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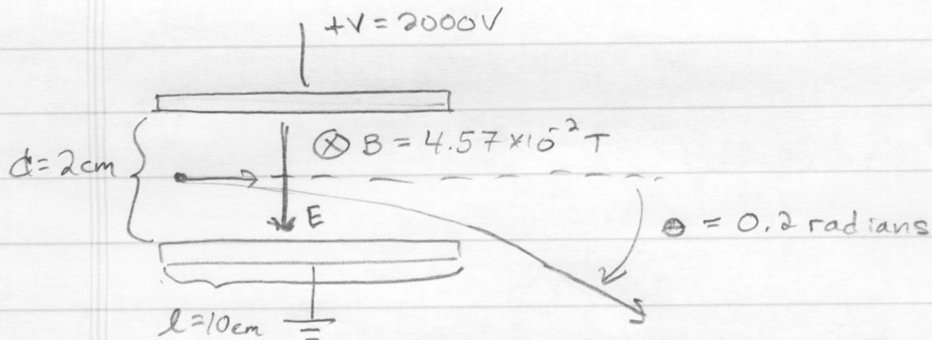
"mystery particle"



(a) Find q/m.

By equation 4.7, $\frac{q}{m} = \frac{V\theta}{B^2 l d} = \frac{(2000V)(0.2\ \text{radians})}{(4.57\times10^{-2}T)^2(10\times10^{-2}m)(2\times10^{-2}m)}$

$= \boxed{9.58\times10^{7}\ C/kg}$

(b) Identify the particle.

The fact that the deflection is downward is a big clue: it must have a positive charge (it is attracted to the negative plate).

Try computing $\frac{q}{m}$ for the proton:

$\frac{q}{m} = \frac{1.6\times10^{-19}C}{1.67\times10^{-27}kg} = \boxed{9.58\times10^{7}\ C/kg}$ ← matches

$\boxed{\text{proton}}$

(c) Find the initial horizontal speed.

By equation 4.6, $V_x = \frac{E}{B} = \frac{V}{dB} = \frac{(2000V)}{(2\times10^{-2}m)(4.57\times10^{-2}T)} = \boxed{2.19\times10^{6}\ m/s}$

(d) Should we treat this relativistically?

$\frac{V_x}{c} = \frac{2.19\times10^{6}m/s}{3\times10^{8}m/s} = .007$, so V_x is small compared to c

$\boxed{\text{No}}$

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Millikan oil drop experiment

$d = 2.00\,cm$, $V = 4000\,V$, $l = 4.00\,mm$, $\rho = 0.8\,g/cm^3$

viscosity of air $= 1.81\times10^{-5}\,kg\,m^{-1}s^{-1}$, $t_{avg}^{o} = 15.9\,s$

Millikan's oil drop experiment is discussed on pages 113 - 119.

(a) Find the radius and mass of the drop.

From equation 4.9, radius $= a = \sqrt{\dfrac{9\eta v}{2\rho g}}$

$$= \left(\frac{9(1.81\times10^{-5}\,kg/m/s)\left(\frac{4\times10^{-3}\,m}{15.9\,s}\right)}{2(0.8)\frac{\times10^{-3}\,kg}{(\times10^{-2}\,m)^3}(9.8\,m/s^2)}\right)^{1/2}$$

Be careful with units and the square root here

$$= \boxed{1.62\times10^{-6}\,m}$$

From equation 4.13, $m = \rho\, \frac{4}{3}\pi a^3 = 0.8\frac{\times10^{-3}\,kg}{(\times10^{-2}\,m)^3}\left(\frac{4}{3}\pi\right)(1.62\times10^{-6}\,m)^3$

$$= \boxed{1.42\times10^{-14}\,kg}$$

(b) Calculate the charge on each drop.

From equation 4.10, $q_1 = \frac{mg}{E}\left(\frac{v+v_1'}{v}\right)$ and $E = \frac{V}{d}$

$$q_1 = \frac{mgd}{V}\left(\frac{v+v_1'}{v}\right) = \frac{(1.42\times10^{-14}\,kg)(9.8\,m/s^2)(2\times10^{-2}\,m)}{4000\,V}\left(\frac{2.52\times10^{-4}\,m/s + 1.11\times10^{-4}\,m/s}{2.52\times10^{-4}\,m/s}\right)$$

$$= \boxed{1.00\times10^{-18}\,C}$$

The other values can be calculated in an analagous manner:

$$\boxed{\begin{aligned} q_2 &= 1.34\times10^{-18}\,C \\ q_3 &= 1.16\times10^{-18}\,C \\ q_4 &= 1.67\times10^{-18}\,C \\ q_5 &= 2.16\times10^{-18}\,C \end{aligned}}$$

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The idea here is to guess the number of charges each drop has such that the fundamental charge is some where between 1.5×10^{-19} C and 2.0×10^{-19} C.

