Physics 222 UCSD/225b UCSB

Lecture 5

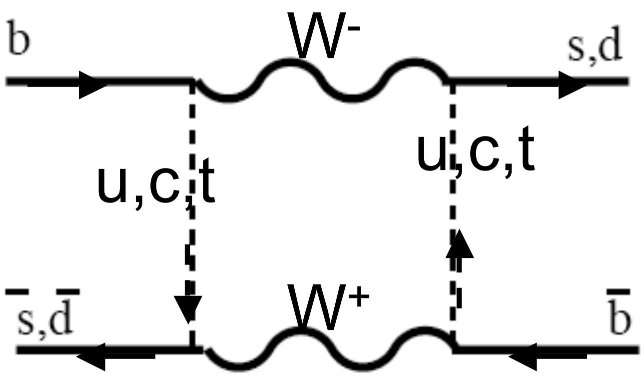
Mixing & CP Violation (1 of 3)

Today we focus on Matter <-> Antimatter Mixing in weakly decaying neutral Meson systems.

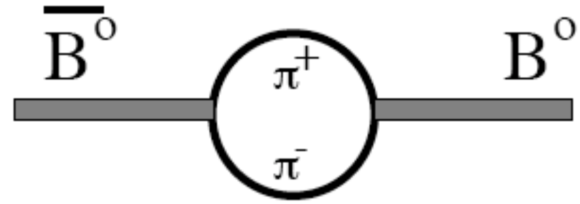
=> K^0, D^0, B^0, B_s^0

Strongly decaying neutral mesons are uninteresting because their decay width is many orders of magnitude larger than the second order weak interaction phenomenon of mixing.

