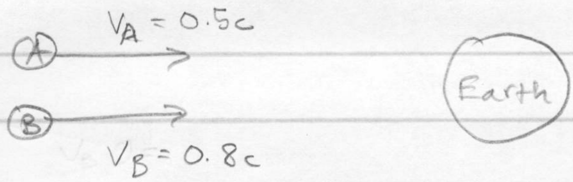
Physics 2D HW 2 Solutions Ch1 # 21 p.1

two space craft



What is V_A as measured by B?

Classically, you would expect B to observe A moving left at $0.8c - 0.5c = 0.3c$. However, since these are relativistic velocities, we know this answer is not quite right.

Equation 1.28 gives the relativistic velocity transformation:

$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

In our case, u_x' is the velocity that B observes for A, v is the velocity of B in the rest frame of the earth, and u_x is the velocity of A in the rest frame of the Earth. In other words, we are taking the Earth's rest frame to be the unprimed frame and B's rest frame to be the primed frame (which has a velocity v in the unprimed frame):

$$V_A' = \frac{V_A - V_B}{1 - \frac{V_A V_B}{c^2}} = \frac{0.5c - 0.8c}{1 - (0.5)(0.8)} = \boxed{-0.5c}$$

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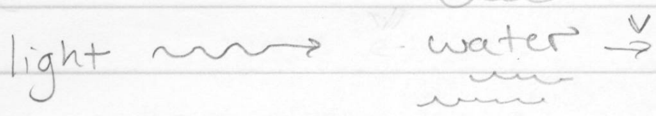
light in a moving medium



(a) show that $u = \frac{c}{n}\left(\frac{1 + nv/c}{1 + v/nc}\right)$

Hint: use inverse velocity transform:

(Eq. 1.30) $$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}}$$

We are given that an observer that is at ~~respect~~ rest with respect to the fluid will measure the speed $u_x' = c/n$. The observed speed in the "lab" frame:

$$u_x = \frac{(c/n) + v}{1 + \frac{(c/n)v}{c^2}} = \boxed{\frac{c}{n}\left(\frac{1 + \frac{vn}{c}}{1 + \frac{v}{nc}}\right)}$$

(b) For $v \ll c$ (use $(1+x)^n \simeq 1 + nx$ for small x)

$$u_x \simeq \frac{c}{n}\left(1 + \frac{vn}{c}\right)\left(1 - \frac{v}{nc}\right) = \frac{c}{n}\left(1 - \frac{v}{nc} + \frac{vn}{c} - \frac{v^2}{c^2}\right)$$

$$= \boxed{\frac{c}{n} + v - \frac{v}{n^2}} - \cancel{\frac{v^2}{nc}} \to 0$$

, b/c we assume that if $v \ll c$ the $v^2 \ll c$ also. notice that none of the other terms "compare" v and c.

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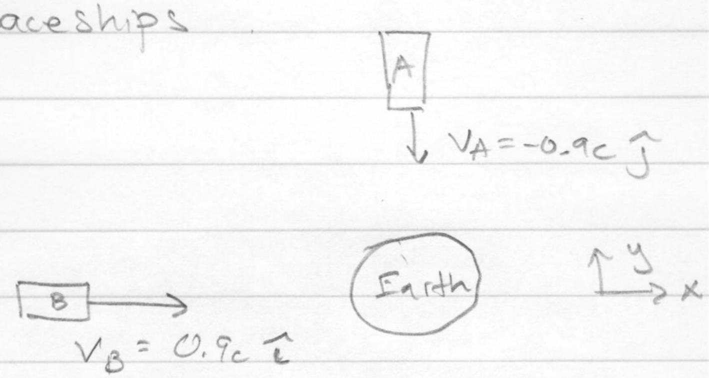
two spaceships



What is V_A as measured by B?

Again, we need to use the velocity transformations:

$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$u_y' = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)}$$

Here the primes indicate the values measured in the primed frame (which we are taking to be B's rest frame) and v is the velocity of that frame in the rest frame of the Earth.

$$u_x' = \frac{0 - 0.9c}{1 - 0} = -0.9c$$

$$u_y' = \frac{-0.9c}{\gamma(1-0)} = -0.392c \quad b/c \quad \gamma = \frac{1}{\sqrt{1-(.9)^2}} = 2.29$$

We get the total velocity from its components in the usual way:

$$u_A' = \sqrt{(u_x')^2 + (u_y')^2} = \boxed{0.982c}$$

Notice that we were lucky that the problem was setup such that the frame we were interested in was moving on the x-axis. This may not always be the case. You should know how to change the transforms or coordinate systems if this were not the case.

