Physics 2D HW9 Solutions Ch. 8 #1 p.1

mass m in 3D box $L_1 \times L_2 \times L_3$

Find six lowest energy states when $L_1 = L, L_2 = 2L, L_3 = 2L$.

Indicate which of these are degenerate.

The 3D particle in a box is covered in Section 8.1

Most of this discussion focuses on the case of $L_1 = L_2 = L_3 = L$.

We have to back up to the formula:

$$E = \frac{1}{2m}\left(|p_x|^2 + |p_y|^2 + |p_z|^2\right) = \frac{\pi^2\hbar^2}{2m}\left(\frac{n_1^2}{L^2} + \frac{n_2^2}{(2L)^2} + \frac{n_3^2}{(2L)^2}\right)$$

$$= \frac{\pi^2\hbar^2}{2mL^2}\left(n_1^2 + \frac{n_2^2}{4} + \frac{n_3^2}{4}\right)$$

We are looking for the 6 <u>lowest</u> states.

n_1	n_2	n_3	E	
1	1	1	$\frac{\pi^2\hbar^2}{2mL^2}\left(1 + \frac{1}{4} + \frac{1}{4}\right)$	
1	2	1	$\frac{\pi^2\hbar^2}{2mL^2}\left(1 + 1 + \frac{1}{4}\right)$	degenerate
1	1	2	$\frac{\pi^2\hbar^2}{2mL^2}\left(1 + \frac{1}{4} + 1\right)$	
1	2	2	$\frac{\pi^2\hbar^2}{2mL^2}\left(1 + 1 + 1\right)$	
1	1	3	$\frac{\pi^2\hbar^2}{2mL^2}\left(1 + \frac{1}{4} + \frac{9}{4}\right)$	degenerate
1	3	1	$\frac{\pi^2\hbar^2}{2mL^2}\left(1 + \frac{9}{4} + \frac{1}{4}\right)$	

Notice that raising the "n" corresponding to y and z is less "energy costly" than in the x direction because the box is bigger in those directions. Also, because some of the directions are the same size, some combinations of n values give the same energy. These states are called "degenerate".

Physics 2D HW 9 Solutions Ch 8 #3 p.1

3D box with $L_1 = L_2 = L_3 = L$, mass m in $n^2 = 11$

What is the energy of the particle?

Equation 8.9 $E = \frac{\pi^2\hbar^2}{2mL^2}\underbrace{(n_1^2+n_2^2+n_3^2)}_{n^2} = \frac{\pi^2\hbar^2}{2mL^2}(1^2+1^2+3^2) = \boxed{\frac{11}{2}\frac{\pi^2\hbar^2}{mL^2}}$

(b) What combinations of ns will give this energy?

	n_1	n_2	n_3
(i)	1	1	3
(ii)	1	3	1
(iii)	3	1	1

all of these give $n^2 = 11$

(c) The wave functions are given by Equation 8.10

$\psi(x,y,z,t) = \left(\frac{2}{L}\right)^{3/2} \sin(k_1 x)\sin(k_2 y)\sin(k_3 z)e^{-i\omega t}$

(i) $k_3 = \frac{3\pi}{L}$, $k_1 = k_2 = \frac{\pi}{L}$

(ii) $k_2 = \frac{3\pi}{L}$, $k_1 = k_3 = \frac{\pi}{L}$

(iii) $k_1 = \frac{3\pi}{L}$, $k_2 = k_3 = \pi/L$

#4 p.1

2D box $L \times L$

What are the wave functions?

The wavefunctions for 1D, 2D and 3D boxes are almost the same except for the normalization constant.

$\psi(x,y) = \left(\frac{2}{L}\right)\sin\left(\frac{n_1\pi}{L}x\right)\sin\left(\frac{n_2\pi}{L}y\right)$

$n_1 = n_2 = 1$ gives the ground state ← not degenerate

$n_1 = 1, n_2 = 2$ or $n_1 = 2, n_2 = 1$ gives the first excited state

↑

degenerate

with $E_n = \frac{\pi^2\hbar^2}{2mL^2}(n_1^2+n_2^2)$

Physics 2D HW9 Solutions Ch. 8 #5 p.1

3D box w/ $L = 2\times10^{-14}$ m, $m = m_p$ (proton)

What is the ground state energy?

$$E_n = \frac{\pi^2\hbar^2}{2mL^2}(n_1^2+n_2^2+n_3^2)$$

$n_1 = n_2 = n_3 = 1 \Rightarrow E = 1.54$ MeV

What are the energyies of the first and second excited states?

$n_1 = 2,\ n_2 = n_3 = 1 \Rightarrow E = 3.08$ MeV

$n_1 = 2,\ n_2 = 2,\ n_3 = 1 \Rightarrow E = 4.63$ MeV

What are the degeneracies?

