Physics 222 UCSD/225b UCSB

Lecture 2

- Weak Interactions
  - Intro and Overview
  - V-A nature of weak current
  - Nuclear beta decay

Weak Interactions

- Some of the most surprising & mysterious phenomena in particle physics:
  - Violates fundamental symmetries
    - C, T, P, CP
  - Changes flavor of quarks and leptons
    - Heavy flavor decay
    - Neutrino oscillations
  - Matter - Antimatter Oscillations
    - K^0, B^0, B_s^0, D^0 oscillations all observed
    - Dazzlingly complex and beautiful phenomena
  - Matter - Antimatter symmetry violation
    - Decay width asymmetries
    - Symmetry violations as a function of proper time of decay
    - Symmetry violations as a function of angular correlations

Charged weak current

- Leptonic:
  - Conserves flavor.
  - Coupling independent of flavor.
- Hadronic:
  - Flavor changing
  - Coupling = leptonic coupling x $V_{qq'}$

$$J^+_\mu = (\bar{\nu}_{eL}\bar{\nu}_{\mu L}\bar{\nu}_{\tau L})\gamma_\mu \begin{pmatrix} e^-_L \\ \mu^-_L \\ \tau^-_L \end{pmatrix} + (\bar{u}_L\bar{c}_L\bar{t}_L)\gamma_\mu \mathbf{V_{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

Cabbibo-Kobayashi-Maskawa (CKM)

- Couplings within family dominate.
- The more off-axial the weaker the coupling.

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix} = \begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix}$$

$$\begin{bmatrix} 0.97383 & 0.2272 & 0.00396 \\ 0.2271 & 0.97296 & 0.04221 \\ 0.00814 & 0.04161 & 0.999100 \end{bmatrix}$$

CKM Matrix

V_{CKM} is unitary by definition.

$\Rightarrow$ 3 angles + 6 phases because of unitarity.

The phases of 5 out of 6 spinors may be adjusted such as to define phase conventions for V_{CKM}.

The phase of the 6th spinor is not available for this. It disappears when $|M|^2$ is formed.

Free parameters: 3 angles + 6 phases - 5 phases = 3 angles ***+ 1 phase***

c = cos; s = sin; x,y,z are angles

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_x c_z & s_x c_z & s_z e^{-i\phi} \\ -s_x c_y - c_x s_y s_z e^{i\phi} & c_x c_y - s_x s_y s_z e^{i\phi} & s_y c_z \\ s_x s_y - c_x c_y s_z e^{i\phi} & -c_x s_y - s_x c_y s_z e^{i\phi} & c_y c_z \end{pmatrix}$$

$$s_x = \lambda,\ s_y = A\lambda^2,\ s_z = O(\lambda^3) \quad \lambda = 0.22,\ A = 0.8$$

Phase shows up at $O(\lambda^3), O(\lambda^4), O(\lambda^5), O(\lambda^6)$

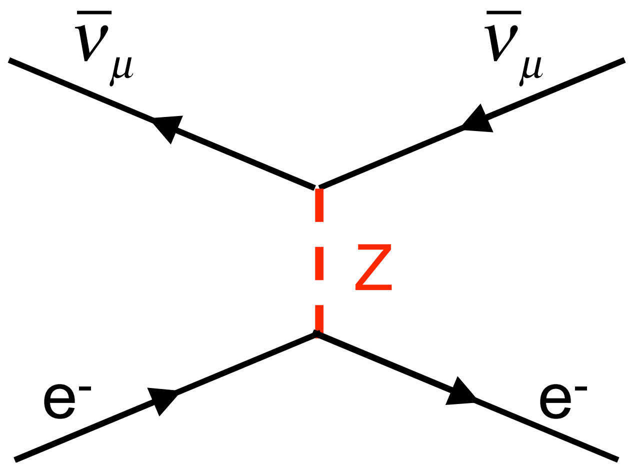
CKM Matrix Phase Convention

- Is admittedly arbitrary.
  - See http://arxiv.org/abs/hep-ph/9708366 if you really want to know the details.
- KISS principle for choice of phase:
  - Dominant processes are chosen to have zero phase.

