PHYS 2B Quiz 1 Solutions

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1 Problem 1

To find x_0, solve for

$$\frac{k(3q)}{x_0^2} + \frac{k(-q)}{(x_0 - a)^2} = 0 \quad (1)$$

The first term is the field due to the charge at the origin; the second due to the charge at distance a from the origin. Sum the two fields to get the net field, and set it to zero.

2 Problem 2

Immediately to the right of the negative charge, the field is very negative, because the influence of the negative charge is strongest at such small distances. As x increases away from the negative charge, the field due to the negative charge dies as $1/(r^2)$ and the field due to the charge 3q begins to dominate. At distance $2.4a$, the fields exactly cancel out. The field will rise to a positive value as x increases further. It peaks then approaches zero as we take x to infinity. To find this peak,

$$\frac{d}{dx}\left(\frac{k(3q)}{x^2} + \frac{k(-q)}{(x - a)^2}\right) = 0 \quad (2)$$

We obtain $3(x-a)^3 = x^3$. Going further, $3^{0.333}(x-a) = x$, from which we solve for $x = 3.3a$.

3 Problem 3

The charge configuration can be split into a point charge $2q$, which has a field that dies as $1/d^2$ (hence $\alpha = 2$), and a dipole with dipole moment qa. The dipole has a field that dies as $1/d^3$, hence $\beta = -2$. The minus sign in β is because the field of the dipole points to the left (because the negative charge is nearer), while the field of the point charge points to the right. The 2 in β is a geometric factor that enters in the derivation of the field of a dipole. See the textbook for that derivation.

4 Problem 4

P_2 is $2^{1/2}a$ away from the negative charge. This results in a field of total magnitude:

$$E = \frac{kq}{(2^{\frac{1}{2}}a)^2} \tag{3}$$

When we resolve components in x and y, we multiply by cos45 or sin45. Both factors are $2^{-1/2}$. Hence, due to the negative charge only,

$$E_x = +\frac{kq}{(2^{\frac{1}{2}}a)^2} \cdot \frac{1}{2^{\frac{1}{2}}} = \frac{kq}{a^2} \cdot (0.4) \tag{4}$$

$$E_y = -\frac{kq}{(2^{\frac{1}{2}}a)^2} \cdot \frac{1}{2^{\frac{1}{2}}} = -\frac{kq}{a^2} \cdot (0.4) \tag{5}$$

The plus and minus signs are because the field points towards the negative charge. The positive charge makes a contribution

$$E_y = +\frac{kq}{a^2} \cdot (3) \tag{6}$$

Sum the E_y components due to both charges to get $\beta = 2.6$.

5 Problem 5

Coloumb's law in its infinitesimal form is

$$dE = \frac{k(dq)}{r^2} \tag{7}$$

where $dq = \lambda_0 \frac{x}{L} dx$. At a point on the rod that is x distance away from the origin, $r^2 = x^2 + L^2$.

To find the total field, integrate this expression:

$$\int dE = \int_0^L dx \frac{k\lambda_0 x}{L(x^2 + L^2)} \cdot \frac{L}{(x^2 + L^2)^{\frac{1}{2}}} \tag{8}$$

The extra factor on the right is there because we are interested in the y component only. This integration has been done in lecture. Take a look.

6 Problem 6

At large distances away, the rod looks like a point charge with total charge

$$Q = \int_0^L dx \lambda_0 \frac{x}{L} = \frac{\lambda_0 L}{2} \tag{9}$$

Apply Coulomb's law for a point charge Q. $\beta = 1/2$.

7 Problem 7

Consider the dotted line that runs from the center of the quadrupole to point P. We are considering the field on this dotted line. Our nearest two charges are a positive charge on the top of the dotted line and a negative charge below. Since field lines only go from positive charge to negative charge, we must conclude that the field points downwards.

Question: What happens when we apply Gauss's law to this configuration of no net charge? Does Gauss's law suggest that the field is zero? Think about it.