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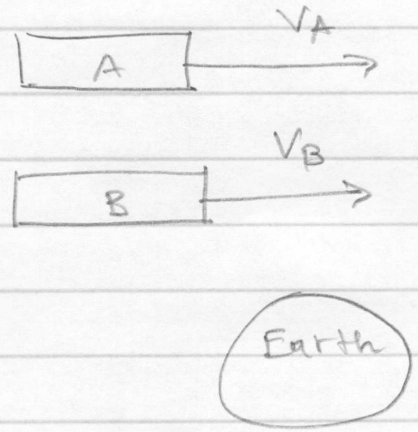
two spaceships A & B with proper lengths L_A and L_B such that $L_A = 3L_B$. Remember, the proper length of something is the length you would measure if you were at rest with respect to it.



Here we will take the Earth's rest frame to be the primed frame. The problem states that $L_A' = L_B'$ and $u_B' = 0.35c$. I chose B because the "shorter" ship must be the slower ship because it will be less contracted in the Earth's frame.

Length contraction $L' = L/\gamma \Rightarrow \dfrac{L_A}{\gamma_A} = \dfrac{L_B}{\gamma_B}$

$$\Rightarrow \gamma_A = \frac{L_A \gamma_B}{L_B} = \frac{3\cancel{L_B}\gamma_B}{\cancel{L_B}} = 3\gamma_B = 3\left(\frac{1}{1-(0.35)^2}\right)^{1/2}$$

$$= 3.20$$

$$\Rightarrow \boxed{u_A' = 0.95c}$$

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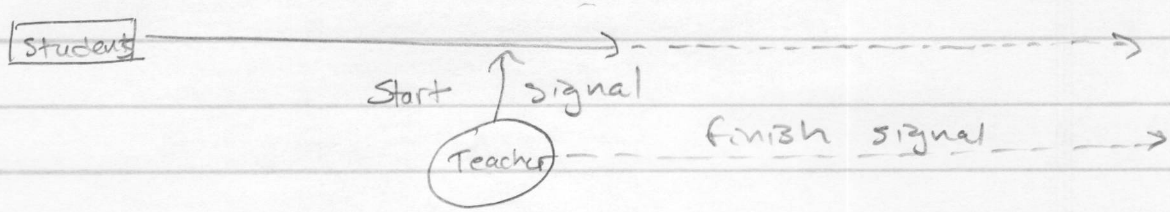
Physics exam on a spaceship:

Show $T = T_0 \sqrt{\frac{1 - v/c}{1 + v/c}}$



This is essentially a time dilation problem. If she could send them a signal instantaneously, she would wait $\Delta t = \gamma \Delta t' = \gamma_v T_0$ but she is going to want to send it sooner because it will take some time for the signal to catch the students. Let T_s be the time after which the teacher sends the signal and T_r be the time at which the students recieve the signal. The time the signal spends going is then $T_r - T_s$.

Now we have

distance light goes $\rightarrow c(T_r - T_s) = v(T_r) \leftarrow$ distance students go

$T_s = T \rightarrow c(\gamma_v T_0 - T) = v(\gamma_v T_0) \leftarrow T_r = \gamma_v T_0$

Solve for $T \rightarrow \gamma_v T_0 - T = \frac{v}{c}\gamma_v T_0$

$$T = \gamma_v T_0 - \frac{v}{c}\gamma_v T_0$$

$$= \gamma_v T_0 (1 - v/c)$$

$$= T_0 \frac{1}{\sqrt{1 - v^2/c^2}} (1 - v/c)$$

$$= T_0 \frac{1}{\sqrt{1 - v/c}\sqrt{1 + v/c}} (1 - v/c)$$

$$= \boxed{T_0 \sqrt{\frac{1 - v/c}{1 + v/c}}}$$ yay!

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electron

$p_{relativistic} = 1.90 p_{classical}$ is given.