Notice: $q_1/6 = 1.67 \times 10^{-19}$ C

$q_2/8 = 1.68 \times 10^{-19}$ C

$q_3/7 = 1.66 \times 10^{-19}$ C

$q_4/10 = 1.67 \times 10^{-19}$ C

$q_5/13 = 1.66 \times 10^{-19}$ C

That's all fine, but now we must check to see if the amount of charge gained or lost is a multiple of this fundamental charge?

$$\left.\begin{aligned} \frac{q_2 - q_1}{2} &= 1.70 \times 10^{-19}\text{ C} \\ \frac{q_2 - q_3}{1} &= 1.80 \times 10^{-19}\text{ C} \\ \frac{q_4 - q_3}{3} &= 1.70 \times 10^{-19}\text{ C} \\ \frac{q_5 - q_4}{3} &= 1.63 \times 10^{-19}\text{ C} \end{aligned}\right\}$$

averaging these gives 1.71×10^{-19} C, but if you include more differences ($q_5 - q_3$, for instance) you can improve the estimate to $\boxed{1.67 \times 10^{-19}\text{ C}}$

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This is exactly the same as the last problem. If you include all possible unique differences (there are 15 because # pairs $= \frac{n(n-1)}{2}$ for $n = 6$) the average value obtained is $\boxed{q = 1.66 \times 10^{-19} C}$.

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This is a "Rutherford Scattering" problem. The discussion in the book is on pages 119-125.

From equation 4.16 we have $\Delta n = \dfrac{k^2 Z^2 e^4 N n A}{4R^2 (\frac{1}{2} m_\alpha V_\alpha^2)^2 \sin^4(\varphi/2)}$

but since we are given 100 cpm @ 20° (with everything else constant) we can calculate

$$\Delta n_{40°} = \Delta n_{20°} \frac{\sin^4(20°/2)}{\sin^4(40°/2)} = \boxed{6.64 \text{ cpm}}$$

The same approach applies to the other angles:

$$\boxed{\begin{aligned} \Delta n_{60°} &= 1.45 \text{ cpm} \\ \Delta n_{80°} &= 0.53 \text{ cpm} \\ \Delta n_{100°} &= 0.26 \text{ cpm} \end{aligned}}$$

(b) Note that the Kinetic energy $(\frac{1}{2} m_\alpha V_\alpha^2)$ is squared and in the denominator $\Rightarrow$ doubling KE $\Rightarrow$ $\frac{1}{4}$ cpm

$\Delta n_{KE \times 2} = \frac{1}{4} \Delta n = \boxed{25 \text{ cpm}}$

(c) The type of material comes into play with Z and N,

So $\Delta n_{Cu} = \Delta n_{Au} \dfrac{Z_{Cu}^2 N_{Cu}}{Z_{Au}^2 N_{Au}} = (100\text{cpm}) \left(\dfrac{29}{79}\right)^2 \left(\dfrac{8.43 \times 10^{22}}{5.9 \times 10^{22}}\right) = \boxed{19 \text{cpm}}$

N is the number of nuclei per unit area and is calculated from the density and thickness.

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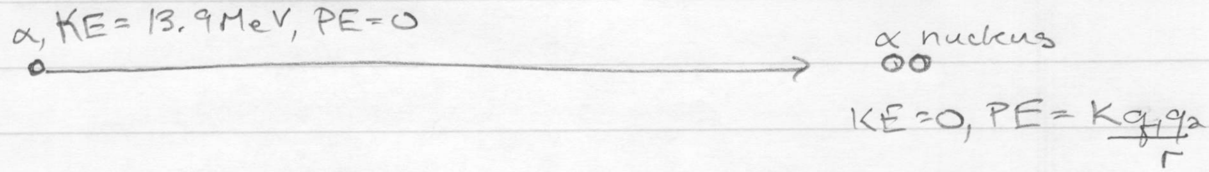
The key realization here is that α particles of 13.9 MeV and <u>less</u> obey Rutherford's scattering formula because they fail to penetrate the nucleus, so we can get an estimate of the size of the nucleus by using conservation of energy:



$$\Rightarrow r = \frac{k q_1 q_2}{KE} = \frac{k(2e)(29e)}{13.9\,\text{MeV}} = \boxed{6.00 \times 10^{-15}\,\text{m}}$$

Be careful with units and put in all constants.

