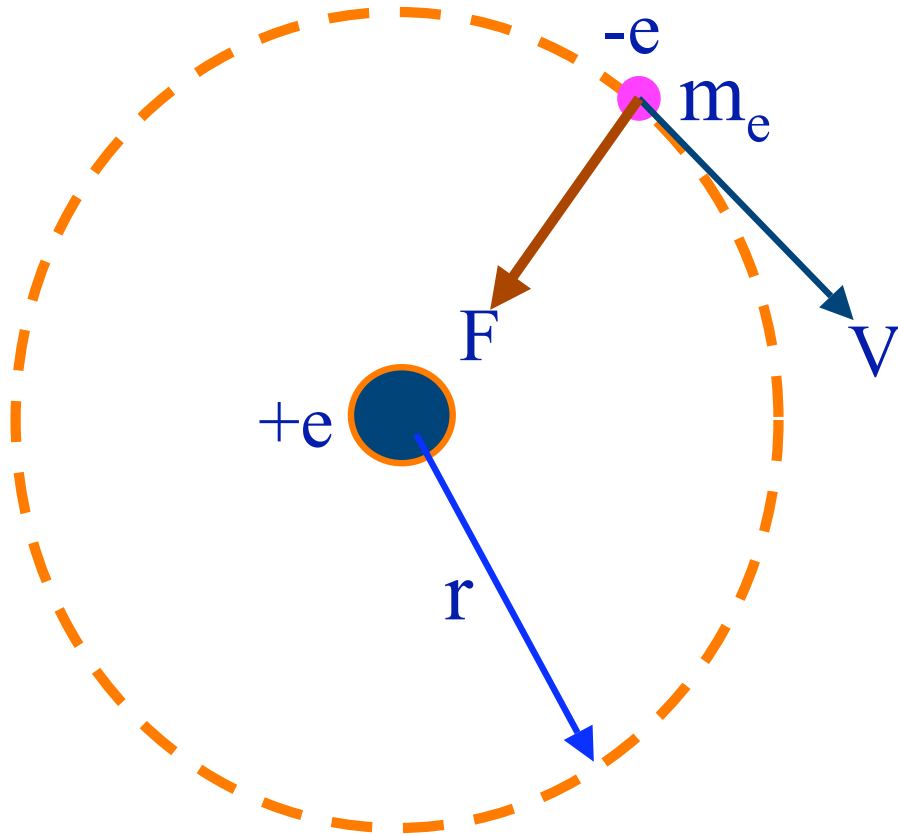
Perhaps the electron rotates around the Nucleus and is bound by their electrical charge

$$F = G \frac{M_1 M_2}{r^2} \Rightarrow \boxed{k \frac{Q_1 Q_2}{r^2}}$$

Laws of E&M destroy this equivalent picture : Why ?



