chapter 33

Problem

42. A series RLC circuit has $R = 75\ \Omega$, $L = 20\ \text{mH}$, and a resonant frequency of 4.0 kHz. (a) What is the capacitance?
(b) What is the impedance of the circuit at resonance? (c) What is the impedance at 3.0 kHz?

Solution

(a) From the expression for resonance in an RLC circuit,

$C = 1/\omega_0^2 L = (2\pi \times 4\ \text{kHz})^{-2}(20\ \text{mH})^{-1} = 79.2\ \text{nF}$. (b) At resonance, $X_L - X_C = 0$, so $Z = R = 75\ \Omega$.

(c) At 3 kHz, $X_L - X_C = \omega L - (1/\omega C) = (2\pi \times 3\ \text{kHz} \times 20\ \text{mH}) - (2\pi \times 3\ \text{kHz} \times 79.2\ \text{nF})^{-1} = -293\ \Omega$,

and $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(75\ \Omega)^2 + (-293\ \Omega)^2} = 303\ \Omega$.

Problem

47. A 2.0-H inductor and a 3.5-μF capacitor are connected in series with a 50-Ω resistor, and the combination is connected to an AC generator supplying 24 V peak at 60 Hz. (a) At the instant the generator voltage is at its peak, what is the instantaneous voltage across each circuit element? Show explicitly that these sum to the generator voltage. (b) If rms voltmeters are connected across each of the three components, what will they read? Do their readings sum to the rms generator voltage? Does this contradict the loop law?

Solution

Before calculating numerical results, we display the general time-dependent voltages in an RLC series circuit, which can be derived from the phasor diagram in Fig. 33-17b (the angle between V_p and the horizontal axis is ωt). Note that the phase constants in Equation 33-3 (for the applied voltage and current, i.e., the resistor voltage) are related to the phase difference in Equation 33-16 by $\phi_V = 0$ and $\phi_I = -\phi$. (See paragraphs following Equation 33-16.)

$$V = V_p \sin\omega t \qquad V_{R,p} = I_p R \qquad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$V_R = V_{R,p} \sin(\omega t - \phi) \qquad V_{C,p} = I_p X_C$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$V_C = V_{C,p} \sin\left(\omega t - \phi - \tfrac{1}{2}\pi\right) \qquad V_{L,p} = I_p X_L$$

$$-\tfrac{1}{2}\pi \le \phi \le \tfrac{1}{2}\pi$$

$$V_L = V_{L,p} \sin\left(\omega t - \phi + \tfrac{1}{2}\pi\right) \qquad I_p = V_p/Z$$

(a) At the instant when $V = V_p = 24$ V, $\omega t = 90°$. From the given values of R, L, C, ω, and V_p, we can calculate ϕ and
the instantaneous voltages:

$V_R = (23.93\ \text{V}) \sin(90° + 4.458°) = 23.86$ V; $V_C = (362.7\ \text{V})\sin 4.458° = 28.19$ V; $V_L = (360.8\ \text{V})\sin 184.458° = -28.05$ V. Four significant figures are necessary to verify the loop law,

$V_R + V_C + V_L = 24.00$ V, in this case. (b) The rms voltages are $1/\sqrt{2}$ times the peak values found in part (a) (approximately 16.9, 256, and 255 volts for R, C, and L respectively). Of course, since the instantaneous voltages have different phases, the sum of the rms voltages does not equal $V_{\text{rms}} = V_p/\sqrt{2} = 17.0$ V.

chapter 34

Problem

2. A parallel-plate capacitor has square plates 10 cm on a side and 0.50 cm apart. If the voltage across the plates is increasing at the rate of 220 V/ms, what is the displacement current in the capacitor?

Solution

The electric field is approximately uniform in the capacitor, so
$\phi_E = EA = (V/d)A$, and $I_D = \varepsilon_0 \partial \phi_E / \partial t = (\varepsilon_0 A/d) \, dV/dt =$
$(8.85 \times 10^{-12} \text{ F/m})(10 \text{ cm})^2 (220 \text{ V/ms})/(0.5 \text{ cm}) = 3.89 \; \mu\text{A}$.

Problem

3. A parallel-plate capacitor of plate area A and spacing d is charging at the rate dV/dt. Show that the displacement current in the capacitor is equal to the conduction current flowing in the wires feeding the capacitor.

Solution

The displacement current is $I_D = \varepsilon_0 \partial \phi_E / \partial t$. For a parallel-plate capacitor,
$E = q/\varepsilon_0 A$, so $I_D = \varepsilon_0 \, \partial(EA)/\partial t = \varepsilon_0 \partial(q/\varepsilon_0)/\partial t = dq/dt$. But dq/dt is just the conduction current (the rate at which charge is flowing onto the capacitor plates); hence $I_D = I$.

