PHYS 2B Quiz 5 Solutions

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1 Problem 1

The particle goes through a superposition of two kinds of motion: straight line motion in the y-direction and a circular orbit in the xz-plane. Because it returns to the y axis for the first time after 6 seconds, we know that the period of one spiral is $6s$. At half a period, the particle has executed a semicircle trajectory to land at $x = -12$; we deduce a radius of orbit of 6. Call the magnitude of the component of the velocity in the xz-plane $v_\perp$. This magnitude does not change at any point in the orbit. The period is the time it takes to execute a full circle, hence $v_\perp = 2\pi r/T = 2\pi$.

Great! Now $v_\perp = v_z$ at $t = 0$, because then there is no component in the x-direction, and we know the total magnitude in the xz-plane is preserved. Because of the angle of 45, we deduce that $v_y = v_z = v_\perp = 2\pi$. In six seconds, the particle travels a distance of $6 * 2\pi = 12\pi m$ in the y-direction.

2 Problem 2

The force on a current-carrying wire in a magnetic fields is $F = I\boldsymbol{L} \times \boldsymbol{B}$ where $\boldsymbol{L}$ is the vector of length L and direction parallel to current flow. $L = 2^{1/2}a$ and the cross product is $LBsin(45) = LB/2^{1/2} = Ba$. Hence the force is $IBa = 1.4 * 2 * 0.7 = 1.96N$.

3 Problem 3

Since the dimension of the loop is of the order of $1m$ while the distance is $10m$, we can make a good approximation that the field of the loop is just the field of a dipole. Looking in the formula sheet, we have

$$B = \frac{\mu_0 \mu}{2\pi r^3} = \frac{2IArea}{10^7(10)^3} = \frac{2(1.4)(a^2/2)}{10^{10}} = 0.7 \times 10^{-10}T \qquad (1)$$

4 Problem 4

We want to superpose magnetic field lines that go in circles around each of the wires. Use the right-hand rule to determine the direction of the field. Call the direction from P to the 2I current x and the direction from P to the I-outgoing current y. I-outgoing contributes one arrow in x; I-incoming contributes 1 in x; 4I contributes 4 in y; 2I contributes 2 in $-y$. The net result is 2 arrows each in x and y. Superpose them to get a net arrow in the up direction.

5 Problem 5

The magnetic field at the center of a circular loop of radius r is $\mu_0 I/2r$. The key point is this: each infinitesimal element on that loop contributes to the field the same amount that every other element does. How do we know this? From the Biot-Savart law, we have $dB \sim (dl \times \hat{r})/r^2$. Each element is equidistant to the center of the loop, hence the denominator is the same for all. $dl \times \hat{r}$ is the same for all too, because it always points in the $\hat{z}$ direction (axis of the loop), no matter which part of the loop you pick.

We can now deduce that: if every element contributes the same amount, then the field is proportional to the number of elements. In particular, the field for one quarter a loop is just one quarter the field for a full loop. The net field from the two quarter-loops is just $1/4 * (\mu_0 I/2a - \mu_0 I/2(2a)) = \mu_0 I/16a$.

The straight length loops make zero contribution to the field. From BS law, we know that $dl \times \hat{r}$ is zero always; dl is either antiparallel or parallel to $\hat{r}$, hence the cross product is proportional to either $sin 180 = 0$ or $sin 0 = 0$.

6 Problem 6

The infinitesimal force on an infinitesimal current-carrying wire is $dF = Idl \times B$. Here $B(l) = \mu_0 I_1/2\pi l$, and the current element dl is always perpendicular to B, hence the cross product is just the product of the magnitudes: $dF = Idl\mu_0 I_1/2\pi l$. Now we sum all the forces due to each element by integration

$$F = \frac{\mu_0 I I_1}{2\pi} \int_a^{2a} dl \frac{1}{l} = \frac{\mu_0 I I_1 ln(2)}{2\pi} = 0.11 \mu_0 I I_1 \tag{2}$$

7 Problem 7

We need to determine the current density j based on the field given at P_1. Use a rectangular Amperean loop: the two horizontal lengths lie 1cm above the bottom surface and 1cm below the top surface. The length of the horizontal section is arbitrary; call it L. The fields are everywhere horizontal, hence the vertical lengths of the loop make no contribution to the integral $\int B \cdot dl$. (dl

is perpendicular to B hence the dot product is zero.) The cross-sectional area enclosed by the loop is $2L$.

$$\oint B \cdot dl = \mu_0 I_{enc} \tag{3}$$

$$BL + (-B)(-L) = \mu_0 j 2L \tag{4}$$

$$j = \frac{B}{\mu_0} = \frac{2}{\mu_0} \tag{5}$$

Now use a different Amperean loop that touches P_2. Use a rectangular loop with the horizontal sides 2cm above the top and 2cm below the bottom surface. Once again, the horizontal length is arbitrary. Now the area enclosed is $4L$.

$$\oint B \cdot dl = \mu_0 I_{enc} \tag{6}$$

$$BL + (-B)(-L) = \mu_0 (\frac{2}{\mu_0}) 4L \tag{7}$$

$$B = 4 \tag{8}$$

8 Problem 8

What is your Amperean loop here? You are given (nearly) half of it. The other half is another $22m$-long horizontal line segment just above the wires. Close off the ends of the two horizontal lines at $x = 0, 22$. We say that the vertical sides make little contribution to the integral because they're so small! By up-down symmetry, the contributions of both top and bottom horizontal lines are equal.

$$\oint B \cdot dl = \mu_0 I_{enc} \tag{9}$$

$$10^{-5} + 10^{-5} = 4\pi 10^{-7} * (8I) \tag{10}$$

$$I = \frac{200}{32\pi} \approx \frac{200}{100} = 2 \tag{11}$$