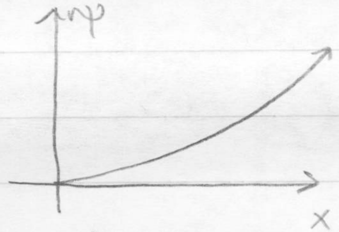
Physics 2D HW 7 Solutions Ch 6 #1 p.1

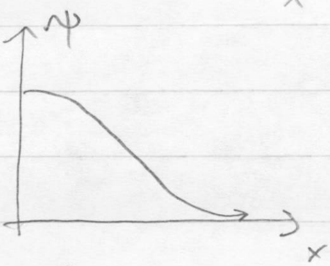
Which of these functions are reasonable wave functions?

(a)



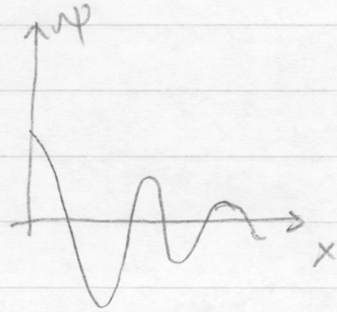
This one is no good because there is no way to normalize it; the area under ψ goes to infinity.

(b)



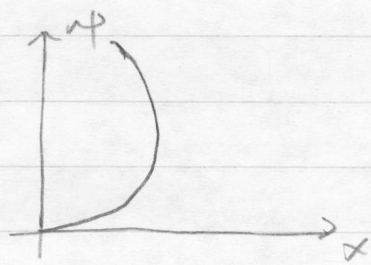
This one looks okay.

(c)



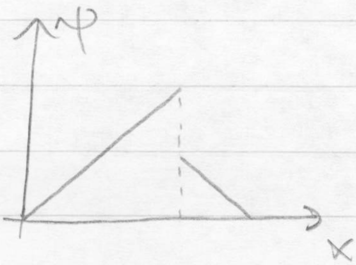
There is no rule against a wave function being negative - this looks okay.

(d)



Well this is not even a function (multivalued) so it is definitely not a wavefunction.

(e)



Discontinuous first derivatives are okay at infinite potentials, but discontinuity of the function itself is unforgivable - not a wavefunction.

Physics 2D HW 7 Solutions Ch 6 # 2 p. 1

$$\psi(x) = \begin{cases} A\cos\left(\frac{2\pi x}{L}\right) & -\frac{L}{4} \le x \le \frac{L}{4} \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine A.

For normalization we require $\int_{-\infty}^{\infty} \psi^* \psi \, dx = 1$

This function is real, so $\psi^* = \psi$ and $|\psi|^2 = \psi^*\psi = \psi^2$.

$$A^2 \int_{-L/4}^{L/4} \cos^2\left(\frac{2\pi x}{L}\right) dx = 1, \quad \text{Let } u = \frac{2\pi x}{L} \Rightarrow du = \frac{2\pi}{L} dx$$

$$@\ x = -\frac{L}{4},\ u = -\frac{\pi}{2}, \quad @\ x = \frac{L}{4},\ u = \frac{\pi}{2}$$

$$A^2 \int_{-\pi/2}^{\pi/2} \cos^2(u) \left(\frac{L}{2\pi}\right) du = 1, \quad \text{note: } \cos^2(u) = \frac{1+\cos(2u)}{2}$$

$$\frac{A^2 L}{4\pi} \int_{-\pi/2}^{\pi/2} \left(1 + \cos(2u)\right) du = 1$$

$$\frac{A^2 L}{4\pi} \left[u + \frac{\sin(2u)}{2} \right]_{-\pi/2}^{\pi/2} = 1$$

$$\frac{A^2 L}{4\pi} \left[\pi \right] = 1 \Rightarrow \boxed{A = \pm \frac{2}{\sqrt{L}}}$$ take + sign for simplicity

(b) What is the probability of $0 \le x \le L/8$?

