PHYSICS 2B FINAL EXAM WINTER QUARTER 2010
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Formulas:

$\sin 30^\circ = \cos 60^\circ = 1/2, \quad \cos 30^\circ = \sin 60^\circ = \sqrt{3}/2, \quad \sin 45^\circ = \cos 45^\circ = \sqrt{2}/2$

$F = k\frac{q_1 q_2}{r^2}$ Coulomb's law ; $k = 9\times 10^9 \mathrm{N\cdot m^2/C^2}$; $\vec{F}_{12} = \frac{kq_1q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$

Electric field due to charge q at distance r: $\vec{E} = \frac{kq}{r^2}\hat{r}$; Force on charge Q: $\vec{F} = Q\vec{E}$

Electric field of dipole: along dipole axis / perpendicular: $E = \frac{2kp}{x^3}$ / $E = \frac{kp}{y^3}$ (p=qd)

Energy of and torque on dipole in E-field: $U = -\vec{p}\cdot\vec{E}$, $\vec{\tau} = \vec{p}\times\vec{E}$

Linear, surface, volume charge density: $dq = \lambda\, ds$, $dq = \sigma\, dA$, $dq = \rho\, dV$

Electric field of infinite: line of charge: $E = \frac{2k\lambda}{r}$; sheet of charge: $E = 2\pi k\sigma = \sigma/(2\varepsilon_0)$

Gauss law: $\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$; Φ = electric flux ; $k = \frac{1}{4\pi\varepsilon_0}$; $\varepsilon_0 = 8.85\times 10^{-12} C^2/Nm^2$

$U_B - U_A = \Delta U_{AB} = -W_{AB} = -\int_A^B \vec{F}\cdot d\vec{l} = -\int_A^B q\vec{E}\cdot d\vec{l} = q\Delta V_{AB} = q(V_B - V_A)$ V=N/C

$V = \frac{kq}{r}$; $V = \int \frac{kdq}{r}$; $V = \frac{kp\cos\theta}{r^2}$ (dipole) ; $E_l = -\frac{\partial V}{\partial l}$; $\vec{E} = -\vec{\nabla}V$

Electrostatic energy: $U = k\frac{q_1q_2}{r}$; Capacitors: $Q = CV$; with dielectric: $C = \kappa C_0$; $\varepsilon_0 = 8.85\, pF/m$

$C = \frac{\varepsilon_0 A}{d}$ parallel plates ; $C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$ cylindrical ; $C = 4\pi\varepsilon_0\frac{ab}{b-a}$ spherical

Energy stored in capacitor: $U = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2$; $U = \int dv\, u_E$; $u_E = \frac{1}{2}\varepsilon_0 E^2$

Capacitors in parallel: $C = C_1 + C_2$; in series: $C = C_1C_2/(C_1 + C_2)$

Elementary charge: $e = 1.6\times 10^{-19} C$

$I = \frac{dq}{dt} = \int \vec{J}\cdot d\vec{A}$; $\vec{J} = ne\vec{v}_d$; $v_d = \frac{eE\tau}{m}$; $\rho = \frac{m}{ne^2\tau}$; $R = \rho\frac{\ell}{A}$; $\vec{E} = \rho\vec{J}, \vec{J} = \sigma\vec{E}$

$V = IR$; $P = VI = I^2R = V^2/R$; $P_{emf} = \varepsilon I$; $R_{eq} = R_1 + R_2$ (series) ; $R_{eq}^{-1} = R_1^{-1} + R_2^{-1}$ (parallel)

Charging capacitor: $Q(t) = C\varepsilon(1 - e^{-t/RC})$; Discharging capacitor: $Q(t) = Q_0 e^{-t/RC}$

Force on moving charge: $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$; force on wire: $d\vec{F} = Id\vec{\ell}\times\vec{B}$

Circular motion: $a = \frac{v^2}{r}$; radius $r = \frac{mv}{qB}$; period $T = \frac{2\pi\, \mathrm{m}}{\mathrm{qB}}$

Magnetic dipole: $\vec{\mu} = I\vec{A}$; torque: $\vec{\tau} = \vec{\mu}\times\vec{B}$; energy: $U = -\vec{\mu}\cdot\vec{B}$

Biot-Savart law: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$; $\mu_0 = 4\pi\times 10^{-7}\frac{N}{A^2}$; Ampere's law: $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$

Long wire: $B = \frac{\mu_0 I}{2\pi r}$; loop, along axis: $B = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}}$; dipole: $\vec{B} = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{x^3}$

solenoid: $B = \mu_0 In$; toroid: $B = \frac{\mu_0 NI}{2\pi r}$; Gauss law for magnetism: $\oint \vec{B}\cdot d\vec{A} = 0$

