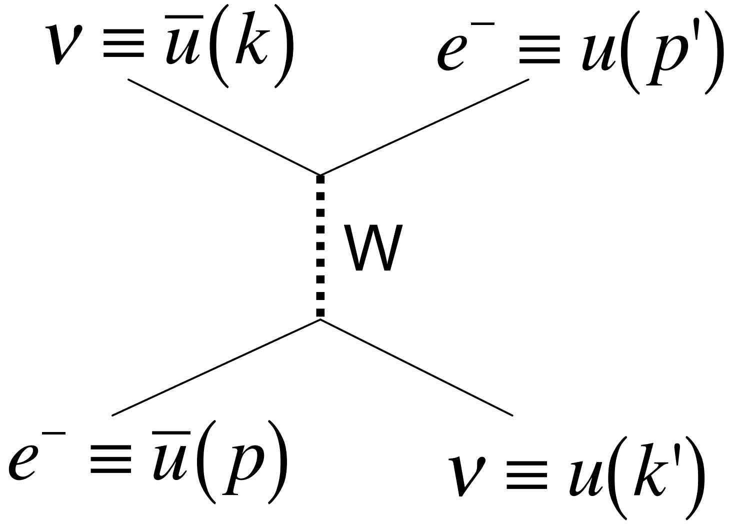
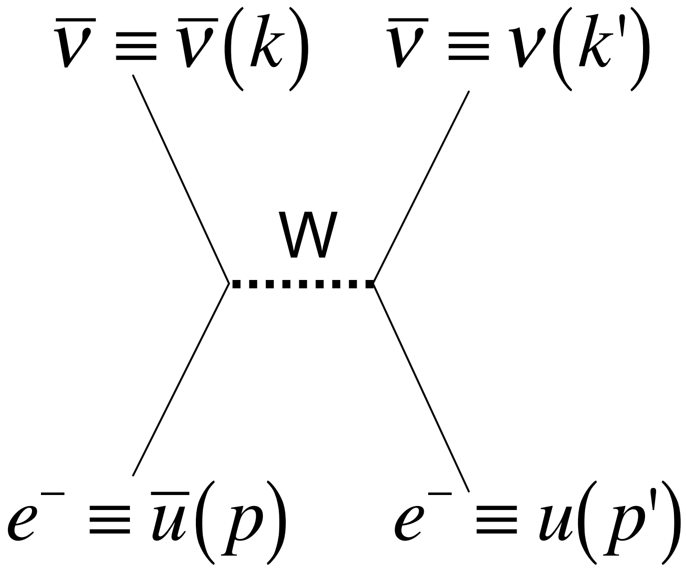
# Physics 222 UCSD/225b UCSB

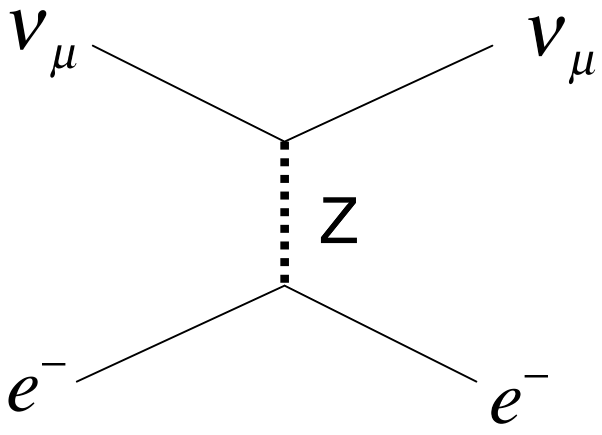
## Lecture 4

- Weak Interactions (continued)
  - Neutrino scattering
  - Unitarity bound
  - GIM mechanism

# Neutrino Electron Scattering





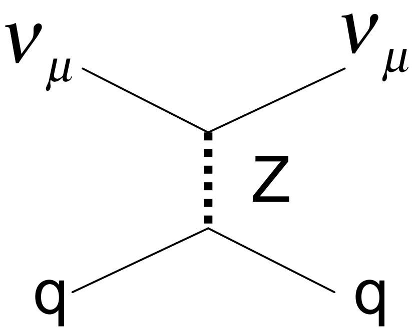


NC is different, e.g.
it couples muon neutrinos
to electrons, i.e. across flavor.