We also know $p_{relativistic} = \gamma m u = \gamma p_{classical}$.

Therefore, $\gamma = 1.90$.

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}} = 1.9 \Rightarrow \boxed{u = 0.85c}$$

Our answer did not depend on the mass or sign of the charge $\Rightarrow$ $\boxed{\text{no change}}$

#3 p.1

Show that $F = m\left(1 - \frac{v^2}{c^2}\right)^{-3/2} \frac{dv}{dt}$ when $\vec{F} \parallel \vec{v}$.

Relativistically, Newton's 2nd Law is

$$F = \frac{dp}{dt} = \frac{d}{dt}(\gamma m v) = m\frac{d}{dt}(\gamma v)$$

$$= m(\gamma' v + v'\gamma)$$ where primes indicate time derivatives.

$$\frac{d}{dt}(\gamma) = \frac{d}{dt}\left(\frac{1}{\sqrt{1-v^2/c^2}}\right) = -\frac{1}{2}\left(\frac{1}{1-v^2/c^2}\right)^{-3/2}\left(\frac{-2v}{c^2}\right)\frac{dv}{dt}$$

$$= \frac{v}{c^2}\left(\frac{1}{1-v^2/c^2}\right)^{-3/2}\frac{dv}{dt}$$

$$\text{So, } F = m\left(\frac{v^2}{c^2}\frac{dv}{dt}\left(\frac{1}{1-v^2/c^2}\right)^{-3/2} + \frac{dv}{dt}\left(\frac{1}{\sqrt{1-v^2/c^2}}\right)\frac{(1-v^2/c^2)}{(1-v^2/c^2)}\right)$$

$$= \boxed{m\left(\frac{1}{1-v^2/c^2}\right)^{-3/2}\frac{dv}{dt}}$$

Be careful when applying the product rule and chain rule.

Get a common denominator by multiplying by $1 = \frac{(1-v^2/c^2)}{(1-v^2/c^2)}$.

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Recall $\vec{F}_B = q\vec{v} \times \vec{B}$

$$F = \frac{dp}{dt} = \frac{d}{dt}(\gamma m v) \text{ from Newton's 2nd law}$$

$F = |q\vec{v} \times \vec{B}| = qvB$ for uniform circular motion in a plane in a static magnetic field

Here's the trick:

$\frac{d}{dt}(\gamma m v) = \gamma m \frac{dv}{dt}$ even though γ is a function of v because γ only depends on v^2, which, like the magnitude of v, does not change for uniform circular motion.

Also, for uniform circular motion, we have $\frac{dv}{dt} = \frac{v^2}{r}$. Putting it all together,

$$qvB = \frac{\gamma m v^2}{r} \Rightarrow v = \frac{qBr}{\gamma m}$$

The distance traveled in one revolution is $2\pi r$, so the period, T, is given by

$$T = \frac{2\pi \cancel{r}\, \gamma m}{qB\cancel{r}}$$

and the frequency is $f = \frac{1}{T} = \frac{qB}{2\pi\gamma m}$

$$= \boxed{\frac{qB}{2\pi m}\left(1 - v^2/c^2\right)^{1/2}}$$

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Show that $E = \gamma mc^2$ and $p = \gamma mu \Rightarrow E^2 = p^2c^2 + (mc^2)^2$

$$E^2 = (\gamma mc^2)^2 = (\gamma mc^2)^2 + p^2c^2 - p^2c^2 \quad \text{, add } 0 = p^2c^2 - p^2c^2$$
$$= (\gamma mc^2)^2 + p^2c^2 - (\gamma mu)^2c^2 \quad \text{, gather } \gamma \text{ terms}$$
$$= m^2c^2\gamma^2(c^2 - u^2) + p^2c^2$$
$$= m^2c^4 \underbrace{\left(\frac{1}{1 - u^2/c^2}\right)\left(1 - \frac{u^2}{c^2}\right)}_{\text{these cancel}} + p^2c^2$$
$$= \boxed{(mc^2)^2 + p^2c^2}$$

Multiplying by 1 and adding zero are commonly used tricks in proofs of this sort.