Faraday law: $\varepsilon = -\frac{d\Phi_B}{dt} = \oint \vec{E}\cdot d\vec{s}$; $\Phi_B = \int \vec{B}\cdot d\vec{A}$ magnetic flux

Mutual inductance: $M = \frac{\Phi_2}{I_1} = \frac{\Phi_1}{I_2}$; $\varepsilon_2 = -M\frac{dI_1}{dt}$; $\varepsilon_1 = -M\frac{dI_2}{dt}$

Self - inductance: $L = \frac{\Phi_B}{I}$; $\varepsilon_L = -L\frac{dI}{dt}$; $L = \mu_0 n^2 A\ell$ for solenoid

Magnetic energy: $U_B = \frac{1}{2}LI^2$; $u_B = \frac{B^2}{2\mu_0}$

RL circuit: $I = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L})$ (rise) ; $I = I_0 e^{-t/\tau_L}$ (decay) ; $\tau_L = L/R$

LC oscillations: $q(t) = q_p\cos(\omega_0 t)$; $I(t) = -\omega_0 q_p \sin(\omega_0 t)$; $\omega_0 = \frac{1}{\sqrt{LC}}$

Alternating emf, RLC: $V = V_p \sin\omega t$; $I = I_p\sin(\omega t - \phi)$; $I_p = V_p/Z$; $\tan\phi = \frac{X_L - X_C}{R}$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$; Voltage amplitudes: $V_R = IR,\ V_L = IX_L,\ V_C = IX_C$; $X_L = \omega L,\ X_C = \frac{1}{\omega C}$

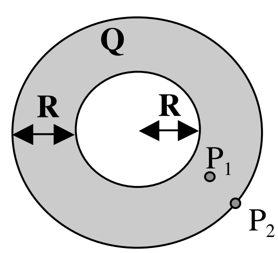
Ampere - Maxwell law: $\oint \vec{B}\cdot dl = \mu_0 I_{enc} + \mu_0\varepsilon_0\frac{d\phi_E}{dt}$; displacement current $I_d = \varepsilon_0\frac{d\phi_E}{dt}$; $\phi_E = \int \vec{E}\cdot d\vec{A}$

Electromag. waves: $\vec{E} = E_p\sin(kx - \omega t)\hat{j}$; $\vec{B} = B_p\sin(kx - \omega t)\hat{k}$; $\frac{E_p}{B_p} = \frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3\times10^8 m/s$

$k = 2\pi/\lambda$, $\omega = 2\pi f$

<u>There are 22 problems. You get 1 point for correct answer, 0 points for incorrect answers, 0.2 points for no answer (up to 10 non-answers). This is Test Form D</u>

Problems 1, 2

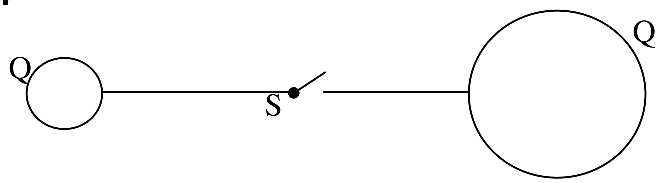


The figure shows a non-conducting spherical shell of inner radius R and outer radius 2R (i.e. radial thickness R) with charge Q uniformly distributed throughout its volume.

Prob 1: The electric field at point P_1 at distance 1.5R from the center is 1N/C. The electric field at point P_2 on the outer surface of the shell, i.e. at distance 2R from the center, is
(a) 1.66N/C ; (b) 2N/C ; (c) 2.33N/C ; (d) 1.33N/C ; (e) 1N/C

Prob 2: Assume this shell now becomes conducting, having the same total charge Q. The electric fields at P_1 and P_2 now (call them E_1 and E_2):
(a) E_1 is same as in prob 1, E_2 is not E_1; (b) Both E_1 and E_2 are 0 ; (c) E_1 is 0, E_2 is same as in prob 1; (d) Both are same as in prob 1; (e) is 0, E_2 is larger than in prob 1;

Problems 3, 4

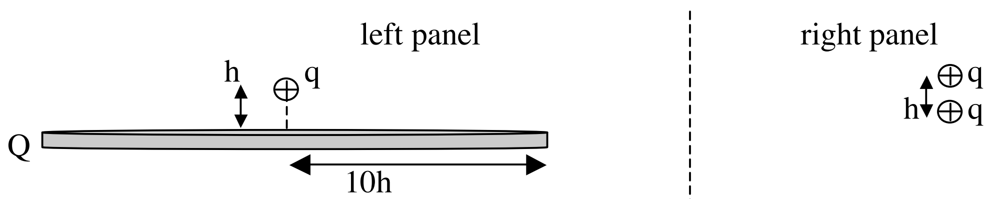


The two metallic spheres shown have radius R and 3R respectively, with R=0.5m, and initially have the same charge Q=2C each. They are at a distance d>>R. They are connected by a metallic wire of resistance 3Ω which has a switch S that is open. Then the switch S is closed.