Probability of being between a and b is given by $\int_a^b |\psi|^2 dx$

$$\frac{2}{L} \int_0^{L/8} \cos^2\left(\frac{2\pi x}{L}\right) dx = \frac{2}{L} \frac{L}{2\pi} \left[u + \frac{\sin(2u)}{2} \right]_0^{\pi/4} = \frac{1}{\pi} \left[\left(\frac{\pi}{4} + \frac{1}{2}\right) - (0 + 0) \right]$$

$$= \boxed{\frac{1}{\pi}\left(\frac{\pi}{4} + \frac{1}{2}\right) \simeq .409}$$ ← Good to check that you get a # btwn 0 and 1.

Physics 2D HW 7 Solutions Ch. 6 # 4 p. 1

$\Psi(x,0)$, $a(k) = (C\alpha/\sqrt{\pi})\, e^{-\alpha^2 k^2}$

Calculate $\Psi(x,t)$ and describe it's evolution.

$\Psi(x,0)$ gives the initial wavefunction in real (x) space. To construct this function (Given in example 6.3) we need to combine plane waves according to

$$\Psi(x,0) = \int_{-\infty}^{\infty} a(k)\, e^{ikx}\, dx \quad \text{where } a(k) = (C\alpha/\sqrt{\pi})\, e^{-\alpha^2 k^2}.$$

Each of these plane waves evolve independently of one another. For a dispersive medium with $\omega = \omega(k)$, the overall time evolution is given by (Eqn. 6.8):

$$\Psi(x,t) = \int_{-\infty}^{\infty} a(k)\, e^{i(kx - \omega(k)t)}\, dk$$

If we then assume that this is a non-relativistic, free particle, $\omega(k) = \frac{\hbar k^2}{2m}$ so we have

$$\Psi(x,t) = \int_{-\infty}^{\infty} (C\alpha/\sqrt{\pi})\, e^{-\alpha^2 k^2}\, e^{i(kx - \frac{\hbar k^2}{2m}t)}\, dk$$

$$= \left(\frac{C\alpha}{\sqrt{\pi}}\right) \int_{-\infty}^{\infty} e^{-\alpha^2 k^2 + ikx - i\frac{\hbar k^2}{2m}t}\, dk$$

Doing this integral is not entirely trivial.

$$\boxed{\Psi(x,t) = \left(\frac{C\alpha}{\beta}\right) e^{-x^2/4\beta^2}} \quad \text{where } \beta^2 = \alpha^2 + \frac{i\hbar t}{2m}$$

This Gaussian grows "shorter and wider" with time.

Physics 2D HW 7 Solutions Ch. 6 # 5 p. 1

particle, $E = 0$, $\psi(x) = Axe^{-x^2/L^2}$

(a) Find $U(x)$.

Schrödinger eqn:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + U(x)\psi(x) = E\psi(x)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left(Axe^{-x^2/L^2}\right) + U(x)\left(Axe^{-x^2/L^2}\right) = 0$$

$$-\frac{\hbar^2 A}{2m}\left[\frac{\partial}{\partial x}\left(e^{-x^2/L^2} + e^{-x^2/L^2}(-2x/L^2)x\right)\right] + \underbrace{U(x)\left(Axe^{-x^2/L^2}\right)}_{} = 0$$

$$-\frac{\hbar^2 A}{2m}\left(e^{-x^2/L^2}\left(\frac{-2x}{L^2}\right) + \left(e^{-x^2/L^2}(-2x/L^2)^2 x + (-4x/L^2)e^{-x^2/L^2}\right)\right) +$$