# Bohr's Bold Model of Atom: Semi Quantum/Classical



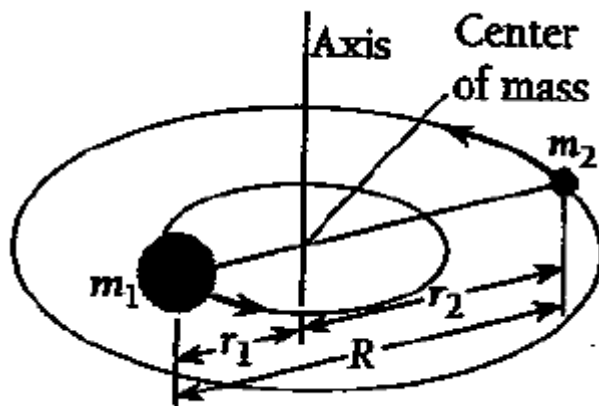
$$U(r) = -k \frac{e^2}{r}$$

$$KE = \frac{1}{2} m_e v^2$$

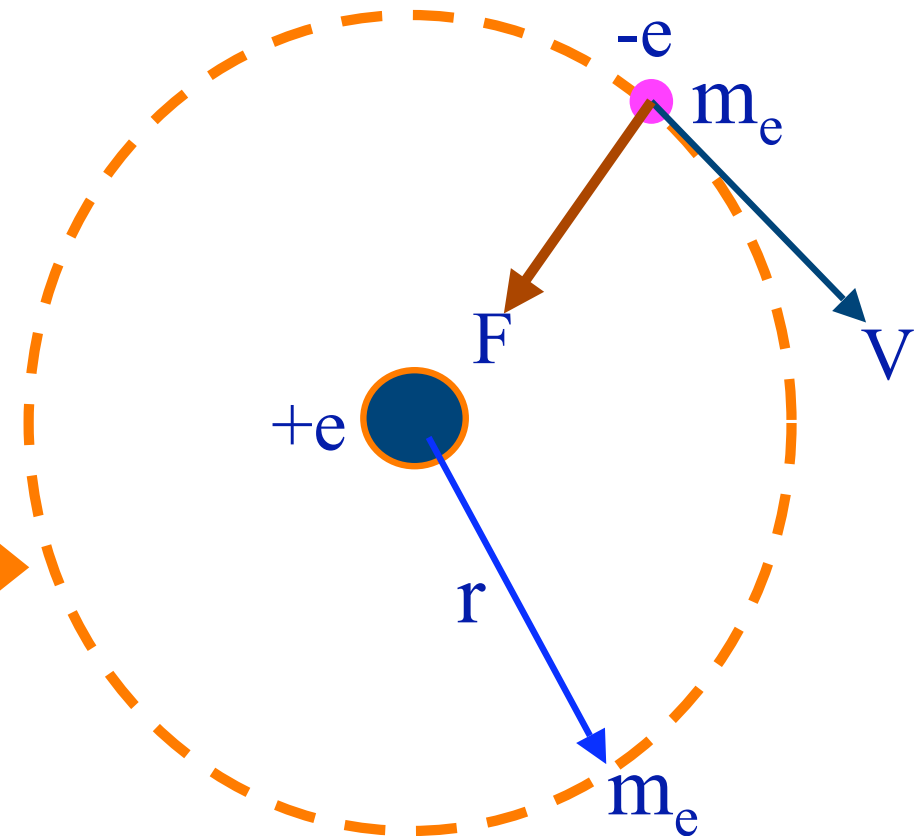
1. Electron in circular orbit around proton with vel= $v$
2. Only stationary orbits allowed . Electron does not radiate when in these stable (stationary) orbits
3. Orbits quantized:
  - $M_e v r = n h/2\pi$  ( $n=1,2,3\dots$ )
4. Radiation emitted when electron “jumps” from a stable orbit of higher energy  $\rightarrow$  stable orbit of lower energy  $E_f - E_i = hf = hc/\lambda$
5. Energy change quantized
  - $f$  = frequency of radiation

# Reduced Mass of 2-body system

General Two body Motion  
under a central force



reduces to



- Both Nucleus &  $e^-$  revolve around their common center of mass (CM)
- Such a system is equivalent to single particle of “reduced mass”  $\mu$  that revolves around position of Nucleus at a distance of ( $e^-$  - N) separation
  - $\mu = (m_e M) / (m_e + M)$ , when  $M \gg m_e$ ,  $\mu = m_e$  (Hydrogen atom)
  - Not so when calculating Muon ( $m_\mu = 207 m_e$ ) or equal mass charges rotating around each other (similar to what you saw in gravitation)

# Allowed Energy Levels & Orbit Radii in Bohr Model

$$E = KE + U = \frac{1}{2} m_e v^2 - k \frac{e^2}{r}$$

Force Equality for Stable Orbit

$\Rightarrow$  Coulomb attraction = CP Force

$$k \frac{e^2}{r^2} = \frac{m_e v^2}{r}$$

$$\Rightarrow KE = \frac{m_e v^2}{2} = k \frac{e^2}{2r}$$

Total Energy  $E = KE + U = -k \frac{e^2}{2r}$

Negative E  $\Rightarrow$  Bound system

This much energy must be added to the system to break up the bound atom

Radius of Electron Orbit :

