

**Formulas:**

$$\sin 30^\circ = \cos 60^\circ = 1/2, \quad \cos 30^\circ = \sin 60^\circ = \sqrt{3}/2, \quad \sin 45^\circ = \cos 45^\circ = \sqrt{2}/2$$

$$F = k \frac{q_1 q_2}{r^2} \quad \text{Coulomb's law} ; k = 9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \quad ; \quad \vec{F}_{12} = \frac{k q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1)$$

$$\text{Electric field due to charge } q \text{ at distance } r : \quad \vec{E} = \frac{kq}{r^2} \hat{r} \quad ; \quad \text{Force on charge } Q : \quad \vec{F} = Q\vec{E}$$

$$\text{Electric field of dipole: along dipole axis / perpendicular: } E = \frac{2kp}{x^3} \quad / \quad E = \frac{kp}{y^3} \quad (p=qd)$$

$$\text{Energy of and torque on dipole in E-field: } U = -\vec{p} \cdot \vec{E} \quad , \quad \vec{\tau} = \vec{p} \times \vec{E}$$

$$\text{Linear, surface, volume charge density: } dq = \lambda ds \quad , \quad dq = \sigma dA \quad , \quad dq = \rho dV$$

$$\text{Electric field of infinite: line of charge: } E = \frac{2k\lambda}{r} ; \quad \text{sheet of charge: } E = 2\pi k\sigma = \sigma/(2\epsilon_0)$$

$$\text{Gauss law: } \Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \quad ; \quad \Phi = \text{electric flux} ; k = \frac{1}{4\pi\epsilon_0} ; \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

$$U_B - U_A = \Delta U_{AB} = -W_{AB} = -\int_A^B \vec{F} \cdot d\vec{l} = -\int_A^B q\vec{E} \cdot d\vec{l} = q\Delta V_{AB} = q(V_B - V_A) \quad V = N/C$$

$$V = \frac{kq}{r} ; V = \int \frac{k dq}{r} \quad ; \quad V = \frac{kp \cos \theta}{r^2} \quad (\text{dipole}) ; \quad E_l = -\frac{\partial V}{\partial l} \quad ; \quad \vec{E} = -\nabla V$$

$$\text{Electrostatic energy: } U = k \frac{q_1 q_2}{r} ; \text{Capacitors: } Q = CV ; \text{with dielectric: } C = \kappa C_0 ; \epsilon_0 = 8.85 \text{ pF/m}$$

$$C = \frac{\epsilon_0 A}{d} \quad \text{parallel plates} \quad ; \quad C = \frac{2\pi\epsilon_0 L}{\ln(b/a)} \quad \text{cylindrical} \quad ; \quad C = 4\pi\epsilon_0 \frac{ab}{b-a} \quad \text{spherical}$$

$$\text{Energy stored in capacitor: } U = \frac{Q^2}{2C} = \frac{1}{2} QV = \frac{1}{2} CV^2 \quad ; \quad U = \int dv u_E \quad ; \quad u_E = \frac{1}{2} \epsilon_0 E^2$$

$$\text{Capacitors in parallel: } C = C_1 + C_2 \quad ; \quad \text{in series: } C = C_1 C_2 / (C_1 + C_2)$$

$$\text{Elementary charge: } e = 1.6 \times 10^{-19} \text{ C}$$

$$I = \frac{dq}{dt} = \int \vec{J} \cdot d\vec{A} \quad ; \quad \vec{J} = ne\vec{v}_d \quad ; \quad v_d = \frac{eE\tau}{m} \quad ; \quad \rho = \frac{m}{ne^2\tau} \quad ; \quad R = \rho \frac{\ell}{A} \quad ; \quad \vec{E} = \rho \vec{J}, \quad \vec{J} = \sigma \vec{E}$$

$$V = IR \quad ; \quad P = VI = I^2 R = V^2 / R \quad ; \quad P_{emf} = \epsilon I \quad ; \quad R_{eq} = R_1 + R_2 \quad (\text{series}) \quad ; \quad R_{eq}^{-1} = R_1^{-1} + R_2^{-1} \quad (\text{parallel})$$

$$\text{Charging capacitor: } Q(t) = C\epsilon(1 - e^{-t/RC}) \quad ; \quad \text{Discharging capacitor: } Q(t) = Q_0 e^{-t/RC}$$