divide by e^{-x^2/L^2}

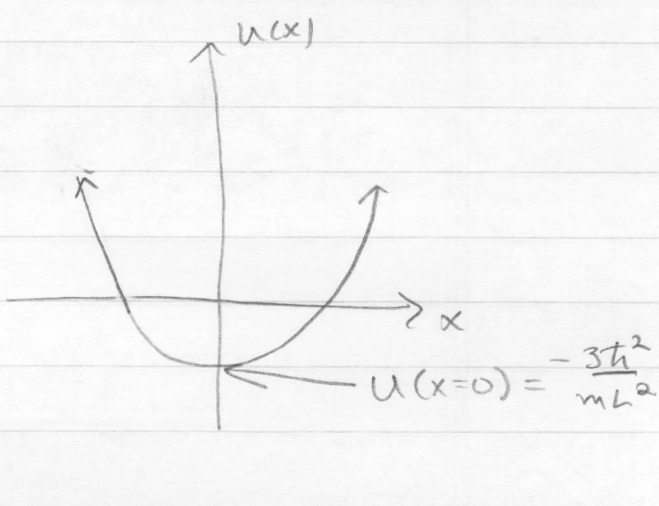
$$-\frac{\hbar^2 A}{2m}\left(\left(\frac{-2x}{L^2}\right) + \left(\frac{4x^3}{L^4} - \frac{4x}{L^2}\right)\right) + U(x)Ax = 0$$

$$-\frac{\hbar^2 A}{2mL^4}\left(-2xL^2 + 4x^3 - 4xL^2\right) + U(x)Ax = 0$$

$$-\frac{\hbar^2}{2mL^4}\left(4x^2 - 6L^2\right) + U = 0 \Rightarrow \boxed{U = \frac{\hbar^2}{2mL^4}\left(4x^2 - 6L^2\right)}$$

(b) Sketch $U(x)$.

This is a parabola because it goes like x^2. It has a minimum when $x = 0$.

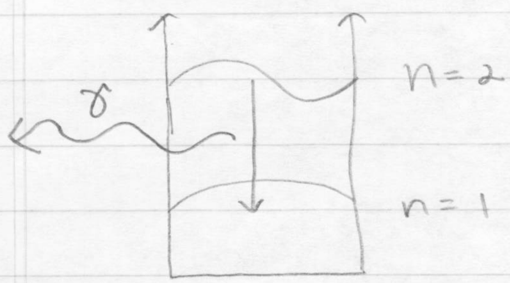


Physics 2D HW 7 Solutions Ch. 6 # 9 p. 1

proton, infinite square well $L = 10^{-5}$ nm

Calculate λ and E for a γ emitted when $n=2 \rightarrow n=1$.

What kind of γ is this?



See section 6.4 (p. 200) for a discussion of the infinite square well. You should understand the results presented there and how to obtain them.

From eqn. 6.17 $E_n = \frac{\hbar^2 k^2}{2m} = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$

$\Rightarrow E_2 - E_1 = \frac{(2^2 - 1^2)\pi^2 \hbar^2}{2 m_p L^2}$, $m_p = 1.67 \times 10^{-27}$ kg (proton)

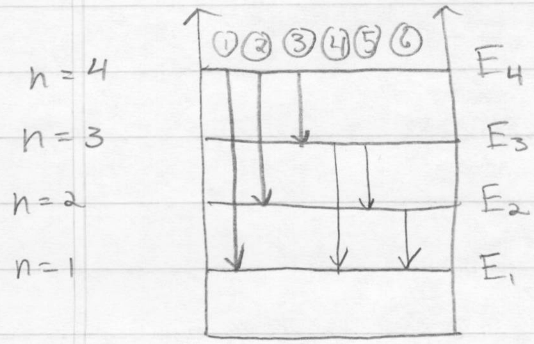
$= \boxed{9.87 \times 10^{-13} \text{ J}}$

$E_\gamma = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E_\gamma} = \boxed{2.01 \times 10^{-13} \text{ m}}$

According to wikipedia, this is a gamma ray.

Physics 2D HW 7 Solutions Ch. 6 # 10 p.1

This system is essentially the same as the previous problem, but with m_e instead of m_p and $L = 0.1 nm$



(a) Draw an energy level diagram up to n=4.

Note that $E_n = \frac{n^2\pi^2\hbar^2}{2m_eL^2}$

So the levels are not evenly spaced as I have drawn.

