Physics 222 UCSD/225b UCSB

Lecture 9

Weak Neutral Currents

Chapter 13 in H&M.

Weak Neutral Currents

- "*Observation of neutrino-like interactions without muon or electron in the gargamelle neutrino experiment*" Phys.Lett.B46:138-140,1973.
- This established weak neutral currents.

$$M = \frac{4G}{\sqrt{2}} 2\rho J_\mu^{NC} J^{NC\mu}$$

$$J_\mu^{NC}(q) = \bar{u}\gamma_\mu \frac{1}{2}\left(c_V - c_A\gamma^5\right)u$$

ρ allows for different coupling from charged current.
$c_v = c_A = 1$ for neutrinos, but not for quarks.

Experimentally: NC has small right handed component.

EWK Currents thus far

- Charged current is strictly left handed.
- EM current has left and right handed component.
- NC has left and right handed component.

=> Try to symmetrize the currents such that we get one $SU(2)_L$ triplet that is strictly left-handed, and a singlet.

Reminder on Pauli Matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \tau_1$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \tau_2$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \tau_3$$

$$\tau_+ = \frac{1}{2}(\tau_1 + i\tau_2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\tau_- = \frac{1}{2}(\tau_1 - i\tau_2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

We will do the same constructions we did last quarter for isospin, using the same formalism. Though, this time, the symmetry operations are identified with a *"multiplet of weak currents"* . The states are leptons and quarks.

Starting with Charged Current

- Follow what we know from isospin, to form doublets:

$$\chi_L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L ; \tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} ; \tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$J_\mu^\pm(x) = \bar{\chi}_L \gamma_\mu \tau_\pm \chi_L$$

$$J_\mu^3(x) = \bar{\chi}_L \gamma_\mu \frac{1}{2} \tau_3 \chi_L = \frac{1}{2} \bar{\nu}_L \gamma_\mu \nu_L - \frac{1}{2} \bar{e}_L \gamma_\mu e_L$$

We thus have a triplet of left handed currents W^+, W^-, W^3 .

Hypercharge, T^3, and Q

- We next take the EM current, and decompose it such as to satisfy:

$$Q = T^3 + Y/2$$

$$j_\mu^{em} = J_\mu^3 + \frac{1}{2} j_\mu^Y$$

- The symmetry group is thus: $SU(2)_L \times U(1)_Y$
- And the generator of Y must commute with the generators T^i, i=1,2,3 of $SU(2)_L$.
- All members of a weak isospin multiplet thus have the same eigenvalues for Y.

Resulting Quantum Numbers

Lepton	T	T^3	Q	Y
ν	1/2	1/2	0	-1
e^-_L	1/2	-1/2	-1	-1
e^-_R	0	0	-1	-2

Quark	T	T^3	Q	Y
u_L	1/2	1/2	2/3	1/3
d_L	1/2	-1/2	-1/3	1/3
u_R	0	0	2/3	4/3
d_R	0	0	-1/3	-2/3

Note the difference in Y quantum numbers for left and right handed fermions of the same flavor.

You get to verify the quark quantum numbers in HW3.

Now back to the currents

- Based on the group theory generators, we have a triplet of W currents for $SU(2)_L$ and a singlet "B" neutral current for $U(1)_Y$.

Basic EWK interaction:

$$-ig\left(J^i\right)^\mu W_\mu^i - i\frac{g'}{2}\left(J^Y\right)^\mu B_\mu$$

- The two neutral currents B and W^3 can, and do mix to give us the mass eigenstates of photon and Z boson.