$$\text{Force on moving charge: } \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad ; \quad \text{force on wire: } d\vec{F} = Id\vec{\ell} \times \vec{B}$$

$$\text{Circular motion: } a = \frac{v^2}{r} \quad ; \quad \text{radius } r = \frac{mv}{qB} \quad ; \quad \text{period } T = \frac{2\pi m}{qB}$$

$$\text{Magnetic dipole: } \vec{\mu} = I\vec{A} \quad ; \quad \text{torque: } \vec{\tau} = \vec{\mu} \times \vec{B} \quad ; \quad \text{energy: } U = -\vec{\mu} \cdot \vec{B}$$

$$\text{Biot - Savart law: } d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{\ell} \times \hat{r}}{r^2} \quad ; \quad \mu_0 = 4\pi \times 10^{-7} \frac{\text{N}}{\text{A}^2} \quad ; \quad \text{Ampere's law: } \oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$\text{Long wire: } B = \frac{\mu_0 I}{2\pi r} \quad ; \quad \text{loop, along axis: } B = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}} \quad ; \quad \text{dipole: } \vec{B} = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{x^3}$$

$$\text{solenoid: } B = \mu_0 I n \quad ; \quad \text{toroid: } B = \frac{\mu_0 N I}{2\pi r} \quad ; \quad \text{Gauss law for magnetism: } \oint \vec{B} \cdot d\vec{A} = 0$$

$$\text{Faraday law: } \epsilon = -\frac{d\Phi_B}{dt} = \oint \vec{E} \cdot d\vec{s} \quad ; \quad \Phi_B = \int \vec{B} \cdot d\vec{A} \quad \text{magnetic flux}$$

Mutual inductance:  $M = \frac{\Phi_2}{I_1} = \frac{\Phi_1}{I_2}$  ;  $\epsilon_2 = -M \frac{dI_1}{dt}$  ;  $\epsilon_1 = -M \frac{dI_2}{dt}$

Self - inductance:  $L = \frac{\Phi_B}{I}$  ;  $\epsilon_L = -L \frac{dI}{dt}$  ;  $L = \mu_0 n^2 A \ell$  for solenoid

Magnetic energy:  $U_B = \frac{1}{2} L I^2$  ;  $u_B = \frac{B^2}{2\mu_0}$

RL circuit:  $I = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L})$  (rise) ;  $I = I_0 e^{-t/\tau_L}$  (decay) ;  $\tau_L = L/R$

LC oscillations:  $q(t) = q_p \cos(\omega_0 t)$  ;  $I(t) = -\omega_0 q_p \sin(\omega_0 t)$  ;  $\omega_0 = \frac{1}{\sqrt{LC}}$

Alternating emf, RLC:  $V = V_p \sin \omega t$  ;  $I = I_p \sin(\omega t - \phi)$  ;  $I_p = V_p / Z$  ;  $\tan \phi = \frac{X_L - X_C}{R}$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$  ; Voltage amplitudes:  $V_R = IR$ ,  $V_L = IX_L$ ,  $V_C = IX_C$ ;  $X_L = \omega L$ ,  $X_C = \frac{1}{\omega C}$

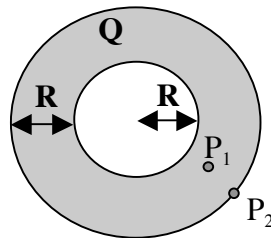
Ampere - Maxwell law:  $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$ ; displacement current  $I_d = \epsilon_0 \frac{d\phi_E}{dt}$ ;  $\phi_E = \int \vec{E} \cdot d\vec{A}$

Electromag. waves:  $\vec{E} = E_p \sin(kx - \omega t) \hat{j}$  ;  $\vec{B} = B_p \sin(kx - \omega t) \hat{k}$  ;  $\frac{E_p}{B_p} = \frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s}$

$k = 2\pi / \lambda$  ,  $\omega = 2\pi f$

**There are 22 problems. You get 1 point for correct answer, 0 points for incorrect answers, 0.2 points for no answer (up to 10 non-answers). This is Test Form A**

**Problems 1, 2**

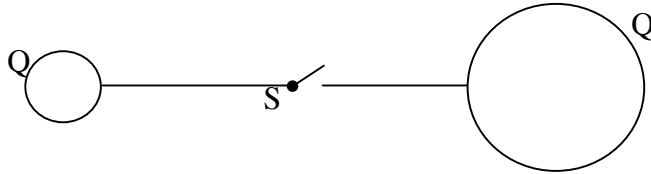


The figure shows a non-conducting spherical shell of inner radius  $R$  and outer radius  $2R$  (i.e. radial thickness  $R$ ) with charge  $Q$  uniformly distributed throughout its volume.