# Historical aside

- If you want to look up the discovery of neutral currents, you might want to start here:

http://cerncourier.com/cws/article/cern/29168



The basic challenge was to distinguish:

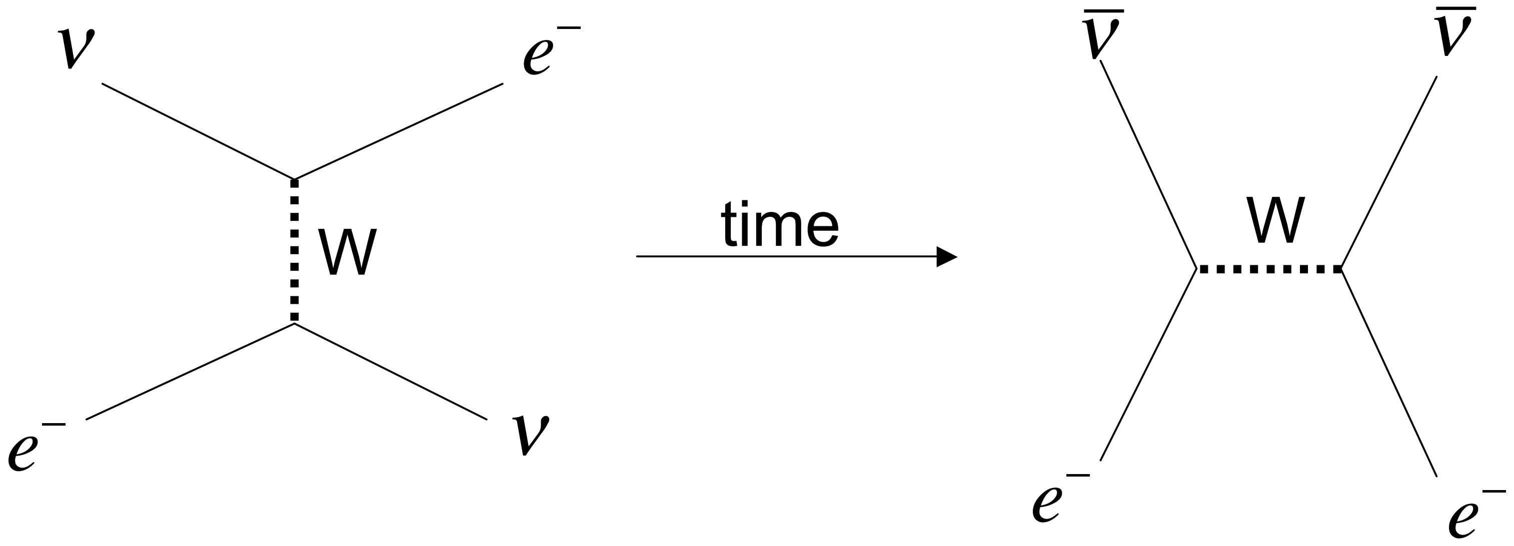
$$\nu_\mu N \rightarrow \nu_\mu X$$

$$\nu_\mu N \rightarrow \mu^- X$$

Where N = nuclean,
and X = hadron.

For neutral current, there is no charged lepton in the final state.

# Neutrino vs Antineutrino CC Scattering with Electron



For antineutrino scattering, the spin of the two incoming particles must couple via V-A because they attach at same vertex.

As a result, only one of the three possible spin combinations is allowed, and we get:

$$\boxed{\sigma\left(\nu_e e^-\right) = 3\sigma\left(\bar{\nu}_e e^-\right)}$$

# Neutrino Electron Scattering

- Matrix Element for CC neutrino-electron:

$$M = \frac{G}{\sqrt{2}} \left[ \bar{u}(k') \gamma^{\mu} \left(1 - \gamma^5\right) u(p) \right] \left[ \bar{u}(p') \gamma_{\mu} \left(1 - \gamma^5\right) u(k) \right]$$

- Cross Section:

$$\sigma\left(\nu_e e^-\right) = \frac{G^2 s}{\pi}$$

***What does it mean for a cross section to increase with center of mass energy ?***

*It’s a sign of a “low energy effective theory”!*

# Overview of “Unitarity bound” Discussion

- Use partial wave analysis to derive the largest possible cross section, $\sigma(s)$, that is compatible with probability conservation.
- Compare this with the calculated cross section.
- Calculate the center of mass energy scale, beyond which the 4-fermion interaction clearly makes no sense because it violates probability conservation.
- Show how the introduction of the W propagator helps to resolve this problem.
- Comment that even with W, issues remain that are related to the longitudinal W polarization.
- Hint that this is fixed only by requiring Gauge Symmetry.