Note that in both cases, you can have 3 combinations of ns will give that energy. $\Rightarrow$ 3-fold degenerate

#7 p.1

The normalization condition is $$\int_0^{L_1}\int_0^{L_2}\int_0^{L_3}|\Psi(x,y,z)|^2 dx\,dy\,dz = 1$$

This is not as bad as it looks because the wavefunctions are real and seperable, so the integral can be rewritten:

$$A^2\int_0^{L_1}\sin^2(k_1x)dx\int_0^{L_2}\sin^2(k_2y)dy\int_0^{L_3}\sin^2(k_3z)dz = 1$$

Obviously all of these integrals will be the same, so

$$\int_0^{L_1}\sin^2(k_1x)dx = \int_0^{L_1}\left(\tfrac{1}{2}-\tfrac{1}{2}\cos(2k_1x)\right)dx = \left[\frac{x}{2}-\frac{1}{4k_1}\sin(2k_1x)\right]_0^{L_1} = \frac{L_1}{2}$$

so we get $A = \sqrt{\frac{2}{L_1}}\sqrt{\frac{2}{L_2}}\sqrt{\frac{2}{L_3}} = \sqrt{\frac{8}{V}}$ where V = volume $(L_1\times L_2\times L_3)$

For $L_1 = L_2 = L_3 = L$, $A = \left(\frac{2}{L}\right)^{3/2}$

Physics 2D HW9 Solutions Ch. 8 #10 p.1

electron $n=4$, $\ell=3$, $m_\ell=3$

What is the orbital angular momentum?

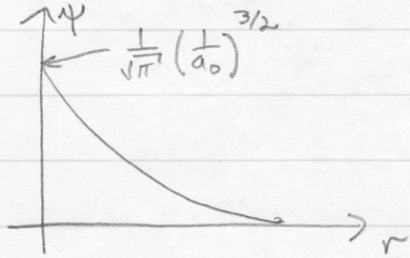
Equation 8.16: $|\vec{L}| = \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{3(3+1)}\,\hbar = 3.65\times10^{-34}$ Js

$L_z = m_\ell \hbar = 3\hbar = 3.16\times10^{-34}$ Js

#12 p.1

ground-state of hydrogen $\psi(r,\theta,\varphi) = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$

(a) Sketch ψ versus r



(b) This is basically a calculus question: look up 3D integrals in spherical polar coordinates. Note that the integrals over θ and φ are easy because the wave function does not depend on those coordinates.

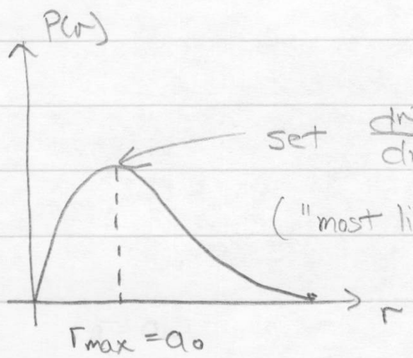
The probability is given by $|\psi|^2 dV$. For any given r, this dV corresponds to a spherical shell, the volume of which is given by $dV = 4\pi r^2 dr$

$$P = |\psi|^2 4\pi r^2 dr$$

Physics 2D HW 9 Solutions Ch. 8 #12 p.2

(c) Sketch $P(r)$ versus r and find r_{max}

Note that $P(r) \to 0$ as $r \to 0$ because $P(r) \propto r^2 e^{-r/a_0}$



set $\frac{dP}{dr} = 0$ to see this

("most likely to be found" $\Leftrightarrow$ highest $P(r)$)

(d) $4\pi \int_0^\infty |\psi|^2 r^2 dr = \frac{4\pi}{\pi a_0^3} \int_0^\infty e^{-2r/a_0} r^2 dr = 1$

use integration by parts, your calculator, computer, integral table or Professor (but not your TA)

(e) $P = \int_{r_1 = a_0/2}^{r_2 = 3a_0/2} \frac{4}{a_0^3} e^{-2r/a_0} r^2 dr = -\frac{17}{2} e^{-3} + \frac{5}{2} e^{-1} \simeq 0.496$

Physics 2D HW9 Solutions Ch. 8 #15 p.1

$\mu = \frac{mM}{m+M}$, Equation 8.38 : $E_n = -\frac{ke^2}{2a_0}\left\{\frac{Z^2}{n^2}\right\}$,

$a_0 = \frac{\hbar^2}{m_e e^2 k}$, change $\mu \leftrightarrow m_e \Rightarrow E_n = -\frac{\mu k^2 e^4}{2\hbar^2}\left\{\frac{Z^2}{n^2}\right\}$

(b) For an $n=3 \to n=2$ transition,

$$E_3 - E_2 = \frac{hc}{\lambda} = -\frac{\mu k^2 e^4 Z^2}{2\hbar^2}\left\{\frac{1}{3^2} - \frac{1}{2^2}\right\}$$

if $Z=1$ (Hydrogen) $\Rightarrow \lambda \simeq 656.3\,\text{nm}$
if $Z=2$ (Helium) $\Rightarrow \lambda \simeq 164.1\,\text{nm}$
} for these, $\mu \simeq m_e$

if $Z=1$ and $\mu = \frac{m_e}{2}$ (positronium) $\Rightarrow \lambda \simeq 1312.6\,\text{nm}$

#16 p.1

Calculate the possible values of the z component of angular momentum for an electron in a d subshell.

d subshell $\Rightarrow \ell = 2 \Rightarrow m_\ell = -2, -1, 0, 1, 2$; $L_z = m_\ell \hbar$

#17 p.1

Calculate L for 4d and 6f electrons in hydrogen.

$L = \sqrt{\ell(\ell+1)}\,\hbar$

4d $\Rightarrow \ell = 2 \Rightarrow L = 2.58 \times 10^{-34}$ Js
6f $\Rightarrow \ell = 3 \Rightarrow L = 3.65 \times 10^{-34}$ Js