**Prob 1:** The electric field at point  $P_1$  at distance  $1.5R$  from the center is  $1N/C$ . The electric field at point  $P_2$  on the outer surface of the shell, i.e. at distance  $2R$  from the center, is  
 (a)  $1N/C$  ; (b)  $1.33N/C$  ; (c)  $1.66N/C$  ; (d)  $2N/C$  ; (e)  $2.33N/C$

**Prob 2:** Assume this shell now becomes conducting, having the same total charge  $Q$ . The electric fields at  $P_1$  and  $P_2$  now (call them  $E_1$  and  $E_2$ ):  
 (a) Both are same as in prob 1; (b)  $E_1$  is same as in prob 1,  $E_2$  is not ; (c)  $E_1$  is 0,  $E_2$  is same as in prob 1; (d)  $E_1$  is 0,  $E_2$  is larger than in prob 1; (e) Both  $E_1$  and  $E_2$  are 0

**Problems 3, 4**



The two metallic spheres shown have radius  $R$  and  $3R$  respectively, with  $R=0.5\text{m}$ , and initially have the same charge  $Q=2\text{C}$  each. They are at a distance  $d \gg R$ . They are connected by a metallic wire of resistance  $3\Omega$  which has a switch  $S$  that is open. Then the switch  $S$  is closed.

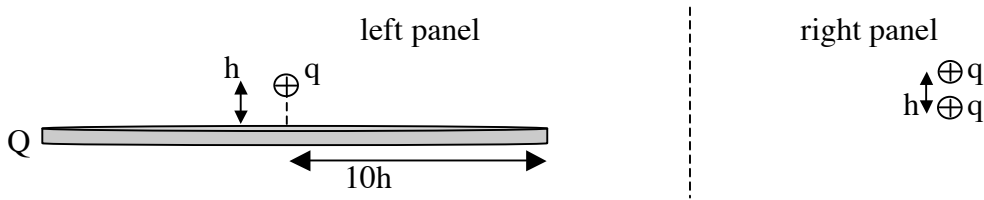
Prob 3: A long time after the switch  $S$  is closed, the charge on the small sphere is

- (a)  $Q$  ; (b)  $Q/2$  ; (c)  $Q/3$  ; (d)  $2Q/3$  ; (e)  $(3/2)Q$

Prob 4: Immediately after the switch  $S$  is closed, the current in the wire (in A) is the product of  $k$  (Coulomb constant) times

- (a)  $0.89$  ; (b)  $0.67$  ; (c)  $0.33$  ; (d)  $0.46$  ; (e)  $0.75$

**Problems 5, 6**



Note: there is no interaction between the charges on the left panel and those on the right panel.

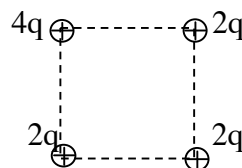
Prob 5: On the left panel, a charge  $q$  is at a very small distance  $h$  from a thin circular charged plate of radius  $10h$  that has total charge  $Q$  uniformly distributed. The force on this charge  $q$  is identical to the force on the upper charge  $q$  of the right panel, where another charge  $q$  is at distance  $h$  below it. The ratio  $Q/q$  is approximately

- (a)  $100$  ; (b)  $10$  ; (c)  $200$  ; (d)  $50$  ; (e)  $1$

Prob 6: suppose the charge  $q$  on the left panel, and the upper charge  $q$  on the right panel, are both moved down a distance  $h/2$ , keeping the circular plate on the left panel and the lower charge on the right panel in the same position. Then the ratio of the forces on the charges we moved, left / right, is

- (a)  $1$  ; (b)  $2$  ; (c)  $4$  ; (d)  $0.5$  ; (e)  $0.25$

**Problems 7, 8**



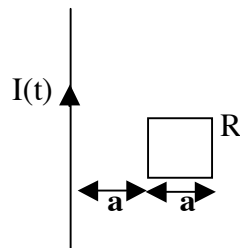
The four charges shown are on the vertices of a square of side length  $a$ .