$E_1 = \frac{\pi^2\hbar^2}{2m_eL^2} = 37.7eV$, $E_2 = (2)^2 E_1 = 151 eV$, $E_3 = 339 eV$, $E_4 = 603 eV$

(b) Find λ for all photons that can be emitted when going from $n=4 \rightarrow n=1$.

I have labeled the six unique transitions in my diagram above. For each transition, to find λ all you have to do is find ΔE and use $\lambda = \frac{hc}{\Delta E}$.

①: $\lambda = \frac{hc}{\Delta E_{4\to1}} = \frac{1240 eVnm}{(603eV - 37.7eV)} = \boxed{2.19 nm}$

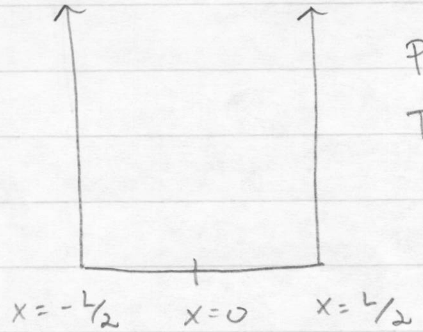
②: $\lambda = 2.75 nm$

③: $\lambda = 4.70 nm$

④: $\lambda = 4.12 nm$

⑤: $\lambda = 6.59 nm$

⑥: $\lambda = 10.9 nm$



Particle in a 1D box

This is a good chance to go back through section 6.4 to make sure you understand the logic they used to solve the 1D particle in a box,

Schrödinger equation: $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\psi(x) = E\psi$

Our $U(x) = \begin{cases} 0 & -\frac{L}{2} \leq x \leq L/2 \\ \infty & \text{elsewhere.} \end{cases}$

The wave function must be zero where $U(x) = \infty$, so we have the condition $\psi(x) = 0$ for $x < -L/2$ and $x > L/2$.

Inside that region, the SE becomes:

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi = -k^2\psi, \quad k = \frac{2mE}{\hbar^2}$$

This is a differential equation with the solution

$$\psi(x) = A\sin(kx) + B\cos(kx)$$

Applying our boundary conditions yields:

$$\psi(-L/2) = A\sin(k(-\tfrac{L}{2})) + B\cos(k(-L/2)) = 0$$

$$\psi(L/2) = A\sin(k(\tfrac{L}{2})) + B\cos(k(L/2)) = 0$$

But $\sin(-x) = -\sin(x)$ and $\cos(-x) = \cos(x)$, so

$$\left.\begin{aligned} -A\sin(\tfrac{kL}{2}) + B\cos(\tfrac{kL}{2}) &= 0 \\ A\sin(\tfrac{kL}{2}) + B\cos(\tfrac{kL}{2}) &= 0 \end{aligned}\right\} \text{add or subtract}$$

$$\Rightarrow \left.\begin{aligned} 2B\cos(\tfrac{kL}{2}) &= 0 \\ 2A\sin(\tfrac{kL}{2}) &= 0 \end{aligned}\right\} \text{so, either } A=0 \text{ or } B=0$$

(can't be both, otherwise no ψ
can't be neither b/c sin, cos $\neq 0$ @ same x)

Physics 2D HW7 Solutions Ch 6 #11 p.2

Notice that the ground state should have zero nodes, but $\sin(kx)$ is zero @ $x=0$, so for the ground state choose $\cos(kx)$. Also, $B \neq 0 \Rightarrow \frac{KL}{2} = n\frac{\pi}{2} \Rightarrow KL = n\pi$, n is integer.