# Partial waves

(to review this, see e.g. Sakurai QM Chapter 7.6)

$$\Psi_i = e^{ikz} = \frac{i}{2kr}\sum_{l=0}^{\infty}(2l+1)\left[(-1)^l e^{-ikr} - e^{ikr}\right]P_l(\cos\theta)$$

$$\Psi_{total} = \Psi_{scattered} + \Psi_i = \frac{i}{2kr}\sum_{l=0}^{\infty}(2l+1)\left[(-1)^l e^{-ikr} - \eta_l e^{2i\delta_l} e^{ikr}\right]P_l(\cos\theta)$$

*Absorption coefficient =1*
*If no energy is absorbed.*

*Allow for arbitrary*
*phase shift.*

Regroup to get scattered wave:

$$\Psi_{scattered} = \Psi_{total} - \Psi_i = \frac{e^{ikr}}{kr}\sum_{l=0}^{\infty}(2l+1)\frac{\eta_l e^{2i\delta_l} - 1}{2i}P_l(\cos\theta)$$

$$\Psi_{scattered} = \frac{e^{ikr}}{r}F(\theta)$$

# Relate to Cross Section

- Scattered outgoing Flux d$\Omega = v_{out}\Psi_{scat}\Psi^{*}_{scat}r^{2}d\Omega$

$$F_{out} = v_{out}\left|F(\theta)\right|^{2}d\Omega$$

$$F_{in} = \Psi_{in}\Psi^{*}_{in}v_{in} = v_{in}$$

- Cross section:

$$d\sigma = \frac{F_{out}}{F_{in}} = \left|F(\theta)\right|^{2}d\Omega$$

$$\sigma = \int \left|F(\theta)\right|^{2}d\Omega$$

$$\sigma = \frac{1}{k^{2}}\sum_{l,m}(2l+1)\left[\frac{\eta_{l}e^{2i\delta_{l}}-1}{2i}\right](2m+1)\left[\frac{\eta_{m}e^{2i\delta_{m}}-1}{2i}\right]^{*}$$

$$\times\int P_{l}(\cos\theta)P_{m}(\cos\theta)d\Omega$$

$$\sigma = \frac{4\pi}{k^{2}}\sum_{l}(2l+1)\left|\left[\frac{\eta_{l}e^{2i\delta_{l}}-1}{2i}\right]\right|^{2} = \frac{4\pi}{k^{2}}\sum_{l}(2l+1)\sin^{2}\delta_{l}$$

# Conclusion from Partial wave excursion

- For our neutrino scattering we had:

$$\frac{d\sigma(v_e e^-)}{d\Omega} = \frac{G^2 s}{4\pi^2}$$

- The angle independence means that only s-wave (l=0) contributes, and we thus have the general bound:

$$\sigma \leq \frac{4\pi}{k^2}$$

***=> Probability conservation is violated at:***

$$\sqrt{s} = k \approx 300GeV$$

# Including the W propagator

- The 4-point Fermi theory thus violates s-matrix unitarity at O(100GeV) energy.
- If we include W propagator the point where s-matrix unitarity is violated is pushed out to $O(10^{11})\ M_W$ .

- However, a number of other problems remain!

# Example WW production

- If you calculate WW production in neutrino scattering, you find:

$$\sigma\left(\nu\nu \to W_L^+ W_L^-\right) \xrightarrow{k^2 \to \infty} \left[\frac{g}{M_w}\right]^4 s$$

- While production of transversely polarized W's remains constant.
- Clearly, here's something problematic about longitudinally polarized massive vector bosons.

# Resolving this in QED

- As an aside, the same problem does not arise for virtual photons in QED because of gauge invariance.
  - For more detailed discussion see Leader & Predazzi, Chapter 2.1

# Aside on Gauge Invariance

- If we were to try and figure out a way to impose gauge invariance to weak interactions, we'd be tempted to postulate that g ~ e.
- It turns out that this works out quantitatively surprisingly well:

$$M_W = \left(\frac{\sqrt{2}g^2}{G}\right)^{\frac{1}{2}} \approx \left(\frac{\sqrt{2}e^2}{G}\right)^{\frac{1}{2}} \approx 106GeV$$

- We'll see later how to work this out correctly.

# Conclusions

- Fermi Theory breaks down at high energies
  - This is a general feature of theories with dimensionful couplings.
- Including W by hand improves high energy behaviour, but does still leave problems, e.g. with νν -> WW for longitudinally polarized Ws.
- Problem with W seems to be related to longitudinal polarization.
- Might be fixed if we could construct a gauge theory of weak interactions.
- We find the EM & Weak almost unify in the most naïve way by setting e = g, and calculating the W mass correctly to within 10%.

