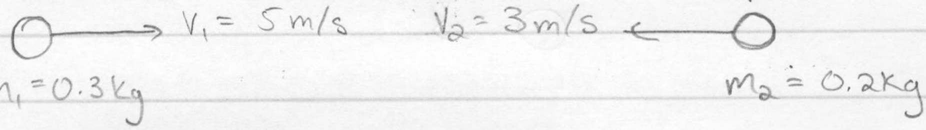
Physics 2D Homework 1 solutions # 3 p.1

two billiard balls



undergo an elastic collision.

Show that if momentum is conserved in the frame of the observer (the frame the problem is originally stated in) then it is also conserved in a frame moving to the left (as I have drawn it) at 2 m/s.

A few things to notice:

1. all speeds are much less than c, the speed of light
   $\Rightarrow$ we can solve this using classical physics
2. elastic collision $\Rightarrow$ energy and momentum are conserved

In the initial frame,

$$p_i = m_1 v_{1i} + m_2 v_{2i} \qquad p_f = m_1 v_{1f} + m_2 v_{2f}$$

$$E_i = \tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 \qquad E_f = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

momentum cons. $\Rightarrow p_i = p_f$

energy cons. $\Rightarrow E_i = E_f$

In the "primed" frame,

$$p_i' = m_1 v_{1i}' + m_2 v_{2i}' \qquad p_f' = m_1 v_{1f}' + m_2 v_{2f}'$$

We want to show $p_i' = p_f'$. (1) $\leftarrow$ this is how I label eqns I want to refer to

Apply the (classical) Galilean transformations:

$v_{ji}' = v_{ji} - v$ for $j = 1, 2$ and v is the primed frames velocity

$v_{jf}' = v_{jf} - v$

Applying these transformations to ①

$$m_1(v_{1i}-v)+m_2(v_{2i}-v) = m_1(v_{1f}-v)+m_2(v_{2f}-v)$$

$$m_1v_{1i}+m_2v_{2i}-(m_1+m_2)v = m_1v_{1f}+m_2v_{2f}-(m_1+m_2)v$$

$$m_1v_{1i}+m_2v_{2i} = m_1v_{1f}+m_2v_{2f} \quad ②$$

② is simply a statement of cons. of momentum in the initial frame, so our assumption that $p_i' = p_f'$ has been shown to be a direct result of conservation of momentum in the initial frame and the Galilean transformations. Alternatively, we could have started with ② and applied the inverse transformations.

Notice that by using this method, we have completely avoided any numerical calculation and proved the point for a general velocity v. Had we been asked to calculate velocities, we would have had to use energy conservation.

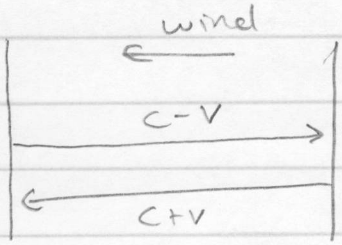
Physics 2D Homework 1 solutions #4 p.1

magical airplane, can fly @ $v=c$ in still air (not speed of light)

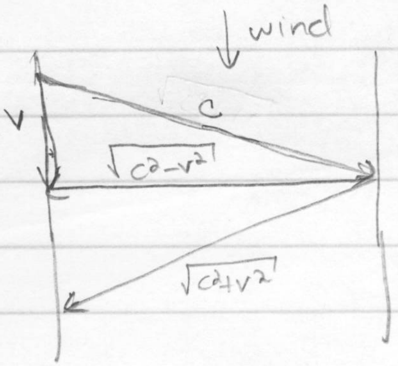
first flies upwind a distance L and then back

then flies back and forth in a cross wind

(a)



$$t_1 = \frac{d}{v} = \frac{L}{c-v} + \frac{L}{c+v} = \boxed{\frac{2L}{c}\left(\frac{1}{1-v^2/c^2}\right)}$$



$$t_2 = \frac{2L}{\sqrt{c^2-v^2}} = \boxed{\frac{2L}{c}\,\frac{1}{\sqrt{1-v^2/c^2}}}$$

(b) $\Delta t = |t_2 - t_1| = \dfrac{2L}{c}\left(\dfrac{1}{\sqrt{1-v^2/c^2}} - \dfrac{1}{1-v^2/c^2}\right)$

if $L = 100\,mi$, $c = 500\,mi/h$, $v = 100\,mi/h$

$\boxed{\Delta t = 0.0084\,h}$

Physics 2D Homework 1 solutions # 5 p.1

At what rate will a clock have to be moving in order to run at half the rate compared to one at rest?

This is a time dilation problem: $\Delta t' = \gamma \Delta t$

Remember γ is always greater than one and we have adopted the convention that primes indicate quantities measured in the frame that is taken to be moving. Therefore, $\Delta t' > \Delta t$.