Mixing - Basic Idea



off shell
intermediate state

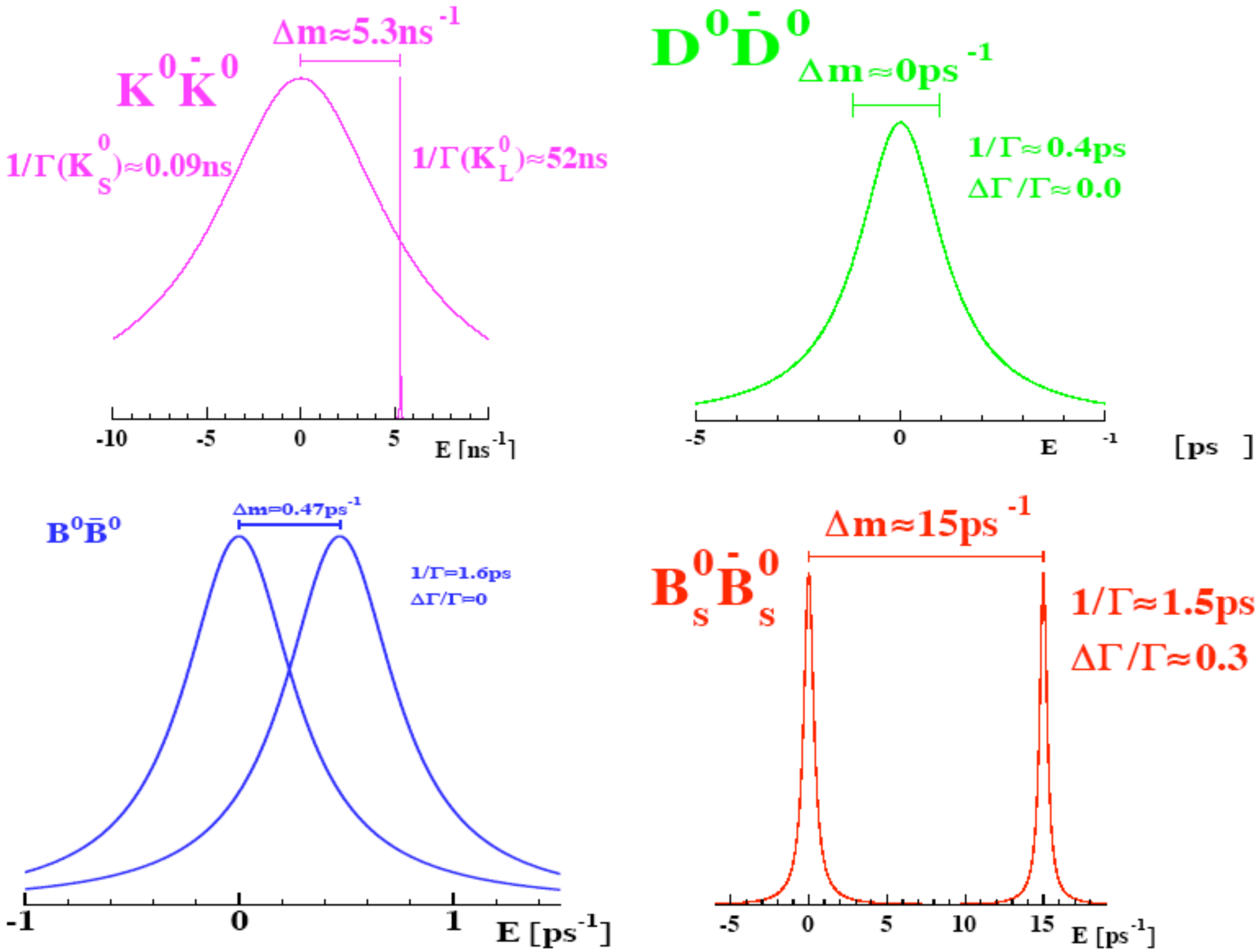


on shell
intermediate state

"Matter" can oscillate into "antimatter" in the sense that the flavor of the initial and final state are each other's antiflavors.

This is possible as a second order weak interaction effect, either by going through a virtual intermediate state (left) or as a "rescattering" via a flavor neutral real intermediate state.

Overview of Δm, ΔΓ, Γ



We will use B^0 system to discuss the formalism.

Two-state Formalism

Basis vectors = flavor eigenstates.

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = (M - \tfrac{i}{2}\Gamma)\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^{\star} & M_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^{\star} & \Gamma_{22} \end{pmatrix}$$

Generic B^0 may be expressed as:

$$|B(t)\rangle = a|B^0\rangle + b|\bar{B}^0\rangle$$

This is a coherent quantum state.
We know its flavor fractions a,b, at time t
only by observing its decay.

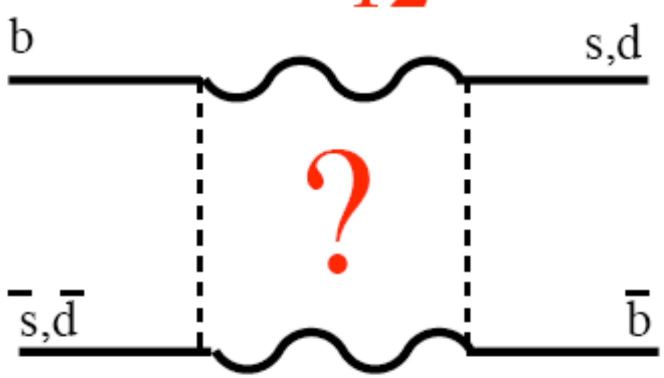
Note: This 2-state formalism is known as the Wigner-Weisskopf approximation.
You can find it justified, e.g. in O.Nachtmann's book Appendix I.

The underlying Physics

$\rightarrow$ CPT theorem guarantees $M_{11} = M_{22}$, $\Gamma_{11} = \Gamma_{22}$.

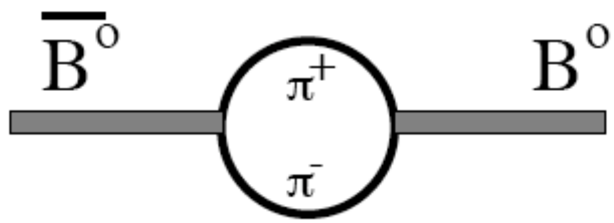
I.e. particle and anti-particle have same mass and width !

M_{12}



off shell
intermediate state

Γ_{12}



on shell
intermediate state

New physics may contribute to off-shell but not on-shell.

Aside:

Analogy to driven Oscillator: there's a phaseshift as you pass through the resonance frequency.

Resonance frequency $\equiv$ mass of the intermediate state

$\rightarrow$ off-shell = "below the resonance"

$\rightarrow$ on-shell = "above the resonance"

This phase shift is the reason why we know that on-shell intermediate states contribute only to Γ_{12} , off-shell only to M_{12} . High mass new particles thus only contribute to M_{12} !!!

