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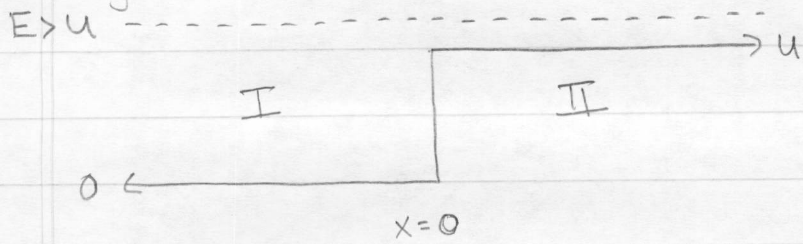

Region I: $U=0$ (free particle) $\Rightarrow \psi_{I}(x) = Ae^{ik_1x} + Be^{-ik_1x}$, $k_1 = \frac{\sqrt{2mE}}{\hbar}$

Region II: also free, but with energy $E-U$: $\psi_{II}(x) = Ce^{ik_2x} + De^{-ik_2x}$

$k_2 = \sqrt{\frac{2m(E-U)}{\hbar^2}}$

A is the amplitude of the wave moving right in Region I
B " " " " " " " left in Region I
C " " " " " " " right in Region II
D " " " " " " " left in Region II

The conditions that we are going to enforce are that the wave function and its first derivative be continuous everywhere (and at $x=0$ in particular).

We are asked to find the reflection coefficient R, so we imagine a wave train (plane wave) coming in from the left, moving right. Part of that wave will be reflected, part transmitted. However, there will be no reflected (left moving) wave in Region II, so we set $D=0$.

$$\Psi_I(0) = \Psi_{II}(0) \Rightarrow A + B = C$$

$$\Psi_I'(0) = \Psi_{II}'(0) \Rightarrow ik_1(A-B) = ik_2 C$$

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The definition of R is $R \equiv \left|\frac{B}{A}\right|^2$ so we need to eliminate C from our equations.

$$K_1 (A-B) = K_2 (A+B)$$

$$K_1 A - K_1 B = K_2 A + K_2 B$$

$$A(K_1 - K_2) = B(K_1 + K_2)$$

$$\Rightarrow \boxed{R = \left|\frac{B}{A}\right|^2 = \frac{(K_1-K_2)^2}{(K_1+K_2)^2}}$$

We know that $R+T=1$ (reflection plus transmission equals everything) so $\boxed{T = 1-R = 1 - \frac{(K_1-K_2)^2}{(K_1+K_2)^2}}$

$E \to U \Rightarrow K_2 \to 0 \Rightarrow \boxed{R = \frac{(K_1-0)^2}{(K_1+0)^2} = 1 \qquad T = 1-1 = 0}$ Complete reflection when $E = U$.

$E \to \infty \Rightarrow K_1 = K_2 \Rightarrow \boxed{R = 0, \ T = 1}$ Complete transmission

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$$K_1 = \frac{\sqrt{2mE}}{\hbar} = \frac{\sqrt{2(938.3\,\text{MeV}/c^2)(25\,\text{MeV})}}{\hbar} = 1.098\times10^{15}\,\text{m}^{-1}$$

$$K_2 = \frac{\sqrt{2m(E-U)}}{\hbar} = \frac{\sqrt{2(938.3\,\text{MeV}/c^2)(25\,\text{MeV}-20\,\text{MeV})}}{\hbar} = 4.908\times10^{14}\,\text{m}^{-1}$$

$$R = \frac{(1.747\times10^{14}\,\text{m}^{-1} - 7.811\times10^{13}\,\text{m}^{-1})^2}{(1.747\times10^{14}\,\text{m}^{-1} + 7.81\times10^{13}\,\text{m}^{-1})^2} = \boxed{0.146}$$

$$T = 1 - R = \boxed{0.854}$$

Since k is proportional to the $\sqrt{\text{mass}}$, and both R and T are given by ratios of Ks, this result will be particle indepenelent. $\boxed{\text{No change.}}$

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To solve this problem, notice that our derivation of R and T in problem 2 did not depend on the sign of U, only that $E > U$, so U could be negative, as it is in this problem.

$E = 54\,\text{eV}$, $U = -10\,\text{eV}$

$K_1 = \sqrt{2mE}/\hbar$, $K_2 = \sqrt{2m(E-U)}/\hbar$ (note: $E-U = 64\,\text{eV}$).

$$R = \frac{(K_1-K_2)^2}{(K_1+K_2)^2} = \boxed{0.180}$$ so 18% of the current is reflected, or $\boxed{1.8\times10^{-5}\,\text{A}}$

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The square barrier is discussed on pages 231-235. Equation 7.9 gives the transmission probability as a function of Energy:

$$T(E) = \left\{ 1 + \frac{1}{4}\left[\frac{U^2}{E(U-E)}\right]\sinh^2(\alpha L) \right\}^{-1}, \quad \alpha = \frac{\sqrt{2m(U-E)}}{\hbar}$$

The conditions that you are given in the problem are that $\frac{2mUL^2}{\hbar^2} \gg 1$ and $E \ll U$

Since $E \ll U$, $\alpha \simeq \frac{\sqrt{2mU}}{\hbar}$ and $\alpha^2 L^2 \approx \frac{2mUL^2}{\hbar^2} \gg 1$

The hyperbolic sine function can be expressed in terms of exponentials as: $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$

So, $\sinh(\alpha L) \simeq \frac{1}{2}e^{\alpha L}$ (b/c $e^{-\alpha L} \simeq 0$, $\alpha^2 L^2 \gg 1$).