W^3 and B mixing

- The physical photon and Z are obtained as:

$$A_\mu = W_\mu^3 \sin\theta_W + B_\mu \cos\theta_W$$

$$Z_\mu = W_\mu^3 \cos\theta_W - B_\mu \sin\theta_W$$

- And the neutral interaction as a whole becomes:

$$-ig\left(J^3\right)^\mu W_\mu^3 - i\frac{g'}{2}\left(J^Y\right)^\mu B_\mu =$$

$$= -i\left(g\sin\theta_W J_\mu^3 + g'\cos\theta_W \frac{J_\mu^Y}{2}\right)A^\mu$$

$$-i\left(g\cos\theta_W J_\mu^3 - g'\sin\theta_W \frac{J_\mu^Y}{2}\right)Z^\mu$$

Constraints from EM

$$ej^{em} = e\left(J_\mu^3 + \frac{J_\mu^Y}{2}\right) = -i\left(g\sin\theta_W J_\mu^3 + g'\cos\theta_W \frac{J_\mu^Y}{2}\right)$$

$$\boxed{\Rightarrow g\sin\theta_W = g'\cos\theta_W = e}$$

$$\Rightarrow g' = \frac{\sin\theta_W}{\cos\theta_W} g$$

We now eliminate g' and write the weak NC interaction as:

$$-i\frac{g}{\cos\theta_W}\left(J_\mu^3 - \sin^2\theta_W j_\mu^{em}\right)Z^\mu \equiv -i\frac{g}{\cos\theta_W} J_\mu^{NC} Z^\mu$$

Summary on Neutral Currents

$$j_\mu^{em} = J_\mu^3 + \frac{1}{2} j_\mu^Y$$

$$J_\mu^{NC} = J_\mu^3 - \sin^2\theta_W \, j_\mu^{em}$$

This thus re-expresses the "physical" currents for photon and Z in form of the "fundamental" symmetries.

Vertex Factors:

$$-ieQ_f\gamma^\mu$$

Charge of fermion

$$-i\frac{g}{\cos\theta_W}\gamma^\mu\frac{1}{2}(c_V^f - c_A^f\gamma^5)$$

c_V and c_A differ according to Quantum numbers of fermions.

Q, c_V,c_A

fermion	Q	c_A	c_V
neutrino	0	1/2	1/2
e,mu	-1	-1/2	$-1/2 + 2\sin^2\theta_W \sim -0.03$
u,c,t	+2/3	1/2	$1/2 - 4/3\sin^2\theta_W \sim 0.19$
d,s,b	-1/3	-1/2	$-1/2 + 2/3\sin^2\theta_W \sim -0.34$

Accordingly, the coupling of the Z is sensitive to $\sin^2\theta_W$.
You will verify this as part of HW3.

Origin of these values

The neutral current weak interaction is given by:

$$-i\frac{g}{\cos\theta_W}\overline{\psi_f}\gamma_\mu\left(\frac{1}{2}(1-\gamma^5)T^3-\sin^2\theta_W Q\right)\psi_f Z^\mu$$

Comparing this with: $$-i\frac{g}{\cos\theta_W}\gamma^\mu\frac{1}{2}(c_V^f-c_A^f\gamma^5)$$

Leeds to:

$$c_V^f=T_f^3-2Q_f\sin^2\theta_W$$

$$c_A^f=T_f^3$$

Effective Currents

- In Chapter 12 of H&M, we discussed effective currents leading to matrix elements of the form:

$$M^{CC} = \frac{4G}{\sqrt{2}} J^{\mu} J_{\mu}{}^{*T} \qquad M^{NC} = \frac{4G}{\sqrt{2}} 2\rho J^{NC\mu} J^{NC}{}_{\mu}{}^{*T}$$

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2} \qquad \rho\frac{G}{\sqrt{2}} = \frac{g^2}{8M_Z^2 \cos^2\theta_W}$$

From this we get the relative strength of NC vs CC:

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W}$$

EWK Feynman Rules

Photon vertex:

$$-ieQ_f\gamma^\mu$$

Z vertex:

$$-i\frac{g}{\cos\theta_W}\gamma^\mu\frac{1}{2}(c_V^f - c_A^f\gamma^5)$$

W vertex:

$$-i\frac{g}{\sqrt{2}}\gamma^\mu\frac{1}{2}(1-\gamma^5)$$

Chapter 14 Outline

- Reminder of Lagrangian formalism
  - Lagrange density in field theory
- Aside on how Feynman rules are derived from Lagrange density.
- Reminder of Noether's theorem
- Local Phase Symmetry of Lagrange Density leads to the interaction terms, and thus a massless boson propagator.
  - Philosophically pleasing …
  - … and require to keep theory renormalizable.
- Higgs mechanism to give mass to boson propagator.

