PHYS 2B Quiz 4 Solutions

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1 Problem 1

From Ohm's law $J_1 = E_1/\rho_1$. The current is $I = J_1 \pi b^2$, i.e. the current density times the cross-sectional area. The same amount of current flows through the narrow and thick cylinders. Hence $I = J_1 \pi b^2 = J_2 \pi (b/2)^2$. This gives $J_2 = 4 * J_1 = 4E_1/\rho_1$. We use Ohm's law again in the second cylinder. $E_2 = \rho_2 J_2 = 4\rho_2 E_1/\rho_1 = 4 * 2.65 * 4/1.68 = 25V/cm$.

2 Problem 2

From problem 1, we know that $J_2 = 4J_1$ or $n_2 e v_2 = 4 n_1 e v_1$. Hence $n_1/n_2 = v_2/(4v_1) = 1.5/2 = 0.75$.

3 Problem 3

When a potential difference of $1.5V$ is applied across a bulb with resistance 6Ω, a current $I = V/R = 1.5/6 = 0.25A$ flows. This causes a power dissipation of $I^2 R = 0.25 * 0.25 * 6 = 0.375W$ in the bulb. In other words, $0.375J$ of energy is continually being drained from the battery per second. The battery lasts for $4500/0.375 = 12000s = 200min = 3.3h$.

4 Problem 4

Identical bulbs have the same resistance R. When connected in series, the potential difference across each bulb is $\varepsilon/2$ according to the potential divider rule. This means a power dissipation of

$$P = \frac{(\varepsilon/2)^2}{R} + \frac{(\varepsilon/2)^2}{R} = \frac{\varepsilon^2}{2R} = 200W \tag{1}$$

When connected in parallel, the potential difference across each bulb is ε. The power dissipated is

$$P = \frac{\varepsilon^2}{R} + \frac{\varepsilon^2}{R} = \frac{2\varepsilon^2}{R} = 200W * 4 = 800W \tag{2}$$

5 Problem 5

In the first scenario, the current that flows is the emf divided by the total resistance $I_1 = \varepsilon/(10 + r)$. Similarly, for the second scenario, $I_2 = \varepsilon/(50 + r)$. The power dissipated in each of the resistors are equal:

$$P = I_2^2 * 50 = I_1^2 * 10 \tag{3}$$

Rearranging,

$$10(50 + r)^2 = 50(10 + r)^2 \tag{4}$$

$$10^{1/2}(50 + r) = 50^{1/2}(10 + r) \tag{5}$$

$$r = \frac{50(10)^{1/2} - 10(50)^{1/2}}{50^{1/2} - 10^{1/2}} = \frac{10(5 - 5^{1/2})}{5^{1/2} - 1} = 22\Omega \tag{6}$$

6 Problem 6

Looks complicated? You have been tricked!

Apply Kirchoff's voltage law on the outermost loop.

$$\varepsilon_1 - \varepsilon_2 - IR_3 = 0 \tag{7}$$

$$I = \frac{8 - 2}{3} = 2A \tag{8}$$

7 Problem 7

Since fully charged capacitor has charge $3C$, we know from the formula for a charging capacitor that $3 = \varepsilon C$ where ε is the emf of the battery. The charge on the capacitor as a function of time is

$$Q(t) = 3(1 - e^{-t/RC}) \tag{9}$$

We know the charge at a time of 1 second:

$$Q(1) = 3(1 - e^{-1/RC}) = 1 \tag{10}$$

$$\frac{2}{3} = e^{-1/RC} \tag{11}$$

We can square the last expression to obtain

$$\frac{4}{9} = e^{-2/RC} \tag{12}$$

That's useful! The charge at a time of 2 seconds is

$$Q(2) = 3(1 - e^{-2/RC}) = 3(1 - \frac{4}{9}) = \frac{15}{9} = 1.67C \tag{13}$$

8 Problem 8

Call the initial capacitance C_0. After the capacitor is fully charged, current stops flowing. The potential drop across the resistor is zero, since $V = IR$ and $I = 0$. That means the full drop occurs across the capacitor; the potential drop across the capacitor is just the emf $6V$. The charge on the capacitor is $Q = C_0V = 6C_0$.

Now the dielectric is inserted instantaneously and the capacitor is beefed up to $3C_0$. Dielectrics weaken the electric field in between the plates and hence reduce the potential difference across it. We assume the dielectric is inserted so quickly that the charge on the capacitor has no time to move. With that same amount of charge, the potential difference is now $V' = Q/C' = 6C_0/3C_0 = 2V$. If we apply Kirchoff's voltage law on the loop, this means that there is now a potential drop of $6 - 2 = 4V$ across the resistor, which results in a current of $I' = V'/R = 4/4 = 1A$.