In this case, we want to solve for γ such that

$$\Delta t' = 2\Delta t$$

which means $\gamma = 2$. Finally, recall $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

So $v = \sqrt{c^2\left(1 - \frac{1}{\gamma^2}\right)}$. Plugging in $\gamma = 2$,

$$v = c\sqrt{\left(1 - \frac{1}{4}\right)} = \boxed{\sqrt{\frac{3}{4}}\, c}$$

This is a sensible answer because $v < c$.

Physics 2D Homework 1 Solutions # 7 p.1

A clock on a spacecraft runs 1s slower per day. What is the relative speed of the aircraft?

Here they want you to use the expansion $(1+x)^n \simeq 1+nx$, $x \ll 1$

So $\gamma = \left(1-\frac{v^2}{c^2}\right)^{-1/2} \simeq \left(1+\frac{v^2}{2c^2}\right)$, $\left(\frac{v}{c}\right)^2 \ll 1$.

$$\Delta t' = \gamma \Delta t = \left(1+\frac{v^2}{2c^2}\right)\Delta t \Rightarrow v = \sqrt{\frac{2c^2(\Delta t' - \Delta t)}{\Delta t}}$$

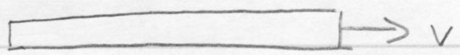
There are $24\,h \times 60\,\frac{min}{h} \times 60\,\frac{s}{min} = 86{,}400\,s$ in one day.

The slow (primed, moving clock) runs $86{,}399\,s$.

$$\Rightarrow v = \sqrt{\frac{2c^2(1s)}{86{,}400s}} = \boxed{0.0048\,c}$$

#8 p.1

meter stick



This is a length contraction problem. "A meter stick" means that its proper length (as measured by an observer at rest with respect to it) is one meter.

If it appears 75 cm long, what is v?

$$L' = \frac{L}{\gamma} \Rightarrow \gamma = \frac{L}{L'} = \frac{1}{\sqrt{1-v^2/c^2}} \Rightarrow v = \sqrt{c^2\left(1-\left(\frac{L'}{L}\right)^2\right)}$$

with $L' = 0.75\,m$, $L = 1\,m \Rightarrow \boxed{v = 0.661c}$

Physics 2D Homework 1 Solutions # 10 p.1

This is a time dilation problem with a little mechanics.

$\Delta t = 2.6 \times 10^{-8}$ s

$\Delta t' = \gamma \Delta t$, $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$, $v = .95c \Rightarrow \gamma = 3.2$

$\Delta t' = (3.2)\, 2.6 \times 10^{-8}\text{s} = \boxed{8.33 \times 10^{-8}\text{ s}}$

Finally, $d = vt = (0.95c)(8.33 \times 10^{-8}\text{s}) = \boxed{23.73\text{m}}$

In our frame of reference, short lived particles that are moving very fast appear to have longer lifetimes and therefore can travel farther than predicted by classical physics. This is a wonderful experimental confirmation of relativity.

#12 p.1

astronaut's heartbeat = 70 beats/min

(a) This observer is at rest w/ respect to her $\Rightarrow$

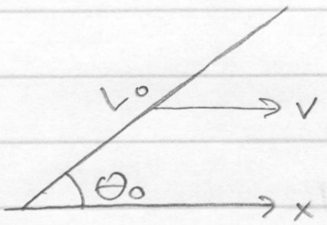
$\boxed{70\text{ beats/min}}$

(b) $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$, $v = .9c \Rightarrow \gamma = 2.29$

$\Delta t' = \gamma \Delta t \Rightarrow \dfrac{1}{\Delta t'} = \dfrac{1}{\gamma \Delta t} = \dfrac{1}{2.29}\left(\dfrac{70\text{ beats}}{\text{min}}\right) = \boxed{\dfrac{30.51\text{ bts}}{\text{min}}}$

Notice that her clock "slowing down" translates into a slower heart rate, which makes sense.

Physics 2D Homework 1 Solutions #14 p.1



(a) This is a length contraction problem, but it is important to remember that contraction only occurs in the direction of movement.

$$L_x' = \frac{L_x}{\gamma} = \frac{L_o \cos\theta_o}{\gamma}, \quad L_y' = L_y = L_o \sin\theta_o$$

$$L' = \sqrt{(L_x')^2 + (L_y')^2} = \sqrt{\left(\frac{L_o \cos\theta_o}{\gamma}\right)^2 + \left(L_o \sin\theta_o\right)^2}$$

$$= \sqrt{L_o^2\left[\left(1-\frac{v^2}{c^2}\right)\cos^2\theta_o + \sin^2\theta_o\right]}$$

$$= \boxed{L_o\sqrt{1-\frac{v^2}{c^2}\cos^2\theta_o}} \quad (\text{b/c } \cos^2\theta_o + \sin^2\theta_o = 1)$$

(b) $$\tan\theta = \frac{L_y'}{L_x'} = \frac{L_o \sin\theta_o\, \gamma}{L_o \cos\theta_o} = \boxed{\gamma \tan\theta_o}$$

The professor and the book sometimes use different notation (primes and no primes). Try thinking physically to avoid being confused.