$$mvr = n\hbar$$

$$\Rightarrow v = \frac{n\hbar}{mr},$$

substitute in KE =  $\frac{1}{2} m_e v^2 = \frac{ke^2}{2r}$

$$\Rightarrow r_n = \frac{n^2 \hbar^2}{mke^2}, \quad n = 1, 2, \dots, \infty$$

$n = 1 \Rightarrow$  Bohr Radius  $a_0$

$$a_0 = \frac{1^2 \hbar^2}{mke^2} = 0.529 \times 10^{-10} \text{ m}$$

In general  $r_n = n^2 a_0; n = 1, 2, \dots, \infty$

Quantized orbits of rotation

# Energy Level Diagram and Atomic Transitions

$$E_n = K + U = \frac{-ke^2}{2r}$$

since  $r_n = a_0 n^2$ ,  $n$  = quantum number

$$E_n = \frac{-ke^2}{2a_0 n^2} = -\frac{13.6}{n^2} \text{ eV}, \quad n = 1, 2, 3, \dots$$

Interstate transition:  $n_i \rightarrow n_f$

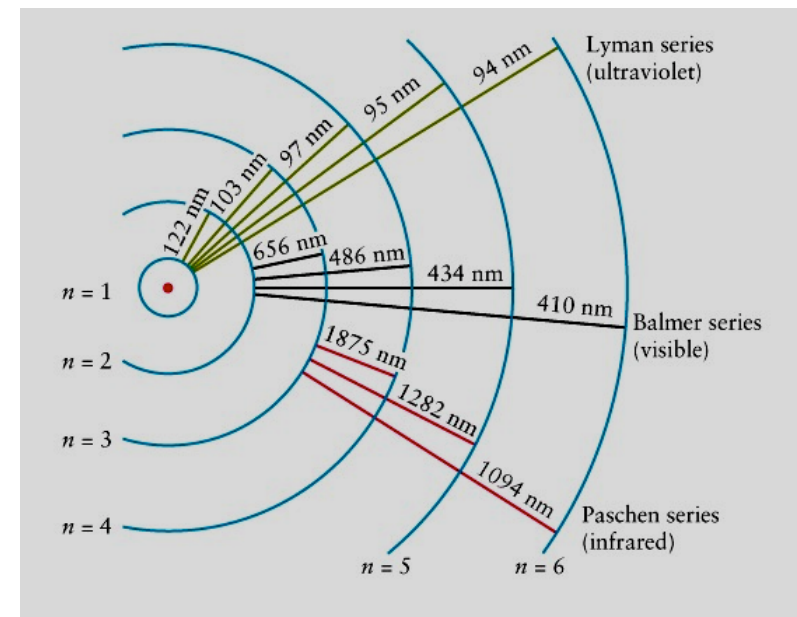
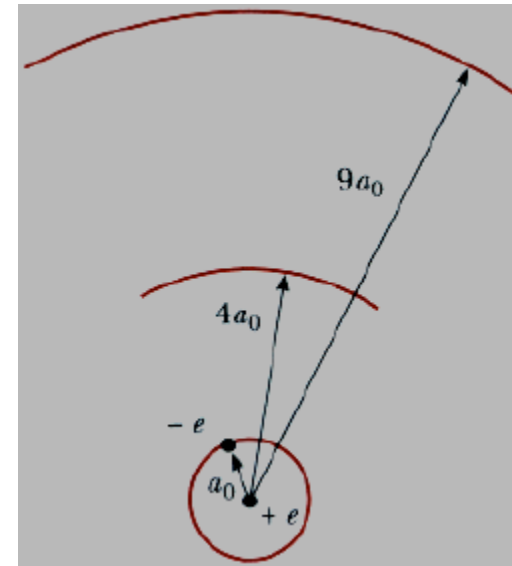
$$\Delta E = hf = E_i - E_f$$

