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Calculate λ for a proton w/ $v = 1\times10^{6}$ m/s.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.626\times10^{-34}\ \text{Js}}{(1.67\times10^{-27}\ \text{kg})(1\times10^{6}\ \text{m/s})} = \boxed{3.97\times10^{-13}\ \text{m}}$$

#2 p.1

Note that the kinetic energy given is much less than the rest mass energy of an e^- (0.511 MeV) so we can ignore relativistic effects.

$$\lambda = \frac{h}{p}, \quad K = \tfrac{1}{2}mv^2 = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK}$$

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{4.136\times10^{-15}\ \text{eVs}}{\sqrt{2(0.511\text{MeV}/c^2)(50\text{eV})}} = \boxed{1.74\times10^{-10}\ \text{m}}$$

Don't forget to plug in the numerical value for c and check to make sure you get the right units.

(b) Note that $\frac{50\text{keV}}{0.5\text{MeV}} = 0.1$, so we are approaching relativistic speeds. The relativistic expression for p in terms of K is $p = \frac{K}{c}(1 + 2mc^2/K)^{1/2}$. This can be derived from $K = E - mc^2 = \sqrt{m^2c^4 + p^2c^2} - mc^2$.

$$\lambda = \frac{h}{p}, \quad p = \frac{50\text{keV}}{c}\left(1 + \frac{2(0.511\text{MeV})}{(50\text{keV})}\right)^{1/2} = 7.72\times10^{-5}\ \frac{\text{eVs}}{\text{m}}$$

$$\lambda = \frac{4.136\times10^{-15}\ \text{eVs}}{7.72\times10^{-15}\ \text{eVs/m}} = \boxed{5.36\times10^{-11}\ \text{m}}$$

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$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.626\times10^{-34}\,Js}{(74kg)(5m/s)} = \boxed{1.79\times10^{-36}\,m}$$ very small

#5 p.1

(a) $\lambda = 10\,nm \Rightarrow p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\,Js}{10\times10^{-9}\,m} = 6.626\times10^{-26}\,\frac{kg\,m}{s}$

$$K = \frac{p^2}{2m} = \frac{(6.626\times10^{-26}\,kgm/s)^2}{2(9.109\times10^{-31}\,kg)} = 2.41\times10^{-21}\,J$$

$$\frac{2.41\times10^{-21}\,J}{} \cdot \frac{1eV}{1.6\times10^{-19}\,J} = \boxed{0.015\,eV}$$

(b) $\lambda = 1\times10^{-10}\,m \Rightarrow p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\,Js}{1\times10^{-10}\,m} = 6.626\times10^{-24}\,kgm/s$

$$K = \frac{p^2}{2m} = \frac{(6.626\times10^{-24}\,kgm/s)^2}{2(9.1\times10^{-31}\,kg)} = 2.41\times10^{-17}\,J$$

$$\frac{2.41\times10^{-17}\,J}{} \cdot \frac{1eV}{1.6\times10^{-19}\,J} = \boxed{1.51\times10^{2}\,eV}$$

(c) $\lambda = 10\times10^{-15}\,m \Rightarrow p = \frac{h}{\lambda} = 6.626\times10^{-20}\,kgm/s$

$$K = \frac{p^2}{2m} = 2.41\times10^{-9}\,J \cdot \frac{1eV}{1.6\times10^{-19}\,J} = \boxed{1.5\times10^{10}\,eV}$$

careful, this is more than the rest mass energy $\Rightarrow$ relativity

$$K = \sqrt{p^2c^2 + m^2c^4} - mc^2$$

$$= \frac{1.97\times10^{-11}\,J}{} \cdot \frac{1eV}{1.6\times10^{-19}\,J} = \boxed{1.23\times10^{8}\,eV}$$ big difference.

The second part of the problem is the same with different masses. Be careful to apply relativistic expressions when necessary.

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$$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\ Js}{1\times10^{-14}m} = 6.626\times10^{-20}\ kgm/s$$

$K = 1.23\times10^{8}eV$ (from the last part of the last problem).

(b) This seems like a large amount of energy for a single particle, but I'm not sure exactly what you are supposed to "expect". This is certainly not how atoms are constructed.