$$\Rightarrow \boxed{K_n = \frac{n\pi}{L}}$$

$\psi_1 = B\cos(K_1 x)$, need to determine B, use normalization

$$\int_{-L/2}^{L/2} (B\cos(k_1 x))^2 dx = B^2 \int_{-L/2}^{L/2} \cos^2(k_1 x)\, dx = B^2\left(\frac{L}{2}\right) = 1$$

b/c $\sin(\pi) = 0$ normalization

$\Rightarrow B = \sqrt{\frac{2}{L}}$ and $\boxed{\psi_1 = \sqrt{\frac{2}{L}} \cos(K_1 x)}$

$$\boxed{|\psi_1|^2 = \frac{2}{L}\cos^2(K_1 x)}$$

For ψ_2, we want 1 node $\Rightarrow A \neq 0 \Rightarrow B = 0$

$$\boxed{\psi_2 = \sqrt{\frac{2}{L}} \sin(K_2 x) \qquad |\psi_2|^2 = \frac{2}{L}\sin^2(K_2 x)}$$

Similarly,

$$\boxed{\psi_3 = \sqrt{\frac{2}{L}} \cos(K_3 x) \qquad |\psi_3|^2 = \frac{2}{L}\cos^2(K_3 x)}$$

Physics 2D HW 7 Solutions Ch. 6 # 16

e^- in infinite square well, $L = 0.3\text{nm}$

We have seen and solved this problem already.

$$n=1 \Rightarrow \psi_1 = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right), \quad |\psi_1|^2 = \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right)$$

$$P_{n=1}(x=0 \to x=L/3) = \int_0^{L/3} |\psi_1|^2 dx = \frac{1}{3} - \frac{\sqrt{3}}{4\pi} \cong 0.196$$

$$P_{n=100}(x=0 \to x=L/3) = \int_0^{L/3} |\psi_{100}|^2 dx = \frac{1}{3} - \frac{\sqrt{3}}{400\pi} \simeq 0.332$$

Classically, we would expect the chance of finding it in the first third of the box to be $1/3$. We can see that as $n \to \infty$, $P_{n\to\infty}(x=0 \to x=L/3) \to 1/3$.

The correspondence principle is satisfied.

#19 p.1

Again, infinite square well, $m = 1\times10^{-3}\,kg$, $L = 1\times10^{-2}\,m$

$E = 1\times10^{-3}\,J$, find n.

$$E_n = \frac{n^2\hbar^2\pi^2}{2mL^2} \Rightarrow n = \sqrt{\frac{2mL^2E}{\hbar^2\pi^2}} = \boxed{4.27\times10^{28}} \text{ very high}$$

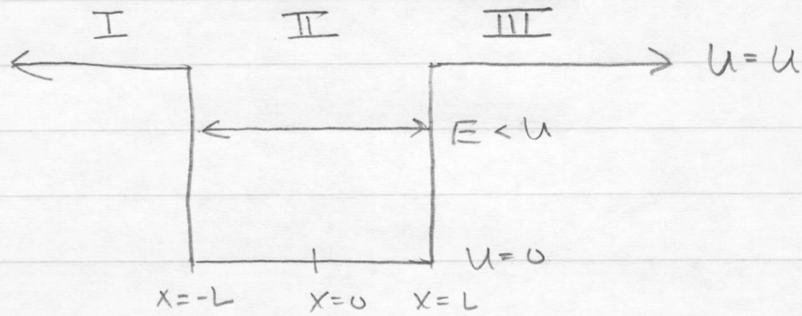
How much energy to go from this $n \to n+1$?

$$\Delta E = \frac{((n+1)^2 - n^2)\hbar^2\pi^2}{2mL^2} = \frac{(2n+1)\hbar^2\pi^2}{2mL^2} = \boxed{4.69\times10^{-32}\,J}$$

This is a tiny amount of energy. This fits with our classical idea of energy being a continuously variable quantity.