$$= \frac{-ke^2}{2a_0} \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$

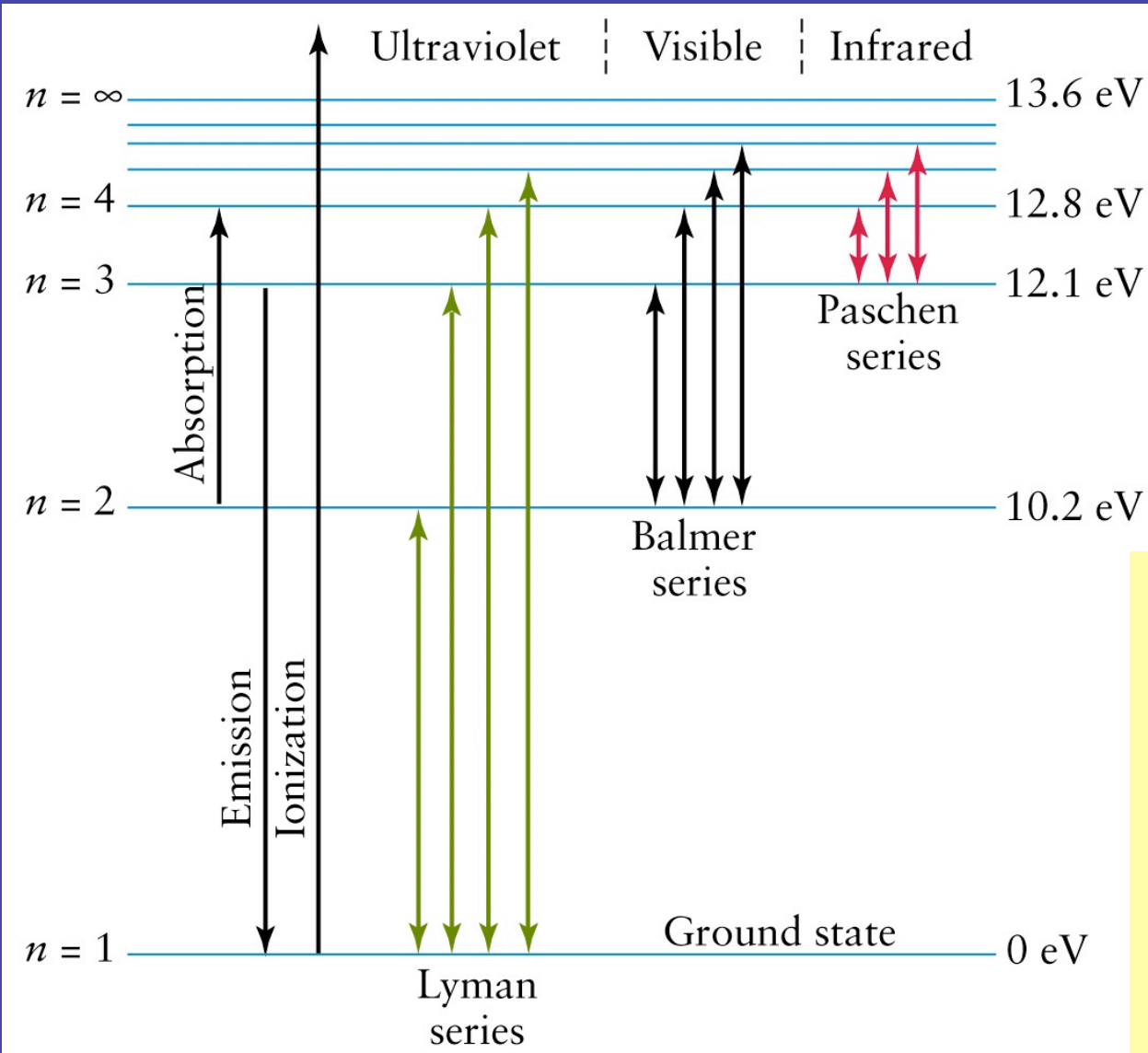
$$f = \frac{ke^2}{2ha_0} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{ke^2}{2hca_0} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$= \mathbf{R} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$