*Sounds like we are on to something!*

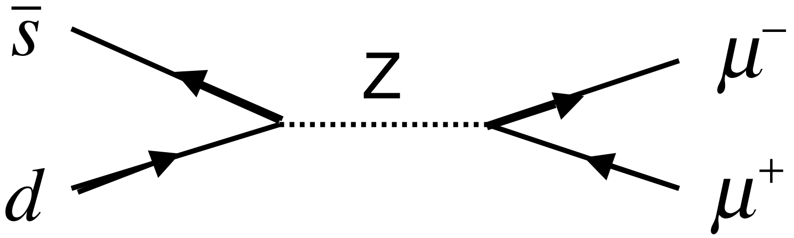
# GIM Mechanism

- Problem:

$$\Gamma\left(K^{+} \to \mu^{+}\nu\right) = 5.1\times10^{7}\,\mathrm{sec}$$

$$\Gamma\left(K^{0} \to \mu^{+}\mu^{-}\right) = 1.4\times10^{-3}\,\mathrm{sec}$$

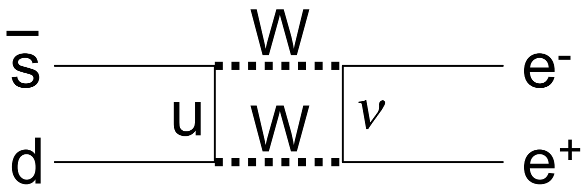
- First obvious conclusion:



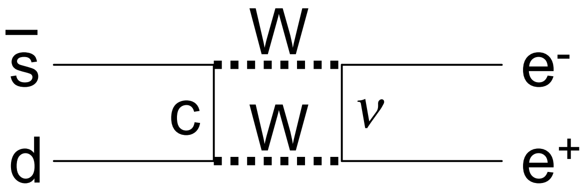
***This diagram doesn't work -> Z couples to same flavor.***

# GIM Mechanism (2)

- 2nd order diagram is not sufficiently small



- Glashow-Illiapolous-Maiani suggested that a c-quark exists, providing another 2nd order diagram to destructively interfere with first.



# GIM Mechanism (3)

- How do we arrange the destructive interference?

Old current:

$$J^{\mu} = \cos\theta \ \ \bar{u}\gamma^{\mu}\left(1-\gamma^{5}\right)d + \sin\theta \ \ \bar{u}\gamma^{\mu}\left(1-\gamma^{5}\right)s$$

New current:

$$J^{\mu} = \cos\theta \ \ \bar{u}\gamma^{\mu}\left(1-\gamma^{5}\right)d + \sin\theta \ \ \bar{u}\gamma^{\mu}\left(1-\gamma^{5}\right)s$$

$$+\cos\theta \ \ \bar{c}\gamma^{\mu}\left(1-\gamma^{5}\right)d - \sin\theta \ \ \bar{c}\gamma^{\mu}\left(1-\gamma^{5}\right)s$$

# GIM Mechanism (4)

- To make this less ad-hoc, propose weak doublets, and a current that is diagonal, with a unitary mixing matrix between doublets to translate from weak to mass eigenstates:

$$J_\mu^+ = (\bar{\nu}_{eL}\bar{\nu}_{\mu L}\bar{\nu}_{\tau L})\gamma_\mu \begin{pmatrix} e_L^- \\ \mu_L^- \\ \tau_L^- \end{pmatrix} + (\bar{u}_L\bar{c}_L\bar{t}_L)\gamma_\mu \mathbf{V_{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

# Concise statement of GIM

$$\sum_{i=1,2,3} \overline{d}'_i d'_i = \sum_{i,j,k=1,2,3} \overline{d}_i U^{T*}_{ij} U_{jk} d_k = \sum_{i=1,2,3} \overline{d}_i d_i$$

- The neutral current couples to the q’ (weak eigenstates) not the q (mass eigenstates).
- The matrix U is unitary because the weak coupling is universal, i.e. same for all weak eigenstates.
- As a result, only flavor conserving neutral currents are allowed.

# Historic Aside

- From this we then find that the K0 to mu+mu- decay is absolutely forbidden if c and s masses are identical.
- In the literature, you sometimes find claims like: *"From the observed rate, one could predict the charm quark mass to be 1-3GeV before its discovery."*
- Not sure this is true, and if true, then they were plain lucky because depending on $V_{cs}$ $V_{cd}$ and $m_c$ versus $V_{ts}$ $V_{td}$ and $m_{top}$ , they could have been way off!

# Outlook on next few lectures

- Next 3 lectures are on heavy flavor physics and CP violation.
  - Mixing phenomenology
    - 2-state formalism in its entirety, maybe as a homework.
  - CP violation in B-system
    - Categorize CP phenomenology
      - CP violation in decay, mixing, and interference between decay and mixing.
  - Experimental aspects of this subject
    - Measuring sin2beta @ Y4S
    - Measuring Bs mixing @ Tevatron
    - The future of this field: LHC-B and SuperBelle

# After that we have choices:

- We can do the topics in two orders. Either way we will cover both:
  - Move on to SUSY for 2-3 lectures, and finish quarter off with EWK symmetry, higgs, et al.
  - Continue with EWK symmetry, higgs, et al., and do SUSY last.
- I don’t have much of a preference myself.
  - It makes more sense to do Standard Model first.
  - However, to start preparing for your seminar talk, it might be useful for me to talk about SUSY first to orient you.