For a more rigorous explanation, see Apendix I of Particle Physics book by O.Nachtmann.

Aside:

- How does this compare with the lepton sector?
- We already talked about neutrino oscillations.
  - The flavor of a neutrino changes over time because:
    - It is created in definite flavor state via weak interaction.
    - It's time evolution is governed by mass eigenstate.
    - Mass and flavor eigenstate are not the same.
- In contrast, here we are studying not the oscillation of the quarks, but of a quark anti-quark bound state.
  - The equivalent in the lepton sector would be an electron anti-muon bound state oscillating into a positron muon bound state.

Clearly, these are very different phenomena!

“Physical Observables” in Mixing

$|M_{12}|$ $|\Gamma_{12}|$ $\mathrm{Arg}(M_{12}/\ \Gamma_{12})$

We have access to these via:

Mass difference: Δm
Width difference: $\Delta\Gamma$
CP violation

The purpose of today’s and the next two lectures is:
⇒ Explain how Δm, $\Delta\Gamma$, and CP violation measure these.
⇒ Show what we presently know from such measurements.

Eigenstates of Hamiltonian

- The eigenstates of the Hamiltonian are the mass eigenstates.
- You obtain them by finding the Eigenvectors of the Schroedinger Equation.
- You find mass and width differences via the eigenvalues.

Solve the Eigenvalue Problem

$$det(H - \mu) = 0$$

$$\begin{aligned} \mu_H &= M - \tfrac{i}{2}\Gamma + Q \\ \mu_L &= M - \tfrac{i}{2}\Gamma - Q \end{aligned}$$

This defines B_H (B_L) as the heavier (lighter) of the two eigenstates.

With a little algebra (see homework) you get:

$$\begin{aligned} Q &= \sqrt{(M_{12}^\star - \tfrac{i}{2}\Gamma_{12}^\star)(M_{12} - \tfrac{i}{2}\Gamma_{12})} \\ &= \sqrt{|M_{12}|^2 + \tfrac{1}{4}|\Gamma_{12}|^2 + i|M_{12}||\Gamma_{12}|cos\phi} \\ \phi &= Arg(\tfrac{\Gamma_{12}}{M_{12}}) \end{aligned}$$

Relating Δm & $\Delta\Gamma$ to Γ_{12} & M_{12}

$$\begin{aligned} (\Delta m)^2 - \frac{1}{4}(\Delta\Gamma)^2 &= 4|M_{12}|^2 - |\Gamma_{12}|^2 \\ \Delta m \Delta\Gamma &= 4Re(M_{12}\Gamma^*_{12}) \end{aligned}$$

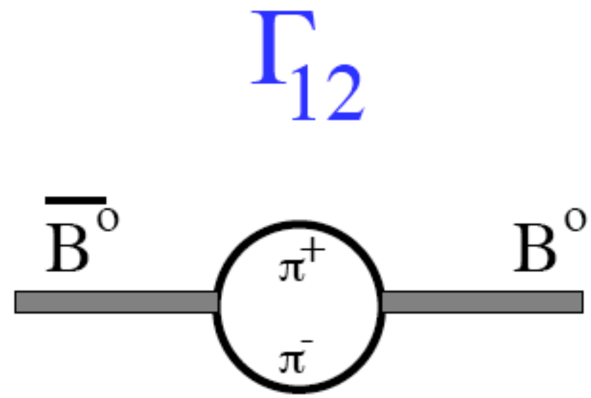
<u>Only physics input so far:</u>
mass and lifetime are same for particle and antiparticle!

⇒$\Delta m>0$ by definition.
⇒ Sign of $\Delta\Gamma$ is chosen by nature.
⇒ If CP is conserved then B_H and B_L are the two CP eigenstates.
⇒ Which one is heavier is chosen by nature. .