Prob 3: A long time after the switch S is closed, the charge on the small sphere is
(a) Q ; (b) Q/3 ; (c) 2Q/3 ; (d) (3/2) Q ; (e) Q/2

Prob 4: Immediately after the switch S is closed, the current in the wire (in A) is the product of k (Coulomb constant) times
(a) 0.75 ; (b) 0.33 ; (c) 0.67 ; (d) 0.46 ; (e) 0.89

Problems 5, 6

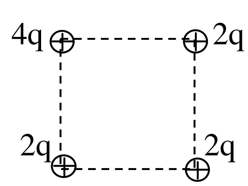


Note: there is no interaction between the charges on the left panel and those on the right panel.

Prob 5: On the left panel, a charge q is at a very small distance h from a thin circular charged plate of radius 10h that has total charge Q uniformly distributed. The force on this charge q is identical to the force on the upper charge q of the right panel, where another charge q is at distance h below it. The ratio Q/q is approximately
(a) 1 ; (b) 50 ; (c) 200 ; (d) 10 ; (e) 100

Prob 6: suppose the charge q on the left panel, and the upper charge q on the right panel, are both moved down a distance h/2, keeping the circular plate on the left panel and the lower charge on the right panel in the same position. Then the ratio of the forces on the charges we moved, left / right, is
(a) 1 ; (b) 0.25 ; (c) 4 ; (d) 2 ; (e) 0.5

Problems 7, 8

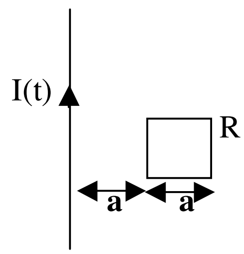


The four charges shown are on the vertices of a square of side length a.

Prob 7: The magnitude of the electric field acting on charge 2q in the lower right corner is 2N/C. The magnitude of the electric field acting on charge 4q is
(a) 0.6N/C ; (b) 1.1N/C ; (c) 1.6N/C ; (d) 2.1N/C ; (e) 2.6N/C

Prob 8: the magnitude of the electric field at distance d=100a from the center of this square is approximately
(a) $2\times10^{-3}\frac{q}{4\pi\varepsilon_0 a^2}$; (b) $\frac{q}{4\pi\varepsilon_0 a^2}$; (c) $10^{-2}\frac{q}{4\pi\varepsilon_0 a^2}$; (d) $2\times10^{-2}\frac{q}{4\pi\varepsilon_0 a^2}$; (e) $10^{-3}\frac{q}{4\pi\varepsilon_0 a^2}$

Problems 9, 10, 11



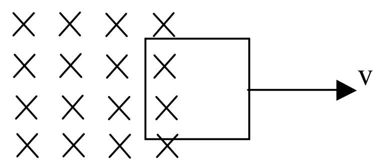
The figure shows a straight vertical long wire carrying a time-dependent current $I(t)=I_0\, t^2/\tau^2$, with I_0=100A and τ=1s. The square loop shown has resistance R=1Ω and sides of length **a**=25cm. Two sides of the square loop are parallel to the long wire, with the closest one at distance **a** from the long wire.

Prob 9: At time t=1s, at the center of the square, the magnetic field due to the long wire has magnitude ? and points ?
(a) 53μT, out of paper ; (b) 68μT, into paper; (c) 29μT, into paper ;
(d) 53μT, into paper ; (e) 29μT, out of paper

Prob 10: the current induced in the square loop at time t=1s has magnitude
(a) 7.1μA ; (b) 6.7μA ; (c) 6.3μA ; (d) 6.5μA ; (e) 6.9μA
Hint: you need to do an integral

Prob 11: the current induced in the square loop at time t=2s flows
(a) alternating, with period π/τ ; (b) counterclockwise; (c) it's unpredictable ;
(d) alternating, with frequency τ; (e) clockwise;