Crudely Categorize Charged Current by theoretical complexity

- Purely leptonic
- Semi-leptonic
- Hadronic

There's also Weak Neutral Currents

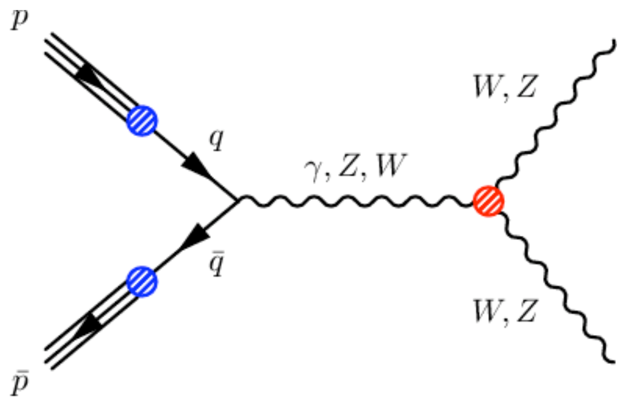


First observed in 1973.

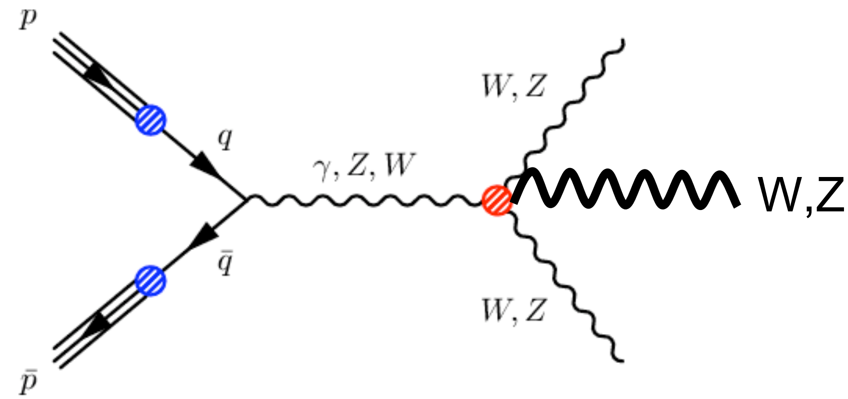
- No flavor changing neutral currents at LO in EWK (FCNC):
  - E.g. BR(K0 -> e+ e-) < 1.4 10^{-7}

    BR(Bd -> mu+mu-) < 1.8 10^{-8}
- Limits on FCNC impose some of the most stringent limits on beyond the standard model physics model building.

And there's boson self-coupling



Example: WZ production involves WWZ triple gauge coupling.



Example: $W^+W^-W^+$ production involves WWWW quartic gauge coupling.

Triple Gauge couplings are well studied, while experimental knowledge of quartic couplings is limited.

and EWK symmetry breaking

We'll walk through this in roughly the order outlined here.

Historical Interlude

- Fermi proposed to explain nuclear beta-decay in analogy to electron-proton scattering.

$$n \to p e^- \bar{\nu}_e$$

$$p e^- \to n \nu_e$$

$$M = G\left(\bar{u}_n \gamma^\mu u_p\right)\left(\bar{u}_{\nu_e} \gamma_\mu u_e\right)$$

He thus envisioned a vector current with a weak coupling constant, G, that we now call "Fermi constant". There was no propagator, nor parity violation in his theory.

We now know:

$$M \propto G\left(\overline{u}_n \gamma^\mu u_p\right)\left(\overline{u}_{\nu_e} \gamma_\mu u_e\right)$$

$$M \propto \frac{g}{\sqrt{2}} J_\mu(1) \frac{g^{\mu\nu} + q^\mu q^\nu / M_W^2}{M_W^2 - q^2} J_\nu(2)$$

At low q^2, we have $G/\mathrm{sqrt}(2) = g^2/(8M_w^2)$
For $G=1.2\ 10^{-5}\ \mathrm{GeV}^{-2}$ we thus get $g=0.36$.