Estimating Γ_{12} / M_{12}

We know from experiment:

	Bd	Bs
$\Delta m/\Gamma$	0.776+-0.008	17.77+-0.12
$\Delta\Gamma/\Gamma$	~0	0.11+-0.08



We estimate from Theory:

B_d	B_s
$\Gamma_{12}/\Gamma < \lvert b \to c\bar{c}d + b \to u\bar{u}d \rvert^2$	$\Gamma_{12}/\Gamma < \lvert b \to c\bar{c}s + b \to u\bar{u}s \rvert^2$
$\Gamma_{12}/\Gamma <\sim O(10^{-3})$	$\Gamma_{12}/\Gamma <\sim O(10^{-2})$

CKM only

=> $|\Gamma_{12}| << |M_{12}|$

With $|\Gamma_{12}| << |M_{12}|$ we get:

$$\boxed{\Delta m = 2|M_{12}| \quad , \quad \Delta\Gamma = 2\frac{Re(M_{12}\Gamma_{12})}{|M_{12}|}}$$

This is now specific to the B system(s), because we used argument about CKM in b-decay.

We have thus discovered the means to measure $|M_{12}|$ and $-2|\Gamma_{12}| \times \cos(Arg(\Gamma^*_{12} M_{12}))$

Only $Im(M_{12}\,\Gamma_{12})$ is missing to extract all three parameters!

We'll discuss how that's done next lecture.

Aside on SM Theory

$$
\begin{aligned}
M_{12} &= \frac{G_F^2}{12\pi^2} m_B m_W^2 \eta_B B_B f_B^2 (V_{tb}V_{td}^*)^2 S_0(x_t) \\
S_0(x_t) &= 2.4\left(\frac{m_t}{170GeV}\right)^{1.52} \\
\Gamma_{12} &= -\frac{G_F^2}{24\pi} m_B m_b^2 B_B f_B^2 (V_{tb}V_{td}^*)^2 \\
&\quad \times \text{Stuff of O(1)}
\end{aligned}
$$

You are asked to use this in HW to estimate the contributions from different quarks in box diagram for K^0 , D^0 , B^0 , $B_s{}^0$.

Also note that "Stuff of O(1)" is nonperturbative QCD, and difficult to calculate, while B_B and f_B (while also nonperturbative) has been done on the lattice.

Relationship between Eigenstates

We have: mass eigenstates = B_H and B_L

flavor eigenstates = B^0 and $\overline{B^0}$

CP eigenstates = B_+ and B_-

Let's first set $|\Gamma_{12}/M_{12}| = 0$:

Define q, p via:

$$\begin{aligned} B_H &= p\,|B^0\rangle + q\,|\overline{B^0}\rangle \\ B_L &= p\,|B^0\rangle - q\,|\overline{B^0}\rangle \end{aligned} \Rightarrow \boxed{\frac{q}{p} = +\frac{M_{12}^*}{|M_{12}|}}$$

Define CP eigenstates:

$$\begin{aligned} CP|B_+\rangle &= +|B_+\rangle \\ CP|B_-\rangle &= -|B_-\rangle \end{aligned} \quad \Rightarrow \quad \begin{aligned} B_+ &= \tfrac{1}{\sqrt{2}}\,|B^0\rangle - \tfrac{1}{\sqrt{2}}\,|\overline{B^0}\rangle \\ B_- &= \tfrac{1}{\sqrt{2}}\,|B^0\rangle + \tfrac{1}{\sqrt{2}}\,|\overline{B^0}\rangle \end{aligned}$$

Where we have used that B^0 is a pseudoscalar meson.

Allow for CP Violation

$$\begin{aligned} B_H &= \cos\tfrac{\phi}{2}\,|B_-\rangle - i\sin\tfrac{\phi}{2}\,|B_+\rangle \\ B_L &= -i\sin\tfrac{\phi}{2}\,|B_-\rangle + \cos\tfrac{\phi}{2}\,|B_+\rangle \end{aligned}$$

- For $\sin\phi \to 0$ we get $|\langle B_+|B_L\rangle|^2 \to 1$
- For $\sin\phi \to 1$ we get $|\langle B_-|B_L\rangle|^2 \to 1$

In the homework, you will calculate explicitly how the CKM Matrix, after a bit of algebra, fully determines the CP violation.

In class, we walk through the basic derivation and concepts, leaving the details for the homework.