Physics 2D Homework 1 Solutions # 16 p.1

It is always good to have a problem with a little humor. This is obviously a doppler shift problem. You also have to know a little about light.

$$f_{obs} = \left(\frac{1+V/c}{1-V/c}\right)^{1/2} f_{source} \quad (1)$$

$$\lambda_{source} = 550\,nm \Rightarrow f_{source} = \frac{c}{\lambda_{source}} = 5.45\times10^{14}\,hz$$

$$\lambda_{obs} = 650\,nm \Rightarrow f_{obs} = \frac{c}{\lambda_{obs}} = 4.61\times10^{14}\,hz$$

Rearranging (1) $\Rightarrow \left(\frac{f_{obs}}{f_s}\right)^2 = \left(\frac{1+V/c}{1-V/c}\right)$

$$\left(1-\frac{V}{c}\right)\left(\frac{f_o}{f_s}\right)^2 = 1+V/c$$

$$\left(\frac{f_o}{f_s}\right)^2 - \frac{V}{c}\left(\frac{f_o}{f_s}\right)^2 = 1+V/c$$

$$\left(\frac{f_o}{f_s}\right)^2 - 1 = \frac{V}{c} + \frac{V}{c}\left(\frac{f_o}{f_s}\right)^2$$

$$\left(\frac{f_o}{f_s}\right)^2 - 1 = \frac{V}{c}\left(1+\left(\frac{f_o}{f_s}\right)^2\right)$$

$$V = c\left(\frac{\left(\frac{f_o}{f_s}\right)^2 - 1}{1+\left(\frac{f_o}{f_s}\right)^2}\right) = \boxed{-4.97\times10^7\,m/s} = \boxed{-1.11\times10^8\,mi/h}$$

Nice car.

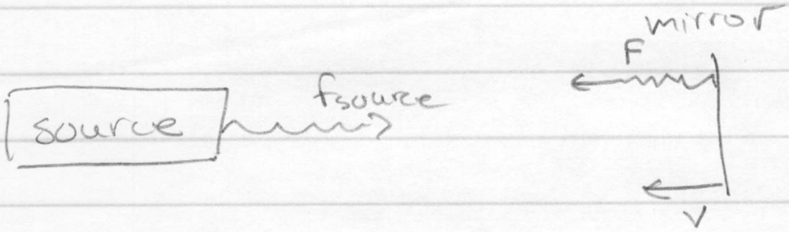
Physics 2D Homework 1 Solutions #18 p.1

Speed trap -

$$f_{obs} = \left(\frac{1+v/c}{1-v/c}\right)^{1/2} f_{source} \quad , \text{ doppler shift}$$

We want to show $f = f_{source}\left(\frac{c+v}{c-v}\right)$

where f is the frequency reflected by the mirror.



The mirror reflects light at the frequency which it observes, or $f_{ref} = \sqrt{\frac{c+v}{c-v}}\, f_{source}$ and is

therefore a new source of waves which are detected by the original source at a frequency

$$f = \sqrt{\frac{c+v}{c-v}}\, f_{ref} = \boxed{\left(\frac{c+v}{c-v}\right) f_{source}}$$

This amounts to applying the doppler shift twice.

(b) Assume $f + f_{source} \simeq 2f_{source}$ (true for $v \ll c$)

Show $f_{beat} = \frac{2v}{\lambda}$.

We have $f(c-v) = f_{source}(c+v)$

$\Rightarrow c(f - f_{source}) = v(f + f_{source}) \simeq v\, 2f_{source}$

But $f_{beat} = f - f_{source} = \frac{2v f_{source}}{c} = \boxed{\frac{2v}{\lambda}}$ (b/c $\lambda = \frac{c}{f_{source}}$)

Physics 2D Homework 1 Solutions # 18 p. 2

(c) $f_{beat} = \frac{2V}{\lambda} = 2(30\,m/s)\,\frac{10\times10^{9}\,Hz}{3\times10^{8}\,m/s} = \boxed{2000\,Hz}$

(d) $f_{beat} = \frac{2Vf_{source}}{c} \Rightarrow \Delta f_{beat} = 2\,\frac{\Delta V\,f_{source}}{c}$

$$\Delta V = \frac{\Delta f_{beat}\,c}{2f_{source}} = \frac{5\,Hz\;3\times10^{8}\,m/s}{2\;10\times10^{9}\,Hz} = \boxed{0.075\,m/s}$$

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two space ships, each with same speed in "lab" frame

relative speed = 0.7c

What speed is each moving at?

Here we will have to apply a Lorentz transformation on the velocities.

If we go into the rest frame of ship 2, then ship 1 is approaching at 0.7c. Let the rest frame of ship 2 be the "primed" frame and the lab frame be the "unprimed" frame.

$u_1' = 0.7c = \frac{u_1 - V}{1-(u_1V/c^2)}$, but $V = -u_1$ so

$$0.7c = \frac{2u_1}{1+u_1^2/c^2} \Rightarrow \boxed{u_1 = 0.408c}$$

Notice that the quadratic equation gives you two answers, but only the one with $u_1 < c$ has physical signifigance.