Reminder of Lagrange Formalism

- In classical mechanics the particle equations of motion can be obtained from the Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

- The Lagrangian in classical mechanics is given by:

$$L = T - V = E_{kinetic} - E_{potential}$$

Lagrangian in Field Theory

- We go from the generalized discrete coordinates $q_i(t)$ to continuous fields $\phi(x,t)$, and thus a Lagrange density, and covariant derivatives:

$$L(q,\dot{q},t) \rightarrow L(\phi,\partial_\mu \phi, x_\mu)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0 \rightarrow \frac{\partial}{\partial x_\mu}\left(\frac{\partial L}{\partial(\partial_\mu \phi)}\right) - \frac{\partial L}{\partial \phi} = 0$$

Let's look at examples (1)

- Klein-Gordon Equation:

$$\frac{\partial}{\partial x_\mu}\left(\frac{\partial L}{\partial(\partial_\mu \phi)}\right) - \frac{\partial L}{\partial \phi} = 0$$

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2$$

$$\partial_\mu \partial^\mu \phi + m^2 \phi = 0$$

Note: This works just as well for the Dirac equation. See H&M.

Let's look at examples (2)

$$\partial_\mu \left(\frac{\partial L}{\partial(\partial_\mu A_\nu)} \right) - \frac{\partial L}{\partial A_\nu} = 0$$

$$L = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu$$

=>

Maxwell Equation:

$$\partial_\mu F^{\mu\nu} = j^\mu$$

$$-\frac{\partial L}{\partial A_\nu} = j^\mu$$

$$\frac{\partial L}{\partial(\partial_\mu A_\nu)} = \frac{\partial}{\partial(\partial_\mu A_\nu)} \left(-\frac{1}{4} \left(\partial_\alpha A_\beta - \partial_\beta A_\alpha \right) \left(\partial^\alpha A^\beta - \partial^\beta A^\alpha \right) \right) =$$

$$= -\frac{1}{2} g^{\alpha\alpha} g^{\beta\beta} \frac{\partial}{\partial(\partial_\mu A_\nu)} \left(\left(\partial_\alpha A_\beta \right)^2 - \left(\partial_\alpha A_\beta \partial_\beta A_\alpha \right) \right) = -\frac{1}{2} 2 \left(\partial^\mu A^\nu - \partial^\nu A^\mu \right) =$$

$$= -F^{\mu\nu}$$

Aside on current conservation

- From this result we can conclude that the EM current is conserved:

$$\partial_\nu \partial_\mu F^{\mu\nu} = \partial_\nu j^\mu$$

$$\partial_\nu \partial_\mu F^{\mu\nu} = \partial_\nu \partial_\mu \left(\partial^\mu A^\nu - \partial^\nu A^\mu \right) =$$

$$= \partial_\mu \partial^\mu \partial_\nu A^\nu - \partial_\nu \partial^\nu \partial_\mu A^\mu = \partial_\mu \partial^\mu (\partial_\nu A^\nu - \partial_\mu A^\mu) = 0$$

- Where I used:

$$\partial_\mu \partial^\mu = \partial_\nu \partial^\nu$$

$$\partial_\nu A^\nu = \partial_\mu A^\mu$$

Aside on mass term

- If we added a mass term to allow for a massive photon field, we'd get:

$$(\partial_\mu \partial^\mu + m^2) A^\nu = j^\nu$$

This is easily shown from what we have done.
Leave it to you as an exercise.