Physics 2D HW 7 Solutions Ch. 6 # 20 p.1

Finite square well



Show that, for $E<U$, $k\tan(kL)=\alpha$, $\alpha=\sqrt{\frac{2m(U-E)}{\hbar^2}}$, $k=\sqrt{\frac{2mE}{\hbar^2}}$

Remember that, for finite potentials, we require that the Ψ and $\frac{d\Psi}{dx}$ be continuous everywhere (and at the boundaries in particular). Inside $-L<x<L$, we have already solved the Schrödinger equation:

$$\Psi(x)=A\sin(kx)+B\cos(kx),\quad k=\sqrt{\frac{2mE}{\hbar^2}}$$

This is the same as the infinite square well.
Outside this region, $U>E$ so k is imaginary and the solutions become exponentials $\Psi(x)=Ce^{\alpha x}+De^{-\alpha x}$

$\alpha=\sqrt{\frac{2m(U-E)}{\hbar^2}}$. If this doesn't look familiar, feel free to review your differential equations.

Now comes the physics: we must set $C=0$ for region III so that the wave function is normalizable when x becomes large and positive (the same is true for D in region I, where x is negative).

Physics 2D HW 7 Solutions Ch. 6 # 20 p. 2

Consider the $n=1$ (ground) state. This state has no nodes and is symmetric about $x=0 \Rightarrow A=0$.

Continuity of ψ @ $x=L \Rightarrow$

$$B\cos(kL) = De^{-\alpha L} \quad (1)$$

Continuity of $\frac{d\psi}{dx}$ @ $x=L \Rightarrow$

$$-Bk\sin(kL) = -\alpha De^{-\alpha L} \quad (2)$$

Dividing (2) by (1) yields

$$-k\tan(kL) = -\alpha$$

or

$$\boxed{k\tan(kL) = \alpha}$$

This equation is significant because only discrete values of k will satisfy it $\Rightarrow$ energy is quantized.

(b) Show that this can be rewritten as $k\sec(kL) = \sqrt{2mU}/\hbar$

Recall $\alpha = \frac{\sqrt{2m(U-E)}}{\hbar}$ and $k = \frac{\sqrt{2mE}}{\hbar}$

So $$\frac{\sqrt{2mE}}{\hbar}\tan(kL) = \frac{\sqrt{2m(U-E)}}{\hbar}$$

square both sides $$\frac{2mE}{\hbar^2}\tan^2(kL) = \frac{2m(U-E)}{\hbar^2}$$

rearrange $$\frac{2mE}{\hbar^2}\left(\tan^2(kL)+1\right) = \frac{2mU}{\hbar^2}$$

square root $k = \frac{\sqrt{2mE}}{\hbar}$ and $\tan^2\theta + 1 = \sec^2\theta$

$$\boxed{k\sec(kL) = \frac{\sqrt{2mU}}{\hbar}}$$

Physics 2D HW 7 Solutions Ch. 6 # 20 p.3

e^- trapped in crystal defect $U = 5eV$, $L = 0.1nm$

Find E_1.

First we need to find k using

$$k\sec(kL) = \sqrt{2mU}/\hbar$$

$$= \sqrt{2 \times 0.511 MeV/c^2 \cdot 5eV}/\hbar$$

$$= 1.145 \times 10^{10} m^{-1}$$

Let $y = kL$

$$y\sec(y) = (1.145 \times 10^{10} m^{-1})(0.1 \times 10^{-9} m)$$

$$y\sec(y) = 1.145$$

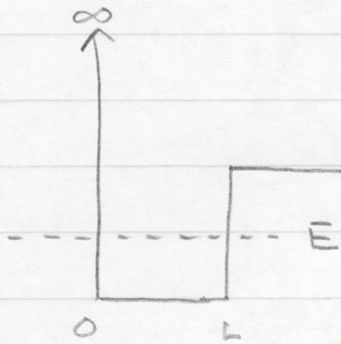
This is impossible to solve analytically, but numerically Mathematica gives me $y = 0.799$. $\Rightarrow k = 7.99 \times 10 \, nm^{-1}$

To find the energy, apply the formula for a free partizle of mass $m = m_e$

$$E = \frac{\hbar^2 k^2}{2m_e} = \boxed{2.432eV}$$

Physics 2D HW 7 Solutions Ch. 6 # 23

Semi-infinite square well



This is not really a new problem - just combine results and apply old techniques.