Problems 12, 13, 14

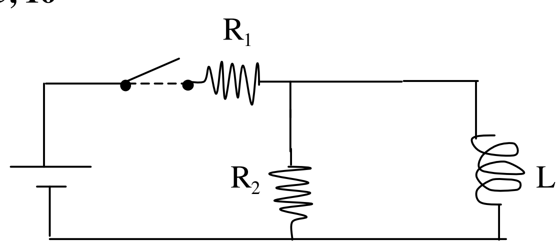


Prob 12: The square loop of wire shown has resistance 30Ω and side length 0.2m. It is being pulled out of a region of constant uniform magnetic field B=2.5T pointing into the paper, at a speed v=40m/s. The induced current in the loop is
(a) 0.33A ; (b) 0.66A; (c) 2.0A ; (d) 1.33A ; (e) 0.5A

Prob 13: The force that needs to be applied to pull the loop out at constant speed is
(a) 0.33N ; (b) 0.66N; (c) 2.0N; (d) 1.33N ; (e) 0.5N

Prob 14: The total energy supplied by the external force in pulling the loop out of the magnetic field region is
(a) 0.13J ; (b) 0.50J ; (c) 0.33J ; (d) 0. 67J ; (e) 0.067J

Problems 15, 16



In the circuit shown, R_1=10Ω, R_2=20Ω and L=5H. The switch has been open for a long time, then it is closed. Immediately after closing the switch the current through R_2 is 2A.

Prob 15: a long time after the switch is closed the current through R_2 is
(a) 1A ; (b) 2A ; (c) 6A ; (d) 0A ; (e) 0.66A

Prob 16: subsequently, the switch is opened again. 1s after the switch is opened the current through R_2 is
(a)1.1A ; (b) 0.011A ; (c) 0.11A ; (d) 0.041A ; (e) 0A

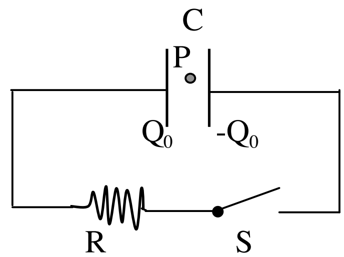
Problems 17, 18

In a series driven RLC circuit, the impedance at resonance is 60Ω, the resonance frequency is ω=2000s^{-1} and the inductance is 10mH.

Prob 17: what is the impedance at frequency ω=4000s^{-1}?
(a) 67Ω ; (b) 74Ω ; (c) 88Ω ; (d) 81 Ω ; (e) 53Ω

Prob 18: at ω=4000s^{-1}, the phase difference between current and voltage is:
(a) current lags voltage by 43° ; (b) current leads voltage by 27°; (c) current lags voltage by 27°; (d) current neither lags nor leads voltage ; (e) current leads voltage by 43°

Problems 19, 20, 21



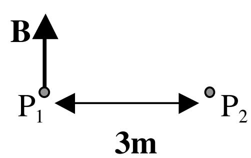
In the circuit shown, the parallel plate capacitor has capacitance 3mF and has round plates of radius a=0.4m. The resistor has resistance 250Ω. Initially the charge on the capacitor is Q_0=5C, with the left capacitor plate having positive charge, and the switch S is open. At time t=0 the switch S is closed.

Prob 19: the current through the resistor at time t=1s is
(a) 1.76A ; (b) 2.36A ; (c) 3.15A; (d) 1.32A ; (e) 6.66A

Prob 20: the magnetic field at point P inside the capacitor, located at distance 0.1m above its symmetry axis, at the moment when the current through the resistor is I=1A, is
$\frac{\mu_0 I}{2\pi a} \times C$, with C=
(a) 2; (b) 0.25 ; (c) 1.25 ; (d) 0.0625 ; (e) 1

Prob 21: at that moment, the magnetic field at point P points
(a) right ; (b) up ; (c) into the paper ; (d) down; (e) out of the paper

Problem 22



The figure shows two points, P_1 and P_2, separated by a distance 3m in the horizontal direction. There is a plane electromagnetic wave propagating in direction normal to the paper. The magnetic field B from this electromagnetic wave at point P_1 is shown at time t_0, pointing up. The frequency of this electromagnetic wave is f=0.5x10^8 cycles/s (Hz) (f=ω/2π). At point P_1 there is a positive charge, and at point P_2 there is a negative charge, both charges are at rest. The forces on these charges due to the fields of the electromagnetic wave at time t_0:

(a) point in same directions, horizontally; (b) point in the opposite direction perpendicular to the paper; (c) information given is insufficient to answer, since one needs to know whether wave is propagating into or out of the paper; (d) point in the opposite direction, horizontally; (e) point in same directions perpendicular to the paper;