# Hydrogen Spectrum: as explained by Bohr



$$E_n = - \left( \frac{ke^2}{2a_0} \right) \frac{Z^2}{n^2}$$

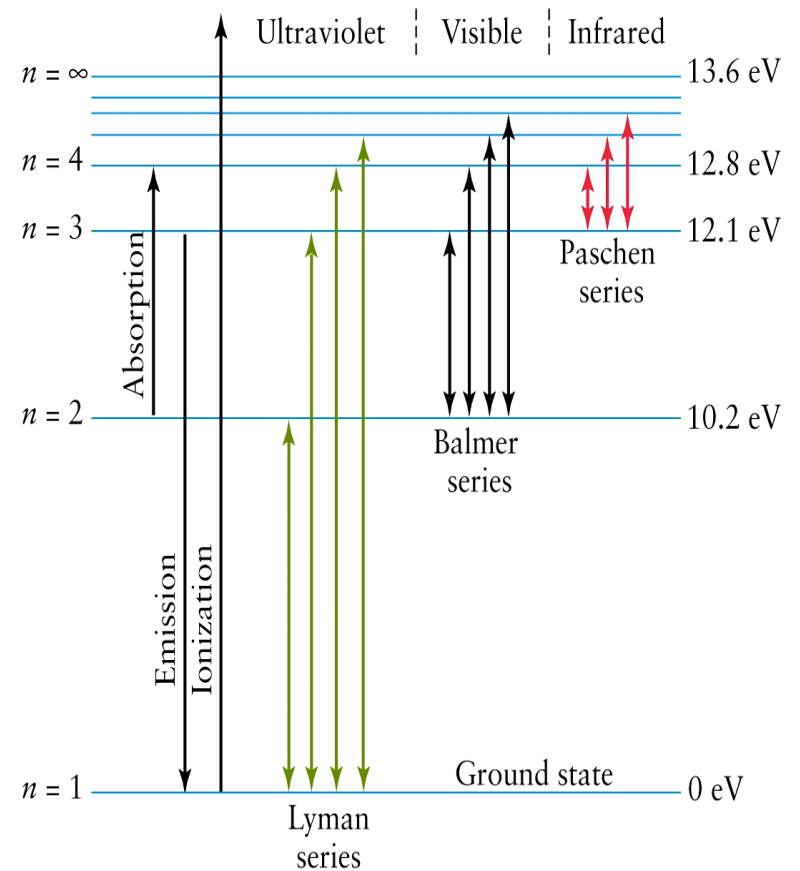
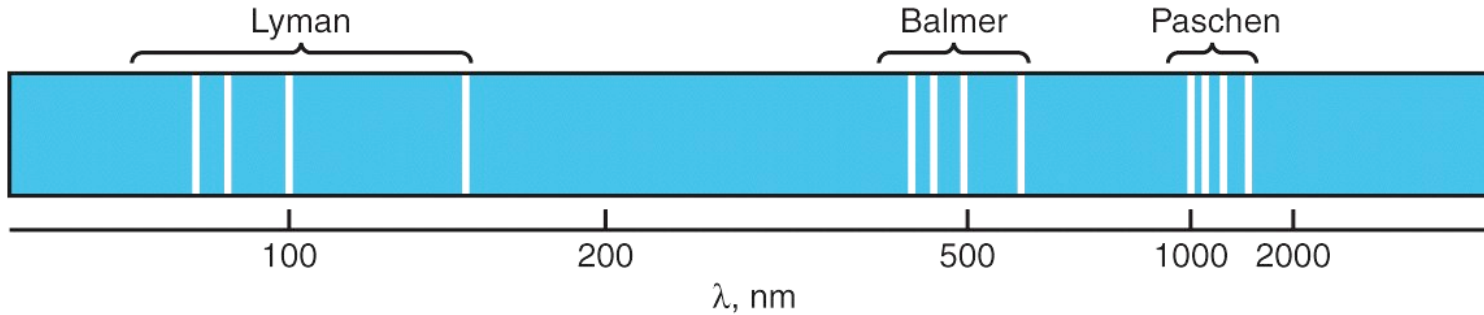
Bohr's "R" same as the Rydberg Constant