#12 p.1

$$p = \frac{hc}{\lambda c} = \frac{1240\ eVnm}{0.1nm} = 12400eV/c$$

energy conservation

$$K = \frac{p^2}{2m} = \frac{(12400eV/c)^2}{2(0.511\times10^6 eV/c^2)} = 150\ eV = e\cdot V \Rightarrow \boxed{V = 150V}$$

#14 p.1

Looking at figure 5.6, it is clear that

$$n\lambda = d\sin\varphi$$

is the condition for constructive interference.

$$\lambda = \frac{h}{p},\quad K = \frac{p^2}{2m} \Rightarrow \lambda = \frac{h}{\sqrt{2mK}} \Rightarrow \boxed{\frac{nh}{\sqrt{2mK}} = d\sin\varphi}$$

(b) $K = 100\ eV$, $n = n_0 \Rightarrow \varphi = 24.1°$, $n = n_0+1 \Rightarrow \varphi = 54.9°$

$$\frac{h((n_0+1)-n_0)}{\sqrt{2mK}} = d(\sin(54.9°) - \sin(24.1°))$$

$$\Rightarrow \boxed{d = 2.99\times10^{-10}m}$$

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$$E = \frac{p^2}{2m}, \quad p = \frac{h}{\lambda}, \quad \hbar = \frac{h}{2\pi}, \quad k = \frac{2\pi}{\lambda} \Rightarrow p = \hbar k$$

$$E = \hbar\omega \Rightarrow \hbar\omega = \frac{\hbar^2 k^2}{2m} \Rightarrow \omega = \frac{\hbar k^2}{2m}$$

By definition, $v_g = \frac{d\omega}{dk} = \frac{2\hbar k}{2m} = \frac{\hbar k}{m} = \frac{p}{m} = v_0$

#17 p.1

$$\omega(k) = \sqrt{c^2k^2 + (mc^2/\hbar)^2}$$

$$\boxed{v_p = \frac{\omega}{k} = \sqrt{c^2 + (mc^2/k\hbar)^2}}$$

$$\boxed{v_g = \frac{d\omega}{dk} = \frac{\frac{1}{2}\, 2kc^2}{\sqrt{c^2k^2 + (mc^2/\hbar)^2}}}$$

$$\boxed{v_p v_g = c^2}$$

If $v_p > c \Rightarrow v_g < g$.

#18 p.1

$$p = mv \Rightarrow \Delta p = m\Delta v = 50\times10^{-3}\text{ kg }(30\text{ m/s} \times 0.001)$$
$$= \boxed{1.5\times10^{-3}\text{ kgm/s}}$$

$\Delta x \Delta p \gtrsim \hbar/2$, minimum uncertainty $\Rightarrow \Delta x \Delta p = \hbar/2$

$$\Rightarrow \Delta x = \frac{\hbar}{2\Delta p} = \frac{6.626\times10^{-34}\text{ Js}}{4\pi(1.5\times10^{-3}\text{ kgm/s})} = \boxed{3.51\times10^{-32}\text{ m}} \text{ small}$$

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$K = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK} = \sqrt{2(938\,MeV/c^2)(1\,MeV)} = 43.3\,MeV/c$

$\Delta p = (43.3\,MeV/c)(0.05) = 2.16\,MeV/c$

$\Delta x \Delta p = \hbar/2 \Rightarrow \Delta x = \frac{\hbar}{2\Delta p} = \frac{6.582\times10^{-16}\,eVs\,(c)}{2(2.16\times10^{6}\,eV)}$

$= \boxed{4.57\times10^{-14}\,m}$

#20 p.1

$\Delta x \Delta p = \hbar/2 \qquad p = \frac{h}{\lambda} \Rightarrow \left|\frac{\Delta p}{\Delta\lambda}\right| = \left|\frac{dp}{d\lambda}\right| = \left|\frac{-h}{\lambda^2}\right| = \frac{h}{\lambda^2}$

$\Rightarrow \Delta p = \frac{h}{\lambda}\frac{\Delta\lambda}{\lambda}, \quad \Delta x = \frac{\hbar}{2\Delta p} = \frac{\hbar}{2}\frac{\lambda}{h}\frac{\lambda}{\Delta\lambda} = \frac{1}{4\pi}\lambda\frac{\lambda}{\Delta\lambda}$

$= \boxed{4.77\times10^{-2}\,m}$

#22 p.1

I will show part a. The others are the same w/ different #s.
The trick here is to set the slit width a equal to Δx.