Problem

5. A parallel-plate capacitor has circular plates with radius 50 cm and spacing 1.0 mm. A uniform electric field between the plates is changing at the rate 1.0 MV/m · s. What is the magnetic field between the plates (a) on the symmetry axis, (b) 15 cm from the axis, and (c) 150 cm from the axis?

Solution

(a) As explained in Example 34-1, cylindrical symmetry and Gauss's law for magnetism require that the **B**-field lines be circles around the symmetry axis, as in Fig. 34-5. For a radius, r, less than the radius of the plates, R, the displacement current is $I_D = \varepsilon_0 d\phi_E / dt = \varepsilon_0 (d/dt) \int \mathbf{E} \cdot dA = \varepsilon_0 \pi r^2 (dE/dt)$, where the integral is over a disk of radius r centered between the plates. Maxwell's form of Ampère's law gives
$\oint \mathbf{B} \cdot d\ell = 2\pi r B = \mu_0 I_D$, where the line integral is around the circumference of the disk. Thus,
$B = \frac{1}{2}\mu_0 \varepsilon_0 r (dE/dt) = r(dE/dt)/2c^2$, where c is the speed of light (Equation 34-16). On the symmetry axis,
$r = 0$, so $B = 0$. (b) For $r = 15 \text{ cm} < R, B = \frac{1}{2}(0.15 \text{ m})(10^6 \text{ V/m} \cdot \text{s})/(3 \times 10^8 \text{ m/s})^2 = 8.33 \times 10^{-13}$ T.
(c) For $r > R$, the displacement current is
$I_D = \varepsilon_0 \pi R^2 (dE/dt)$, so $B = (dE/dt)R^2/2c^2 r$. At $r = 150$ cm, $B = (10^6 \text{ V/m} \cdot \text{s}) \times$
$(50 \text{ cm})^2 / 2(3 \times 10^8 \text{ m/s})^2 (150 \text{ cm}) = 9.26 \times 10^{-13}$ T.
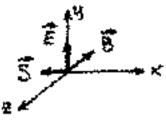
Problem

7. At a particular point the instantaneous electric field of an electromagnetic wave points in the $+y$ direction, while the magnetic field points in the $-z$ direction. In what direction is the wave propagating?

Solution

For electromagnetic waves in vacuum, the directions of the electric and magnetic fields, and of wave propagation, form a right-handed coordinate system, as shown. (The vector relationship is summarized in Equation 34-20b.) Therefore, the
given wave is headed in the $-x$-direction.

Problem 7 Solution.

Problem

8. The fields of an electromagnetic wave are $\mathbf{E} = E_p \sin(kz + \omega t)\hat{\mathbf{j}}$ and $\mathbf{B} = B_p \sin(kz + \omega t)\hat{\mathbf{i}}$. Give a unit vector in the direction of propagation.

Solution

In an electromagnetic wave, when $\mathbf{E}$ is parallel to $\hat{\mathbf{j}}$ and $\mathbf{B}$ to $\hat{\mathbf{i}}$ $\mathbf{S}$ is parallel to $\mathbf{E} \times \mathbf{B}$ or $\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}$. (See previous solution.)

Problem

9. The electric field of a radio wave is given by $\mathbf{E} = E \sin(kz - \omega t)(\hat{\mathbf{i}} + \hat{\mathbf{j}})$. (a) What is the peak amplitude of the electric field? (b) Give a unit vector in the direction of the magnetic field at a place and time where $\sin(kz - \omega t)$ is positive.

Solution

(a) The peak amplitude is the magnitude of $E(\hat{\mathbf{i}} + \hat{\mathbf{j}})$, which is $E\sqrt{2}$. Note that $\hat{\mathbf{i}} + \hat{\mathbf{j}} = \sqrt{2}\,\hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is a unit vector $45°$ between the positive x and y axes. (b) When $\mathbf{E}$ is parallel to $\hat{\mathbf{n}}$ (for $\sin(kz - \omega t)$ positive) $\mathbf{B}$ points $45°$ into the second quadrant (so that $\mathbf{E} \perp \mathbf{B}$, and $\mathbf{E} \times \mathbf{B}$ is in the $+z$ direction). Thus, $\mathbf{B}$ is parallel to the unit vector $(-\hat{\mathbf{i}} + \hat{\mathbf{j}})/\sqrt{2}$.