Weak interactions is weak because M_W is large compared to, say the mass of the proton.

weak interaction violates parity

- Basic structure of the weak interaction Matrix Element:

$$(vertex)_\mu (propagator)^{\mu\nu} (vertex)_\nu$$

$$\gamma^\mu (1-\gamma^5) \gamma_\mu (1-\gamma^5) = \gamma^\mu \gamma_\mu + \gamma^\mu \gamma_\mu \gamma^5 \gamma^5 - 2\gamma^\mu \gamma_\mu \gamma^5$$

scalar

pseudoscalar

Matrix element has mixed parity.
Parity is thus not conserved.

Form of Charged Current

- Charge Raising Current:

$$J^{\mu} = \overline{u}_{\nu}\gamma^{\mu}\frac{1}{2}\left(1-\gamma^{5}\right)u_{e}$$

- Charge Lowering Current:

$$J_{\mu}^{T*} = \overline{u}_{e}\gamma_{\mu}\frac{1}{2}\left(1-\gamma^{5}\right)u_{\nu}$$

Any Matrix element needs to be a product of raising and lowering current in order to conserve charge !!!

Nuclear beta decay

- ^{14}O -> $^{14}N^{*}$ + e^{+} + electron-neutrino
- I.e., u -> d + e^{+} + electron-neutrino
- First Q:
  - Can we successfully describe a nuclear transition using our formalism derived for partons?
- Answer: “Conserved Vector Current” (CVC)
  - Isospin symmetry guarantees that QCD does not modify the weak vector currents because they are in isospin triplet with EM current, whose charge does not get modified by QCD, after all.

$$\overline{\psi}_n \gamma^\mu \psi_p \qquad \overline{\psi}_p \gamma^\mu \psi_p \qquad \overline{\psi}_p \gamma^\mu \psi_n$$

Axial Vector part of current

- Initial and final nuclear states have $J^P = 0^+$
  - Both nuclei have J=0
  - Both nuclei have same parity

  ⇒We can safely assume that the wave function of the nucleus is unchanged, and thus ignore the axial vector part of the weak current in this transition.
- This turns out to be important because axial vectors receive ~20% modification of effective current from nuclear physics QCD, while vector currents don't.
    - CVC = conserved vector current
    - PCAC = partially conserved axial vector current
- Better use vector current transitions when trying to measure G !!!

Calculating T_{fi}

$$T_{fi} = \frac{-i4GV_{ud}}{\sqrt{2}} \int \left[\overline{\psi}_n(x)\gamma_\mu \frac{1}{2}\left(1-\gamma^5\right)\psi_p(x) \right] \left[\overline{\psi}_\nu(x)\gamma^\mu \frac{1}{2}\left(1-\gamma^5\right)\psi_e(x) \right] d^4x$$

Now simplify:

$$u_p = \sqrt{2m}\begin{pmatrix} \chi \\ 0 \end{pmatrix}$$

Nuclean spinors are non-relativistic:

$$\chi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} or \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\gamma^\mu \rightarrow \gamma^0$$

Leptonic current has free particle wave function:

$$\overline{\psi}_\nu(x)\gamma^\mu \frac{1}{2}\left(1-\gamma^5\right)\psi_e(x) = \overline{u}_\nu(p_\nu)\gamma^\mu \frac{1}{2}\left(1-\gamma^5\right)v_e(p_e)e^{-i(p_\nu + p_e)x}$$