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protons

Given $E = \gamma mc^2 = 400\, mc^2 \Rightarrow \gamma = 400$

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \Rightarrow \boxed{u = 0.999997c} \quad \text{fast}$$

What is this energy in MeV?

$$m_p = 938.272 \frac{\text{MeV}}{c^2} \Rightarrow E = 400\left(938.272 \frac{\text{MeV}}{c^2}\right)c^2$$
$$= 375309 \text{ MeV}$$

The kinetic energy is then $K = E - mc^2 = \boxed{374371 \text{ MeV}/c^2}$

The trick here is to use the right units for proton mass, namely $\frac{\text{MeV}}{c^2}$ and to recognize that the kinetic energy is the total energy minus the rest mass energy.

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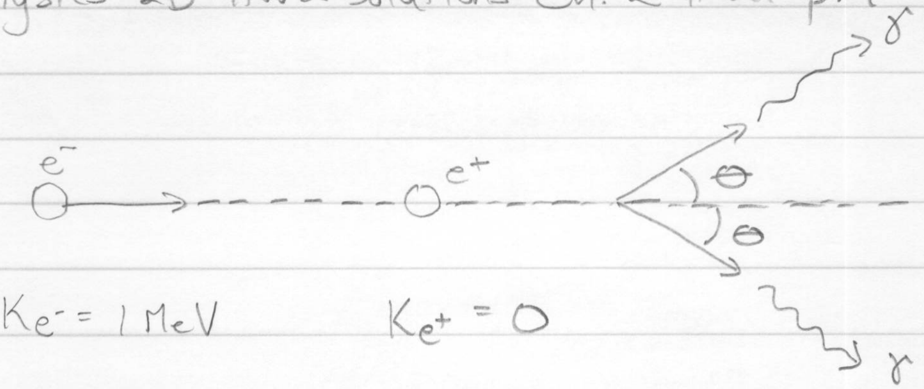

$K_{e^-} = 1\,MeV$ $\quad K_{e^+} = 0$ $\quad E_\gamma = p_\gamma c$

Find E_γ, p_γ and θ:

Relativistic energy and momentum must be conserved and the γ rays share the final energy equally.

$E_i = K_{e^-} + m_{e^-}c^2 + m_{e^+}c^2$, note: $m_{e^-} = m_{e^+}$

$=$

$E_f = 2E_\gamma \Rightarrow K_{e^-} + 2m_{e^-}c^2 = 2E_\gamma$

$$\boxed{E_\gamma = \frac{K_{e^-} + 2m_{e^-}c^2}{2}} = \boxed{1.011\,MeV}$$

$$p_\gamma = \frac{E_\gamma}{c} = \boxed{\frac{K_{e^-} + 2m_{e^-}c^2}{2c}} = \boxed{1.011\,MeV/c}$$

Conservation of momentum $\Rightarrow \cos\theta\, 2p_\gamma = p_{e^-}$

The easiest way to get the momentum of a relativistic particle if you know the total E is:

$$E^2 = p^2c^2 + (mc^2)^2 \Rightarrow p = \sqrt{\frac{E^2}{c^2} - (mc^2)^2}$$

In our case $p_{e^-} = \sqrt{\frac{E_i^2}{c^2} - (m_{e^-}c^2)^2} = 1.422\,MeV/c$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{1.422}{2(1.011)}\right) = \boxed{45.3^\circ}$$

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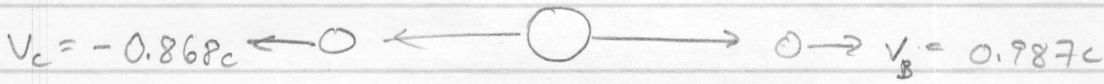
unstable particle decays

$A \to B + C$



$m_A = 3.34 \times 10^{-27}$ Kg

What are m_B and m_c ?

Both relativistic energy and momentum are conserved, A is initially at rest.

$$m_A c^2 = E_B + E_c = \gamma_B m_B c^2 + \gamma_c m_c c^2$$

$$0 = \gamma_B m_B u_B + \gamma_c m_c u_c$$

two equations in two unknowns, solve

note $\gamma_A, \gamma_B, u_A, u_B$ and m_A are all given.

$$\boxed{\begin{aligned} m_B &= 2.512 \times 10^{-28} \text{ Kg} \\ m_C &= 8.825 \times 10^{-28} \text{ Kg} \end{aligned}}$$

Use your favorite method for solving simultaneous equations.