**Prob 7:** The magnitude of the electric field acting on charge  $2q$  in the lower right corner is  $2\text{N/C}$ . The magnitude of the electric field acting on charge  $4q$  is  
 (a)  $0.6\text{N/C}$  ; (b)  $1.1\text{N/C}$  ; (c)  $1.6\text{N/C}$  ; (d)  $2.1\text{N/C}$  ; (e)  $2.6\text{N/C}$

**Prob 8:** the magnitude of the electric field at distance  $d=100a$  from the center of this square is approximately

(a)  $10^{-3} \frac{q}{4\pi\epsilon_0 a^2}$  ; (b)  $10^{-2} \frac{q}{4\pi\epsilon_0 a^2}$  ; (c)  $\frac{q}{4\pi\epsilon_0 a^2}$  ; (d)  $2 \times 10^{-3} \frac{q}{4\pi\epsilon_0 a^2}$  ; (e)  $2 \times 10^{-2} \frac{q}{4\pi\epsilon_0 a^2}$

**Problems 9, 10, 11**



The figure shows a straight vertical long wire carrying a time-dependent current  $I(t)=I_0 t^2/\tau^2$ , with  $I_0=100\text{A}$  and  $\tau=1\text{s}$ . The square loop shown has resistance  $R=1\Omega$  and sides of length  $a=25\text{cm}$ . Two sides of the square loop are parallel to the long wire, with the closest one at distance  $a$  from the long wire.

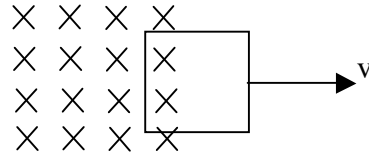
**Prob 9:** At time  $t=1\text{s}$ , at the center of the square, the magnetic field due to the long wire has magnitude ? and points ?  
 (a)  $29\mu\text{T}$ , into paper ; (b)  $29\mu\text{T}$ , out of paper ; (c)  $53\mu\text{T}$ , out of paper ;  
 (d)  $53\mu\text{T}$ , into paper ; (e)  $68\mu\text{T}$ , into paper

**Prob 10:** the current induced in the square loop at time  $t=1\text{s}$  has magnitude  
 (a)  $6.3\mu\text{A}$  ; (b)  $6.5\mu\text{A}$  ; (c)  $6.7\mu\text{A}$  ; (d)  $6.9\mu\text{A}$  ; (e)  $7.1\mu\text{A}$

**Hint:** you need to do an integral

**Prob 11:** the current induced in the square loop at time  $t=2\text{s}$  flows  
 (a) counterclockwise ; (b) clockwise ; (c) alternating, with period  $\tau$  ;  
 (d) alternating, with frequency  $\pi/\tau$  ; (e) it's unpredictable

**Problems 12, 13, 14**

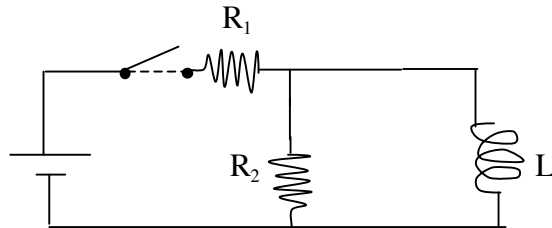


Prob 12: The square loop of wire shown has resistance  $30\Omega$  and side length  $0.2\text{m}$ . It is being pulled out of a region of constant uniform magnetic field  $B=2.5\text{T}$  pointing into the paper, at a speed  $v=40\text{m/s}$ . The induced current in the loop is  
 (a)  $0.66\text{A}$  ; (b)  $1.33\text{A}$  ; (c)  $2.0\text{A}$  ; (d)  $0.5\text{A}$  ; (e)  $0.33\text{A}$

Prob 13: The force that needs to be applied to pull the loop out at constant speed is  
 (a)  $0.66\text{N}$  ; (b)  $1.33\text{N}$  ; (c)  $2.0\text{N}$  ; (d)  $0.5\text{N}$  ; (e)  $0.33\text{N}$

Prob 14: The total energy supplied by the external force in pulling the loop out of the magnetic field region is  
 (a)  $0.13\text{J}$  ; (b)  $0.067\text{J}$  ; (c)  $0.67\text{J}$  ; (d)  $0.33\text{J}$  ; (e)  $0.50\text{J}$

**Problems 15, 16**



In the circuit shown,  $R_1=10\Omega$ ,  $R_2=20\Omega$  and  $L=5\text{H}$ . The switch has been open for a long time, then it is closed. Immediately after closing the switch the current through  $R_2$  is  $2\text{A}$ .

