

# PHYS 2B Quiz 7 Solutions

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## 1 Problem 1

The induced current  $I_2$  is  $\varepsilon_{induced}/R = -M(dI_1/dt)/R$ . Note that all these quantities are constant. Hence, if you know the induced current at an earlier time, you know it at a later time; they're equal. Ans: 1mA.

## 2 Problem 2

We are given that the magnitude of

$$I_2 = \frac{MdI_1/dt}{R} \quad (1)$$

$$0.001 = \frac{2 * dI_1/dt}{200} \quad (2)$$

$$\frac{dI_1}{dt} = 0.001 * 200/2 = 0.1A/s \quad (3)$$

Hence the current at 1s is just one times the rate. Ans: 0.1A.

## 3 Problem 3

To calculate the mutual inductance, first calculate the amount of flux that a current  $I_o$  in the circle causes to penetrate the square. Call this flux  $\Phi_{o \rightarrow \square}$ . That's easier than passing a current through the square loop and asking the amount of flux that penetrates the circular loop.

If the square is small enough, the magnetic field over the entire square is well approximated by its value in the middle of the square. We know that. Refer to the formula sheet for a magnetic field at the center of a current-carrying circular loop.

$$M = \frac{\Phi_{o \rightarrow \square}}{I_o} = \frac{(\mu_0 I_o / (2 * 1))(0.1^2)}{I_o} = \frac{4\pi * 10^{-7} * 0.01}{2} = 2\pi * 10^{-9} H \quad (4)$$

## 4 Problem 4

(Ignoring pesky minus signs.)

$$I_o = \frac{\varepsilon_{ind}}{R_o} = \frac{M(dI_{\square}/dt)}{R_o} = \frac{2\pi * 10^{-9} * (100/1)}{0.1} = 2\pi * 10^{-6} A \quad (5)$$

## 5 Problem 5

In the steady state just before switch  $S_2$  is closed, there is no potential across the inductor before  $dI/dt = 0$ . That means that we have an equipotential line starting from the right side of  $R_1$  through the inductor and back to the bottom of the cell. If you connect any kind of resistor between two points of equal potential, no current will flow through the resistor at any time.  $R_2$  is one such resistor. Ans: 0A.

To put it another way, the inductor acts like a path of no resistance. Putting  $R_2$  that won't change a thing. Electrons strictly follow the path of zero resistance and will ignore any other alternatives.

## 6 Problem 6

When switch  $S_1$  is opened, the right half of the circuit, consisting of only  $R_2$  and the inductor, is now isolated. Energy stored in the inductor will gradually be lost through heating the resistor and the current decays with a time constant  $L/R_2 = 1/2 = 0.5s$ . Initially the current is  $\varepsilon/R_1 = 1/1 = 1A$ . The current after one second of decay is  $1 * \exp(-1/0.5) = \exp(-2) = 0.14A$ .

## 7 Problem 7

First find the capacitance  $C$ . After one quarter a cycle, the energy in the capacitor has completely moved to the inductor. Note: half a cycle is when the energy moves back to the capacitor for the first time, but with electric fields pointing in opposite direction. Three quarter cycle is when the energy has moved back to the inductor for a second time, but with magnetic fields pointing in opposite direction. A full cycle is when the energy returns to the capacitor with electric fields pointing in the original direction.

The oscillations go like  $\cos(t/\sqrt{LC})$ . One quarter a cycle is an angle (in radians) of  $\pi/2 = t/\sqrt{LC} = 3/\sqrt{0.002 * C}$ . Hence  $C = (6/\pi)^2/0.002 \approx 36/0.02 = 1800$ .

Initially there is a total energy of  $Q^2/(2C) = 16/3600 \approx 0.0044J$ . Total energy is conserved. When all the energy is transferred to the inductor, we have  $0.0044 = LI^2/2$  or  $I = \sqrt{0.0088/0.002} = \sqrt{4.4} = 2.1A$

## 8 Problem 8

The answer is buried in question 7. Dig: 0.0044*J*.