<u>#11 p.1</u>

Here we have begun talking about the Bohr model of the atom. The discussion is on pages 125-139.

The Balmer series is transitions to the $n=2$ level.

$$n=3 \rightarrow n=2 \Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \Rightarrow \lambda = \boxed{656\,\text{nm}}$$

$$n=4 \rightarrow n=2 \Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) \Rightarrow \lambda = \boxed{486\,\text{nm}}$$

$$n=5 \rightarrow n=2 \Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{5^2}\right) \Rightarrow \boxed{\lambda = 434\,\text{nm}}$$

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Equation 4.35 : $r_n = n^2 \frac{a_0}{Z}$, $a_0 = 0.592 \times 10^{-10}$ m

Note: $Z=1$ for hydrogen (charge of the nucleus).

$$r_1 = (1)^2 (0.529 \times 10^{-10} m) = \boxed{0.529 \times 10^{-10} m}$$

$$r_2 = \boxed{2.12 \times 10^{-10} m}$$

$$r_3 = \boxed{4.77 \times 10^{-10} m}$$

(b) Find the velocity of the e^- in these orbits.

From equation 4.26 : $\frac{1}{2} m_e v^2 = ke^2/2r$

$$\Rightarrow v = \sqrt{\frac{ke^2}{m_e r}}$$

$$v_1 = \sqrt{\frac{(9 \times 10^9 Nm^2/C^2)(1.6 \times 10^{-19} C)^2}{(9.11 \times 10^{-31} kg)(0.529 \times 10^{-10} m)}} = \boxed{2.19 \times 10^6 m/s}$$

$$\boxed{\begin{aligned} v_2 &= 1.09 \times 10^6 m/s \\ v_3 &= 7.28 \times 10^5 m/s \end{aligned}}$$

These velocities are small compared to the speed of light so the relativistic corrections are therefore expected to be small.

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An $n=1$ hydrogen absorbs a γ and goes $n=3$.
What is E_γ?

$$\frac{hc}{\lambda} = hcR\left(\frac{1}{1^2}-\frac{1}{3^2}\right) = 13.6\,eV\left(\frac{1}{1^2}-\frac{1}{3^2}\right) = \boxed{12.1\,eV}$$

all possible transitions from $3 \to 1$:

$n=3 \to n=2$ $\quad \Delta E = 13.6\,eV\left(\frac{1}{2^2}-\frac{1}{3^2}\right) = \boxed{1.89\,eV}$

$n=3 \to n=1$ $\quad \boxed{\Delta E = 12.1\,eV}$ (by symmetry)

$n=2 \to 1$ $\quad \Delta E = 13.6\,eV\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = \boxed{10.2\,eV}$

#21 p.1

The paschen series is the transitions to the $n=3$ state.
The shortest wavelength (highest energy) is achieved by transitioning from the $n=\infty$ state:

$$\Delta E = 13.6\,eV\left(\frac{1}{3^2}-\frac{1}{\infty^2}\right) = \boxed{1.51\,eV}$$

Note: $hc = 1240\,eV\,nm \Rightarrow \dfrac{1240\,eV\,nm}{1.51\,eV} = \boxed{821\,nm}$

The longest wavelength (lowest energy) is given by:

$$\Delta E = 13.6\,eV\left(\frac{1}{3^2}-\frac{1}{4^2}\right) = \boxed{0.66\,eV}$$

and $\dfrac{1240\,eV\,nm}{0.66\,eV} = \boxed{1876\,nm}$

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Find the potential energy and kinetic energy of an e^- in the ground state of hydrogen.

$E = K + U = \frac{1}{2}mv^2 - \frac{ke^2}{r}$ and, from Eq. 4.26,

$\frac{1}{2}mv^2 = \frac{1}{2}ke^2/r$ so $E = \frac{1}{2}ke^2/r - ke^2/r = -\frac{1}{2}\frac{ke^2}{r} = U/2$.

$U = 2E = 2(-13.6\,eV) = \boxed{-27.2\,eV}$, $K = E - U = \boxed{13.6\,eV}$

#25 p. 1

$$\Delta E = (13.6\,eV)\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 0.66\,eV$$

$$\Delta E = hf \Rightarrow f = \frac{\Delta E}{h} = \frac{0.66\,eV}{4.136\times10^{-15}\,eV\,s} = \boxed{1.60\times10^{14}\,Hz}$$

Compare to frequency of $n=3$ and $n=4$ Bohr orbits.