Concept for measuring Δm

- Measure the flavor of the meson at production.
  - Refered to as “flavor tagging”
- Measure the flavor at decay.
  - Via flavor specific final state.
- Measure meson momentum and flight distance.
  - Calculate proper time from this.
- Put it all together to measure probability for finding a meson with opposite flavor at decay from the flavor at production.
  - Do this as a function of proper time.

Mixing

Probability for meson to keep its flavor:

$$
\begin{aligned}
|\langle f|H|B^0\rangle|^2 &= |\langle f|B^0(t)\rangle|^2 \\
&= \tfrac{1}{4|p|^2}|\langle f|B_L(t)\rangle + \langle f|B_H(t)\rangle|^2 \\
&= \tfrac{1}{4|p|^2}|pAe^{(-im_L-\Gamma_L/2)t} + pAe^{(-im_H-\Gamma_H/2)t}|^2 \\
&= \tfrac{1}{4}|A|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m t)
\end{aligned}
$$

Probability for meson to switch flavor:

$$
\begin{aligned}
|\langle \bar{f}|H|B^0\rangle|^2 &= |\langle \bar{f}|B^0(t)\rangle|^2 \\
&= \tfrac{1}{4|p|^2}|\langle \bar{f}|B_L(t)\rangle + \langle \bar{f}|B_H(t)\rangle|^2 \\
&= \tfrac{1}{4|p|^2}|q\bar{A}e^{(-im_L-\Gamma_L/2)t} - q\bar{A}e^{(-im_H-\Gamma_H/2)t}|^2 \\
&= \tfrac{1}{4}|\tfrac{q}{p}|^2|\bar{A}|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m t)
\end{aligned}
$$

Anatomie of these Equations (1)

Unmixed:

$$|\langle f|H|B^0\rangle|^2 = \frac{1}{4}\boxed{|A|}^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m t)$$

Mixed:

$$|\langle \bar{f}|H|B^0\rangle|^2 = \frac{1}{4}\boxed{\left|\frac{q}{p}\right|}^2\boxed{|\bar{A}|}^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m t)$$

|q/p| =1 unless there is CP violation in mixing itself.

$|A| = |\bar{A}|$ unless there is CP violation in the decay.

We will discuss both of these in more detail in next lecture!

Anatomie of these Equations (2)

Unmixed:

$$|\langle f|H|B^0\rangle|^2 = \frac{1}{4}|A|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} \boxed{+} 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta mt)$$

Mixed:

$$|\langle \bar{f}|H|B^0\rangle|^2 = \frac{1}{4}|\frac{q}{p}|^2|\bar{A}|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} \boxed{-} 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta mt)$$

cosΔmt enters with different sign for mixed and unmixed!

$$\frac{\text{Unmixed - Mixed}}{\text{Unmixed + Mixed}} = \frac{2e^{-(\Gamma_H+\Gamma_L)t/2}}{e^{-\Gamma_L t}+e^{-\Gamma_H t}}\cos\Delta mt$$

Assuming no CP violation in mixing or decay.

Will explain when this is a reasonable assumption in next lecture.

Anatomie of these Equations (3)

Unmixed:

$$|\langle f|H|B^0\rangle|^2 = \frac{1}{4}|A|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m t)$$

Mixed:

$$|\langle \bar{f}|H|B^0\rangle|^2 = \frac{1}{4}\left|\frac{q}{p}\right|^2|\bar{A}|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m t)$$

Now assume that you did not tag the flavor at production, and there is no CP violation in mixing <u>or</u> decay, i.e. |q/p|=1 and |A| = |A|

$$|\langle f|H|B^0\rangle|^2 + |\langle \bar{f}|H|B^0\rangle|^2 = \frac{1}{2}|A|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t})$$

All you see is the sum of two exponentials for the two lifetimes.

Summary of today

- We discussed the basic formalism for matter <-> antimatter oscillations.
- We showed how this is intricately related to:
  - Mass difference of the mass eigenstates
  - Lifetime difference of the mass eigenstates
  - CP violation in the decay amplitude
  - CP violation in the mixing amplitude
- We discussed how the formalism simplifies in the B-meson system due to natures choice of M_{12} and Γ_{12} .
- We showed how one can measure $\cos\Delta m$.

Next Lecture

- Discuss CP violation in the B system in detail.
- Justify what simplifying assumptions are ok under what conditions.
- Come back to lifetime difference measurement.