Positron spinor

Consider energy release

- Energy of e is O(1MeV).
  - –> de Broglie wavelength ~ 10^{-11}cm >> $R_{nucleus}$
  - –> we can consider x-dependence of leptonic current to be trivially integrable.
- We then end up with:

$$T_{fi} \approx \frac{-iG}{\sqrt{2}}\left[\bar{u}_\nu(p_\nu)\gamma^0\frac{1}{2}\left(1-\gamma^5\right)v_e(p_e)\right]\int \psi_n^{T*}(x)\psi_p(x)e^{-i(p_\nu+p_e)x}d^4x$$

$$e^{-i(p_\nu+p_e)x} \approx 1 \Rightarrow \int \psi_n^{T*}(x)\psi_p(x)e^{-i(p_\nu+p_e)x}d^4x \approx 2me^{-i(E_p-E_n)}\left(2\sqrt{\frac{1}{2}}\right)$$

Isospin factor (see homework)

Following the usual procedures

- We then follow the usual procedure to go from T_{fi} to M to dΓ and get:

$$d\Gamma \approx G^2 |V_{ud}|^2 \sum_{spins} \left| \overline{u}_\nu(p_\nu)\gamma^0 \frac{1}{2}\left(1-\gamma^5\right) v_e(p_e) \right|^2$$

$$\times \frac{d^3 p_e}{(2\pi)^3 2E_e} \frac{d^3 p_\nu}{(2\pi)^3 2E_\nu} 2\pi\delta\left(E_0 - E_e - E_\nu\right)$$

$$\sum_{spins} \left| \overline{u}_\nu(p_\nu)\gamma^0 \frac{1}{2}\left(1-\gamma^5\right) v_e(p_e) \right|^2 = 8E_e E_\nu \left(1 + v_e \cos\theta\right)$$

$$\frac{d\Gamma}{dp_e} = \frac{G^2 |V_{ud}|^2}{\pi^3} p_e^2 \left(E_0 - E_e\right)^2$$

For more detail,
See H&M p.260

Kurie plot and neutrino mass

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$$\frac{1}{p_e}\sqrt{\frac{d\Gamma}{dp_e}} = \frac{G|V_{ud}|}{\sqrt{\pi^3}}(E_0 - E_e)$$

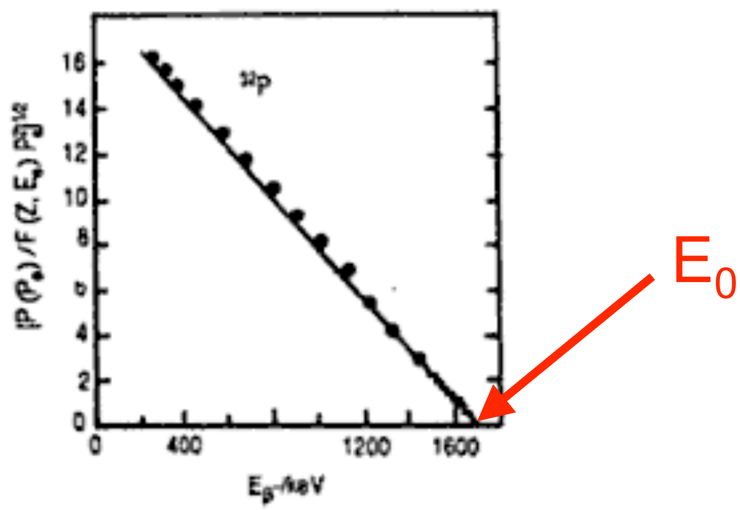

Fig. 4.22 The Fermi-Kurie plot for ^{32}P (from F.T. Porter, F. Wagner, Jr., and M.S. Freedman,[18] reproduced with author's permission)

The endpoint of this plot does not reach E_0 if the neutrino is massive. This has been used in tritium beta decay to set limits on neutrino masses.

(See link to 17keV Neutrino story on last quarter's website)

Measuring V_{ud}

- Comparisson of beta decay and muon decay allows for precision measurement of $|V_{ud}|$

$$|V_{ud}| = 0.9736 \pm 0.0010$$

Precision of 1/1000 => challenge in nuclear physics.