Inside the well, we have a free particle:

$$\psi(x) = A\sin(kx) + B\cos(kx), \quad k = \sqrt{2mE}/\hbar$$

Outside the well, we have a decaying exponential:

$$\psi(x) = Ce^{-\alpha x}, \quad \alpha = \sqrt{2m(U-E)}/\hbar$$

Because $U = \infty$ @ $x = 0$, we require $\psi(0) = 0 \Rightarrow B = 0$.

@ $x = L$ we require ψ and $\frac{d\psi}{dx}$ to be continuous:

$$A\sin(kL) = Ce^{-\alpha L} \quad (1)$$

$$kA\cos(kL) = -\alpha Ce^{-\alpha L} \quad (2)$$

Dividing (2) by (1) yields

$$\boxed{k\cot(kL) = -\alpha}$$

which quantizes our energy levels. Using the same tricks as before we can rearrange this to obtain

$$\frac{kL}{\sin(kL)} = \sqrt{\frac{2mUL^2}{\hbar^2}} \quad (3)$$

which only has solutions for $\sqrt{\frac{2mUL^2}{\hbar^2}} \geq 1$.

In other words, for any given L, there will be no k that satisfies (3) if $\sqrt{\frac{2mUL^2}{\hbar^2}} < 1$.

Physics 2D HW 7 Solutions Ch. 6 # 24 p.1

Quantum oscillator

$\psi(x) = C x e^{-\alpha x^2}$

Use the Schrödinger equation to obtain α in terms of m and ω.

SE: $-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x) + U(x)\psi(x) = E\psi(x)$, $U(x) = \frac{1}{2} m \omega^2 x^2$

Plugging in $U(x)$ and $\psi(x)$

$$\frac{d\psi}{dx} = -2\alpha x \, C e^{-\alpha x^2} + C e^{-\alpha x^2}$$

$$\frac{d^2\psi}{dx^2} = (2\alpha x)^2 C e^{-\alpha x^2} - 6\alpha C e^{-\alpha x^2}$$

$$-\frac{\hbar^2}{2m}\left((2\alpha x)^2 - 6\alpha\right) + \frac{1}{2} m \omega^2 x^2 = E \quad \text{, multiply by } -\frac{2m}{\hbar^2}$$

$$(2\alpha)^2 x^2 - 6\alpha - \frac{m^2\omega^2 x^2}{\hbar^2} = -\frac{2mE}{\hbar^2} \quad \text{, add } \frac{m^2\omega^2 x^2}{\hbar^2}$$

$$(2\alpha)^2 x^2 - 6\alpha = \left(\frac{m\omega}{\hbar}\right) x^2 - \frac{2mE}{\hbar^2}$$

If this is true for all x, then $2\alpha = \frac{m\omega}{\hbar}$ and $6\alpha = \frac{2mE}{\hbar^2}$.

$\Rightarrow \boxed{\alpha = \frac{1}{2} m\omega/\hbar}$ and $\boxed{E = \frac{3}{2}\hbar\omega}$

(b) Normalization: $\int_{-\infty}^{\infty} |\psi|^2 dx = 1$

$\int_{-\infty}^{\infty} (C x e^{-\alpha x^2}) dx = 1$ Do this however you see fit.

$$\boxed{C = \left(\frac{32\alpha^3}{\pi}\right)^{1/4}}$$

Physics 2D HW 7 Solutions Ch. 6 # 29 p. 1

$\psi(x) = Ce^{-x}(1-e^{-x})$, $x>0$. Find C.

Normalization: $\int_{-\infty}^{\infty} |\psi|^2 dx = 1$

$$\int_0^{\infty} (Ce^{-x}(1-e^{-x}))^2 dx = 1 \Rightarrow \boxed{C = \sqrt{12}\ nm^{-1/2}}$$

Note the only trick here is to realize that the lower bound for integration is 0 instead of $-\infty$ because $\psi = 0$ for $x<0$.