Anytime you have an expression $(A+B)$ and you are told $A \gg B$ you can say $A+B \simeq A$. Likewise,

$$T(E) \cong \left\{ 1 + \frac{1}{4}\left[\frac{U}{E}\right]\left(\frac{1}{2}e^{\alpha L}\right)^2 \right\}^{-1} = \left\{ 1 + \frac{U}{16E}e^{2\alpha L} \right\}^{-1}$$

Since αL is large and $U \gg E$, we can get rid of this

$$\boxed{T(E) \simeq \frac{16E}{U} e^{-2\alpha L}}$$

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The second part of this problem amounts to plugging in numbers for $e^{-2\alpha L}$, $\alpha = \frac{\sqrt{2m(U-E)}}{\hbar}$

(1) $e^{-2\alpha L} \simeq 0.9$

(2) $\simeq 0.36$

(3) $= 0.41$

(4) $\simeq 0.00$ ← notice that the correspondence principle kicks in again

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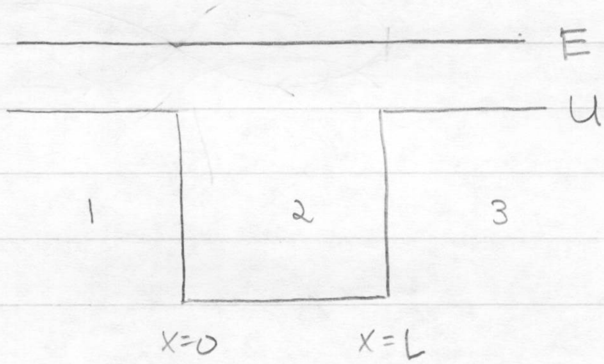
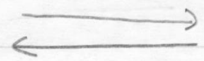
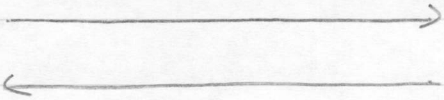


(a)



path of wave reflected @ x = 0



path of wave reflected @ x = L

The path length difference is $2L$. Normally, you would think that the condition $\lambda_2 = 2L$ would give you constructive interference when these waves recombine. However, the second wave undergoes an inversion (or 180° phase shift) when reflecting off the $x = L$ wall ⇒ you get destructive interference.

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In the three regions, you can think of the particle as free with different energies, either E or E-U.

$$\Psi_1 = Ae^{ik_1x} + Be^{-ik_1x} \qquad k_1 = \sqrt{2m(E-U)}/\hbar$$
$$\Psi_2 = Ce^{ik_2x} + De^{-ik_2x} \to 0 \qquad k_2 = \sqrt{2mE}/\hbar$$
$$\Psi_3 = Ee^{ik_3x} + \cancel{Fe^{-ik_3x}} \qquad k_3 = k_1$$

For a particle incident from the left (moving right), there will be no left moving wave in Region 3 $\Rightarrow F=0$.

Continuity equations:

$$\Psi_1(0) = \Psi_2(0) \Rightarrow A+B = C+D$$
$$\Psi_2(L) = \Psi_3(L) \Rightarrow Ce^{ik_2L} + De^{-ik_2L} = Ee^{ik_1L}$$
$$\Psi_1'(0) = \Psi_2'(0) \Rightarrow ik_1A - ik_1B = ik_2(C-D)$$
$$\Psi_2'(L) = \Psi_3'(L) \Rightarrow ik_2(Ce^{ik_2L} - De^{-ik_2L}) = ik_1Ee^{ik_1L}$$

We want to show that $B=0$ (no left moving wave in Region 1) when $\lambda_2 = 2L \Rightarrow k_2 = \frac{2\pi}{\lambda_2} = \frac{\pi}{L}$

So we can plug in $k_2L = \pi$ in our continuity equations. (note that $e^{\pm i\pi} = -1$ from $e^{ix} = \cos(x) + i\sin(x)$)

$$A+B = C+D = A-B \Rightarrow 2B = 0 \Rightarrow \boxed{B=0}$$
$$-C-D = Ee^{ik_1L} = \frac{1}{k_1}\frac{\pi}{L}(D-C) = \frac{(A-B)}{\frac{\pi}{L}(C-D)}\frac{\pi}{L}(D-C) = B-A$$
$$k_1(A-B) = \frac{\pi}{L}(C-D)$$
$$\frac{1}{k_1}\frac{\pi}{L}(D-C) = Ee^{ik_1L} \checkmark$$

I know the logic here is hard to follow, but you can figure it out from the boxed equations.