$\Delta x \Delta p = \hbar/2$

$a\Delta p = \hbar/2 \Rightarrow a = \frac{\hbar}{2\Delta p}, \quad p = \gamma m v$, need v

$K = (\gamma - 1)mc^2 \Rightarrow \gamma = \frac{K + mc^2}{mc^2} = \frac{1}{\sqrt{1 - v^2/c^2}}$

Given $K = 0.01\times10^{6}\,eV$, you can solve for $\gamma = 1.02$ and $v = 5.91\times10^{7}\,m/s$. $p = \gamma m v = 5.49\times10^{-23}\,kg\,m/s$.

$\Delta p = 0.01 \times p = 5.49\times10^{-25}\,kg\,m/s$

$a = \hbar/2\Delta p = \boxed{9.61\times10^{-11}\,m}$

(b) $\boxed{6.94\times10^{-12}\,m}$

(c) $\boxed{9.8\times10^{-14}\,m}$

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$$\Delta E \Delta t \geq \frac{\hbar}{2} \Rightarrow \Delta E \geq \frac{\hbar}{2\Delta t} = \frac{\hbar}{2(0.1\,ns)} = \boxed{3.29\times10^{-6}\,eV < 5\,eV \quad \text{no}}$$

#26 p.1

Here ΔE is estimated as half the width at half max

$$\Rightarrow \Delta E \simeq 50\,MeV$$

$$\Rightarrow \Delta t \simeq \frac{\hbar}{2\Delta E} = \frac{6.582\times10^{-16}\,eVs}{2(50\times10^{6}\,eV)} = \boxed{6.582\times10^{-24}\,s}$$

28 p.1

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.626\times10^{-34}\,Js}{(1.67\times10^{-27}\,kg)(0.4\,m/s)} = \boxed{9.93\times10^{-7}\,m}$$

(b) $$\sin\theta = \frac{\lambda}{2D} = \frac{9.93\times10^{-7}\,m}{2(1\times10^{-3}\,m)} = 4.96\times10^{-4} \ll 1 \Rightarrow \sin\theta \simeq \theta$$

$$y = R\theta = (10\,m)(4.96\times10^{-4}) = \boxed{4.96\,mm}$$

(c) The particle interferes with itself $\Rightarrow$ it went through both slits.

29 p.1

With one slit open, $P_1 = |\psi_1|^2$ or $P_2 = |\psi_2|^2$. With both slits open $P = |\psi_1 + \psi_2|^2$. At a maximum, $P = (|\psi_1| + |\psi_2|)^2$. At a minimum, $P = (|\psi_1| - |\psi_2|)^2$. We are told $P_1/P_2 = 25 \Rightarrow |\psi_1|/|\psi_2| = 5$

$$\Rightarrow \frac{P_{max}}{P_{min}} = \frac{(|\psi_1| + |\psi_2|)^2}{(|\psi_1| - |\psi_2|)^2} = \frac{(5|\psi_2| + |\psi_2|)^2}{(5|\psi_2| - |\psi_2|)^2} = \frac{6^2}{4^2} = \boxed{\frac{36}{16}}$$

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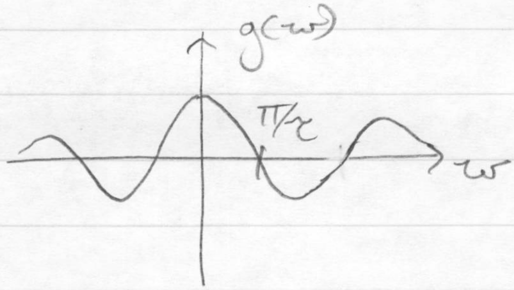
$$\Delta E \Delta t = \frac{\hbar}{2} \Rightarrow \Delta m c^2 \Delta t = \frac{\hbar}{2} \Rightarrow \frac{\Delta m}{m} = \frac{\hbar}{2(mc^2)\Delta t}$$

$$= \boxed{2.81 \times 10^{-8}}$$

#34 p.1

$$g(\omega) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} V(t)(\cos(\omega t) - i\sin(\omega t))\,dt = \left(\frac{2}{\pi}\right)^{1/2} V_0 \frac{\sin(\omega t)}{\omega}$$



(b) $\Delta\omega \simeq \pi/\tau$, $\Delta t = \tau \Rightarrow \boxed{\Delta\omega \Delta t = \pi}$

(c) $\Delta\omega = \dfrac{\pi}{\Delta t} \Rightarrow \Delta f = \dfrac{1}{2\Delta t} = \dfrac{1}{1\mu s} = \boxed{1 \times 10^{6}\ Hz}$

(d) $\Delta f = \dfrac{1}{2\Delta t} = \dfrac{1}{1ns} = \boxed{1 \times 10^{9} Hz}$