(b) The probability is a maximum when $\frac{d|\psi|^2}{dx} = 0$ and

$\frac{d^2|\psi|^2}{dx^2} > 0$. $\frac{d|\psi|^2}{dx} = 0$ has two solutions

$x = 0$ and $x = \ln(2)$, but only $\boxed{x = \ln(2)}$ gives a positive second derivative.

(c) $$\langle x \rangle = \int_{-\infty}^{\infty} \psi^* x \psi\, dx = \int_0^{\infty} x|\psi|^2 dx = 12\left\{\frac{1}{4} - 2\left(\frac{1}{9}\right) + \frac{1}{16}\right\} = \boxed{\frac{13}{12}\ nm}$$

Note that the most likely position and average position need not be the same.

Physics 2D HW7 Solutions Ch. 6 # 31 p. 1

e^- w/ $\psi(x) = Ce^{-|x|/x_0}$, $x_0 = \text{const}$ $C = \frac{1}{\sqrt{x_0}}$

Find $\langle x \rangle$, $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ and $P(\langle x \rangle - \Delta x < x < \langle x \rangle + \Delta x)$.

$\langle x \rangle = \int_{-\infty}^{\infty} x |\psi|^2 dx = 0$ b/c the integrand is odd.

$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\psi|^2 dx = \frac{x_0^2}{2}$

$\Delta x = \frac{x_0}{\sqrt{2}}$

$P(\langle x \rangle - \Delta x < x < \langle x \rangle + \Delta x) = \int_{\langle x \rangle - \Delta x}^{\langle x \rangle + \Delta x} |\psi|^2 dx = \boxed{1 - e^{-\sqrt{2}}} \simeq 0.757$.

Note that this is not a function of x_0.

<u>#32 p. 1</u>

Quantum oscillator - ground state

Calculate $\langle x \rangle$, $\langle x^2 \rangle$ and Δx.

$\psi = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} e^{-\frac{1}{2}m\omega x^2/\hbar}$

$$\boxed{\begin{aligned} \langle x \rangle &= \int_{-\infty}^{\infty} x |\psi|^2 dx = 0 \\ \langle x^2 \rangle &= \int_{-\infty}^{\infty} x^2 |\psi|^2 dx = \frac{\hbar}{2m\omega} \\ \Delta x &= \sqrt{\frac{\hbar}{2m\omega}} \end{aligned}}$$

Physics 2D HW 7 Solutions Ch 6 # 37 p.1

$\Psi(x,0) = C\,[\psi_1 + \psi_2]$ infinite square well

Normalization: $\int_{-\infty}^{\infty} \Psi^*(x,0)\,\Psi(x,0)\,dx = 1$

$$C^2 \int_{-\infty}^{\infty} (\psi_1^* + \psi_2^*)(\psi_1 + \psi_2)\,dx = 1$$

$$C^2 \Big[\int_{-\infty}^{\infty} |\psi_1|^2 dx + \int_{-\infty}^{\infty} |\psi_2|\,dx + 2\int_{-\infty}^{\infty} \psi_1 \psi_2\,dx \Big] = 1$$

equal to 1 b/c of normalization   0 by orthogonality

$$\Rightarrow \boxed{C = \frac{1}{\sqrt{2}}}$$

$$\boxed{\Psi(x,t) = \frac{1}{\sqrt{2}} \left\{ \psi_1 e^{-iE_1 t/\hbar} + \psi_2 e^{-iE_2 t/\hbar} \right\}}$$

evolution of energy eigenstates

$\hat{E} = i\hbar \frac{\partial}{\partial t}$ but $\hat{E}\Psi \neq \text{constant} \cdot \Psi$

$\Rightarrow \Psi$ is not an energy eigenstate.

$$\langle E \rangle = \int_{-\infty}^{\infty} \Psi^* \hat{E} \Psi\,dx = \boxed{\frac{1}{2}(E_1 + E_2)}$$