Prob 15: a long time after the switch is closed the current through  $R_2$  is  
 (a)  $2\text{A}$  ; (b)  $1\text{A}$  ; (c)  $0.66\text{A}$  ; (d)  $6\text{A}$  ; (e)  $0\text{A}$

Prob 16: subsequently, the switch is opened again.  $1\text{s}$  after the switch is opened the current through  $R_2$  is  
 (a)  $1.1\text{A}$  ; (b)  $0.11\text{A}$  ; (c)  $0.011\text{A}$  ; (d)  $0.041\text{A}$  ; (e)  $0\text{A}$

**Problems 17, 18**

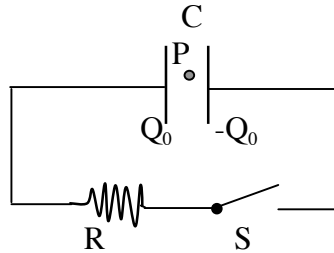
In a series driven RLC circuit, the impedance at resonance is  $60\Omega$ , the resonance frequency is  $\omega=2000\text{s}^{-1}$  and the inductance is  $10\text{mH}$ .

Prob 17: what is the impedance at frequency  $\omega=4000\text{s}^{-1}$ ?  
 (a)  $74\Omega$  ; (b)  $67\Omega$  ; (c)  $81\Omega$  ; (d)  $88\Omega$  ; (e)  $53\Omega$

**Prob 18:** at  $\omega=4000\text{s}^{-1}$ , the phase difference between current and voltage is:

- (a) current leads voltage by  $43^\circ$ ; (b) current lags voltage by  $43^\circ$  ; (c) current leads voltage by  $27^\circ$ ; (d) current lags voltage by  $27^\circ$  ; (e) current neither lags nor leads voltage

**Problems 19, 20, 21**



In the circuit shown, the parallel plate capacitor has capacitance  $3\text{mF}$  and has round plates of radius  $a=0.4\text{m}$ . The resistor has resistance  $250\Omega$ . Initially the charge on the capacitor is  $Q_0=5\text{C}$ , with the left capacitor plate having positive charge, and the switch  $S$  is open. At time  $t=0$  the switch  $S$  is closed.

**Prob 19:** the current through the resistor at time  $t=1\text{s}$  is

- (a)  $1.32\text{A}$  ; (b)  $3.15\text{A}$  ; (c)  $6.66\text{A}$  ; (d)  $2.36\text{A}$  ; (e)  $1.76\text{A}$

**Prob 20:** the magnetic field at point  $P$  inside the capacitor, located at distance  $0.1\text{m}$  above its symmetry axis, at the moment when the current through the resistor is  $I=1\text{A}$ , is

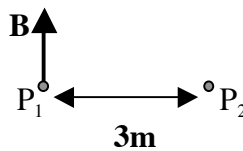
$$\frac{\mu_0 I}{2\pi a} \times C, \text{ with } C=$$

- (a)  $0.25$  ; (b)  $1.25$  ; (c)  $1$  ; (d)  $0.0625$  ; (e)  $2$

**Prob 21:** at that moment, the magnetic field at point  $P$  points

- (a) up ; (b) down ; (c) out of the paper ; (d) into the paper ; (e) right

**Problem 22**



The figure shows two points,  $P_1$  and  $P_2$ , separated by a distance  $3\text{m}$  in the horizontal direction. There is a plane electromagnetic wave propagating in direction normal to the paper. The magnetic field  $B$  from this electromagnetic wave at point  $P_1$  is shown at time  $t_0$ , pointing up. The frequency of this electromagnetic wave is  $f=0.5 \times 10^8$  cycles/s (Hz) ( $f=\omega/2\pi$ ). At point  $P_1$  there is a positive charge, and at point  $P_2$  there is a negative charge, both charges are at rest. The forces on these charges due to the fields of the electromagnetic wave at time  $t_0$ :

- (a) point in the same direction, horizontally; (b) point in opposite directions, horizontally; (c) point in the same direction perpendicular to the paper; (d) point in opposite directions perpendicular to the paper; (e) information given is insufficient to answer, since one needs to know whether wave is propagating into or out of the paper