**R**

derived empirically from photographs of the spectral series



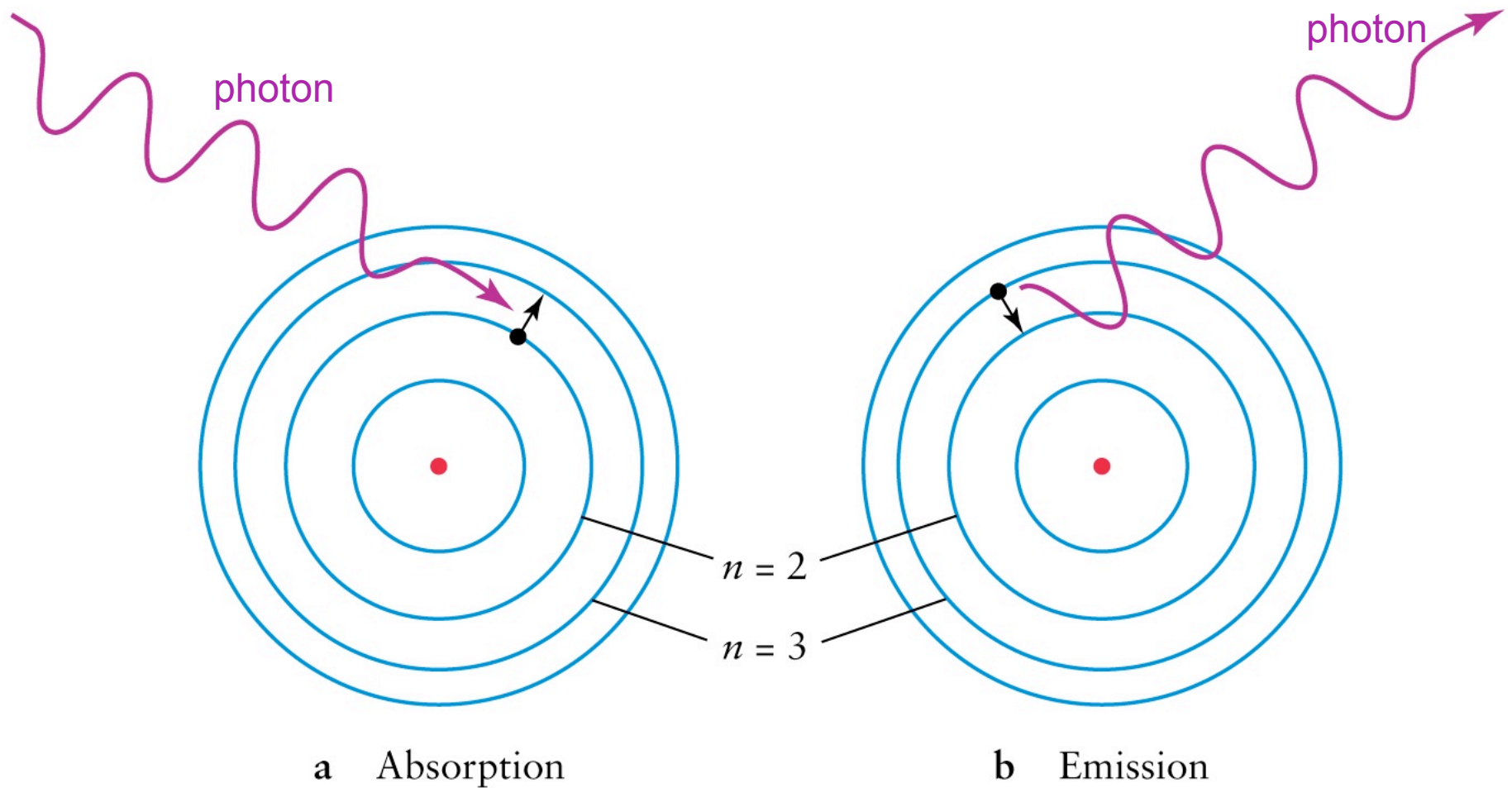
# Another Look at the Energy levels



$$E_n = - \left( \frac{ke^2}{2a_0} \right) \frac{Z^2}{n^2}$$

Rydberg Constant

# Bohr's Atom: Emission & Absorption Spectra



## Some Notes About Bohr Like Atoms

- Ground state of Hydrogen atom ( $n=1$ )  $E_0 = -13.6 \text{ eV}$
- Method for calculating energy levels etc applies to all Hydrogen-like atoms  $\rightarrow -1e$  around  $+Ze$ 
  - Examples :  $\text{He}^+$ ,  $\text{Li}^{++}$
- Energy levels would be different if replace electron with Muons
- Bohr's method can be applied in general to all systems under a central force (e.g. gravitational instead of Coulombic)