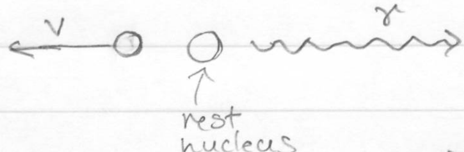
$$f_n = \frac{v}{d} = \frac{v}{2\pi r_n} = \frac{\sqrt{ke^2/mr_n}}{2\pi n^2 a_0}$$

$$\boxed{\begin{aligned} f_3 &= 2.44\times10^{14}\,Hz \\ f_4 &= 1.03\times10^{14}\,Hz \end{aligned}}$$

The γ frequency is between these two.

#29 p. 1

This problem should be solved using momentum and energy conservation.



Energy

$E_i = \Delta E_{3\to1}$

$E_F = \frac{1}{2}Mv^2 + E_\gamma$

Momentum

$P_i = 0$

$P_F = E_\gamma/c - Mv$

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We need to eliminate E_γ, the unknown.

$$\Rightarrow \quad v^2 + 2cv - \frac{2\Delta E_{3\to1}}{M} = 0$$, a quadratic equation.

$$v = \frac{-2c \pm \sqrt{4c^2 + 4\left(\frac{2\Delta E_{3\to1}}{M}\right)}}{2}$$ If we make the approximation

$$\frac{\Delta E_{3\to1}}{Mc^2} \ll 1 \text{ gives } -2c \pm 2c\left(1 + \frac{\Delta E_{3\to1}}{Mc^2}\right) = v$$

We choose the + sign to get a positive v

$$\Rightarrow \quad v = \frac{\Delta E_{3\to1}}{Mc^2}\, c = \frac{hcR\left(\frac{1}{1^2} - \frac{1}{3^2}\right)}{Mc} = \boxed{\frac{8hR}{9M}}$$

#32 p. 1

How to do this problem is completely spelled out in the statement. Because these corrections are small, you should use as many decimal places as you have available to you.

$$\boxed{\begin{aligned} \lambda_{1H} &= 656.4691\text{nm} \\ \lambda_{2H} &= 656.2925\text{nm} \\ \lambda_{3H} &= 656.2325\text{nm} \end{aligned}}$$

Yay for Harold Urey! (Recognize that name?)

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Nothing new here; just the same formulas with new "strange" numbers!

$$r_n = \frac{n^2 a_0'}{Z} = \frac{(1)^2(0.529\times10^{-10}\,m)}{(207)(82)} = \boxed{3.11\times10^{-15}\,m}$$

↑ this comes from the fact that $m_\mu = 207\, m_e$ in a_0'

$$E_n = \frac{ke^2}{2a_0'}\frac{Z^2}{n^2}, \quad E_1 = \frac{ke^2}{2a_0'}Z^2 = \frac{(9\times10^{9}\,Nm^2/C^2)(1.6\times10^{-19}\,C)^2\,82^2}{2(0.529\times10^{-10}\,m/207)}$$

$$= 3.03\times10^{-12}\,J$$

$$= \boxed{18.9\,MeV}$$

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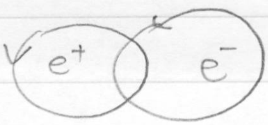
positronium



$$r_n' = \frac{n^2 a_0'}{Z} = \frac{n^2}{1}\left(\frac{\hbar^2}{\mu e^2 k}\right) \qquad \mu = \frac{m_{e^-} m_{e^+}}{m_{e^-}+m_{e^+}} = m_{e^-}/2$$

$\Rightarrow \boxed{r_n = 2 r_n}$ μ in the denominator, everything else is constant; r_n is the hydrogen radii

Likewise $\boxed{E_n' = E_n/2}$

#37 p.1 Correspondence principle

$$\Delta E = \frac{ke^2}{2a_0}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right) = \frac{ke^2}{2a_0}\left(\frac{2n-1}{(n-1)^2 n^2}\right) = hf$$

$$\Rightarrow \boxed{f = \frac{ke^2}{2ha_0}\left(\frac{2n-1}{(n-1)^2 n^2}\right)} \rightarrow \frac{ke^2}{2ha_0}\left(\frac{2n}{n^4}\right) \text{ as } n\to\infty$$

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Classical frequency: $f = \frac{v}{2\pi r} = \left(\frac{1}{2\pi}\right)\left(\frac{ke^2}{m}\right)^{1/2}\left(\frac{1}{r^{3/2}}\right)$

but $r = \frac{n^2 \hbar^2}{4\pi^2 m k e^2} \Rightarrow f = \frac{4\pi^2 m k^2 e^4}{h^3 n^3} = \boxed{\frac{ke^2}{2h a_0} \frac{2}{n^3}}$