PHYS 2B Quiz 2 Solutions

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1 Problem 1

From Gauss's law, we know the electric field of a uniformly charged sphere with charge density ρ

$$\frac{\rho 4\pi r^3}{3\epsilon_0} = E 4\pi r^2 \tag{1}$$

$$E = \frac{\rho r}{3\epsilon_o} \tag{2}$$

We are informed that

$$E(r = \frac{R}{2}) = 5 = \frac{\rho R}{6\epsilon_o} \tag{3}$$

Rewriting ρ as the total charge Q divided over the volume of a sphere, and dividing both sides by two, we obtain

$$2.5 = \frac{Q}{4\pi\epsilon_0 (2R)^2} \tag{4}$$

This gives us the Coulomb field at $r = 2R$ and the magnitude of the field (2.5).

2 Problem 2

The gain in kinetic energy from R to 2R is the loss in electrostatic potential energy:

$$\frac{1}{2}mv_1^2 = kQq(\frac{1}{R} - \frac{1}{2R}) = \frac{kQq}{2R} \tag{5}$$

Similarly for R to 3R,

$$\frac{1}{2}mv_2^2 = kQq(\frac{1}{R} - \frac{1}{3R}) = \frac{2kQq}{3R} \tag{6}$$

Hence,

$$\frac{v_2^2}{v_1^2} = \frac{\frac{2}{3}}{\frac{1}{2}} \tag{7}$$

Taking the square root, we obtain $v_2 = 1.15v_1$.

3 Problem 3

Exploit the spherical symmetry of the problem so that we can use Gauss' law:

$$\frac{Q_{enclosed}}{\epsilon_0} = E4\pi r^2 \tag{8}$$

For the electric field to be zero, the total charge enclosed must be zero. The total charge has two contributions from the inner and outer parts of the sphere:

$$0 = Q_{enclosed} = \rho_2 \frac{4\pi R^3}{3} + \rho_1 \frac{4\pi((2R)^3 - R^3)}{3} \tag{9}$$

Dividing out common factors, we obtain $\rho_2 = -7\rho_1$.

4 Problem 4

Same reasoning as in problem 3, now with cylindrical symmetry:

$$0 = Q_{enclosed} = \rho_2 \pi R^2 L + \rho_1 \pi((2R)^2 - R^2)L \tag{10}$$

We get $\rho_2 = -3\rho_1$.

5 Problem 5

The plate has area charge density $\sigma = q/a^2$ because the charge is uniformly distributed in volume. Apply Gauss's law with a small (i.e. much smaller than the plate) square pillbox that has area A and width $3a/100$. This pillbox encloses σA amount of charge.

$$\frac{\sigma A}{\epsilon_0} = 2EA \tag{11}$$

The extra factor of 2 on the right is because the electric field is equal in magnitude both above and below the plate. Rearranging, the electric field is $E = q/(2\epsilon_0 a^2)$. This is the electric field of an infinite charged plate; the formula is a good approximation for small distances above or below the plate.

6   Problem 6

For a conducting surface, the charge is no longer uniformly distributed in volume. Any extra charge resides on the top and bottom surfaces. This is a consequence of the mobility of charge inside a conductor. The key point is that the projected area density of charge is still the same as in problem 5. We employ the same method to obtain the same answer.

7   Problem 7

The two conducting spheres are connected by a conducting wire. Any connected conductor must be at the same electrostatic potential. We estimate the potential at the surfaces of the spheres by the formula for a charge distribution with spherical symmetry $V = kQ/r$. Strictly speaking, the symmetry has been broken in the direction of the other ball. However, if the second ball is very far away, it is not a bad approximation.

$$V = \frac{kq_1}{R} = \frac{kq_2}{1.5R} \tag{12}$$

Equality of potential at the surface of balls with different radii implies that the electric fields are different:

$$\frac{E_2}{E_1} = \frac{\frac{kq_2}{(1.5R)^2}}{\frac{kq_1}{R^2}} = \frac{\frac{kq_1}{1.5R^2}}{\frac{kq_1}{R^2}} = \frac{1}{1.5} = 0.67 \tag{13}$$

In the second equality, we used the first equation (equality of potential).

In the limit that the smaller ball gets really small, we note this huge difference in electric fields. Pointy parts of a conductor have larger electric fields than smoother parts. This is why lightning rods are so pointy. The electric field at the tip of a rod is so strong that the air breaks down and a current can flow from the rod to the sky.

8   Problem 8

The charges form the vertices of a square of length $2a$. To evaluate the potential at a point, we superpose the potentials of all four charges. At point 1, we note that the contributions from the positive and negative charge to the right cancel, because they are equidistant to point 1 and have opposite charges. Hence,

$$V_1 = \frac{kq}{a} + \frac{kq}{a} = \frac{2kq}{a} \tag{14}$$

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At point 2, the contributions directly above and below also cancel, for the same reason. We have contributions from the two positive charges to the left, which are at distance $5^{.5}a$ away.

$$V_2 = \frac{kq}{5^{.5}a} + \frac{kq}{5^{.5}a} = \frac{2kq}{5^{.5}a} \tag{15}$$

$$\frac{V_2}{V_1} = \frac{1}{5^{.5}} = 0.45 \tag{16}$$

If necessary, please come to office hours to clarify any of these problems.