Astrophysical MHD I: Convection and Magnetic Fields

Examine:

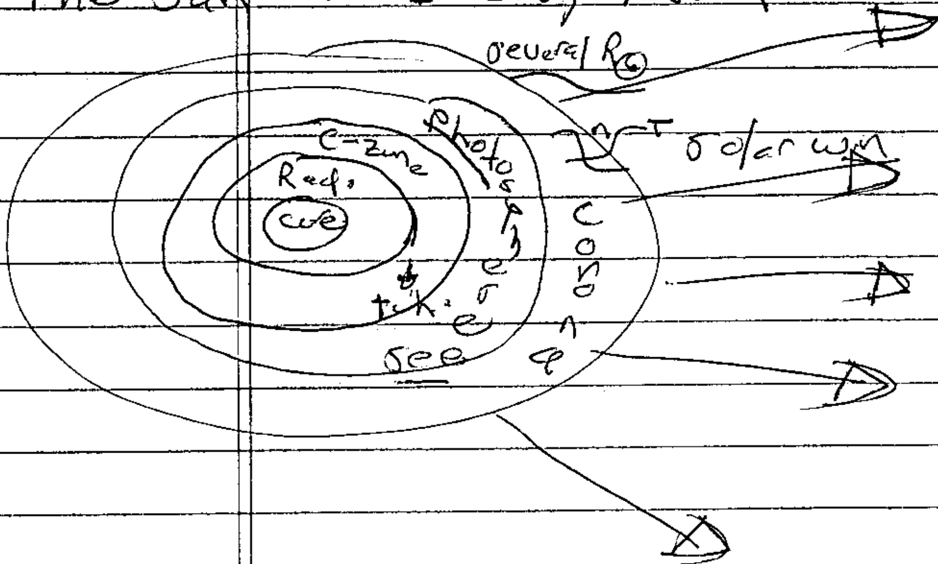
- → sunspots
- → magnetic field eruptions / solar prominences
- → clumping of matter in ISM

Theme: Buoyancy (Rayleigh - Benard convection) + Magnetic Fields

i.e.
- convection ✓
- convection with $\underline{B}_0 \parallel g$ ✓
- exclusion of magnetic field by convective motions ✓

⇒ - magnetic buoyancy instability. ✓

The Sun → 1 easy lesson



Photosphere $R_\odot$

Rad. zone $.75 R_\odot$

$T_{core} \sim 10^7 K$
$T_{Rad} \sim 10^6 K$
$T_{CZ} \sim 10^5 K$
$T_{photo} \sim 10^3 - 10^4$

→ heat flux driven system, i.e. Q = const.

→ source → nuclear burn in core (fusion which works)

→ radiation zone → heat transported by radiation (photons)
$Q = Q_{rad}$.

→ convection zone → sufficient absorption to destabilize convection (gas motion) due heating.
$Q = Q_{conv}$.

~~Tachocline ~ base of convection zone~~
→ (G) → rotation on cylinders ⇒ solid body ?!

Convection zone ⇒

~~– Rayleigh – Benard paradise~~

– site of (at least part of) solar dynamo i.e. → helical motion ⇒ ⟨ rotation / buoyancy
→ STF
→ Parker ⟩ cartoons

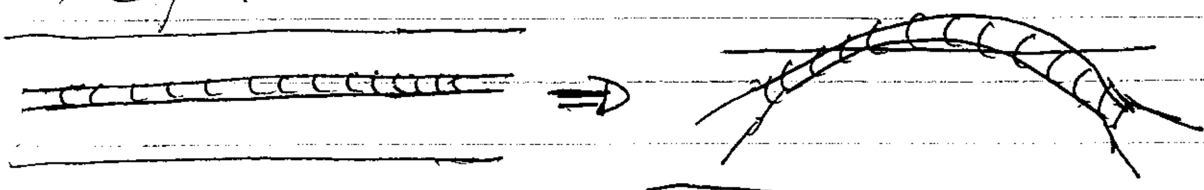
$\nabla P = \rho g$ magnetic buoyancy

$\nabla P = \underline{J} \times \underline{B}$ plasma interchange

→ Magnetic Buoyancy Instability

another (Interchange)

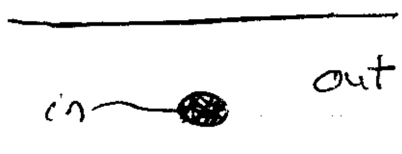
{ why fields erupt ? ! }



i.e. flux tubes rise! — c.f. picture from Chaudhuri

simple story

why! — compare inside tube / outside tube



$$p + \frac{B^2}{8\pi} \cong \text{const.}$$

rising flux tube

total pressure balance $\left\{ \; P_{out} = P_{in} + \dfrac{B^2}{8\pi} \right.$

$\therefore \; P_{out} > P_{in}$, but $\left\{ \begin{array}{l} P = P_0 (\rho/\rho_0)^{\gamma} \\ p = k \rho T \end{array} \right.$

buoyancy! $\left\{ \begin{array}{l} \Longleftrightarrow \; \rho_{out} > \rho_{in} \; ! \\ \Rightarrow \text{ tube rises.} \end{array} \right.$

{ N.B. This is a "lack of equilibrium", rather than an instability, strictly speaking.

Note: → suggests problem of magnetic buoyancy, i.e.

⇒ convection with magnetic field as the "stuff" to be convected. — contrast confinement.

Sketch RT → HW

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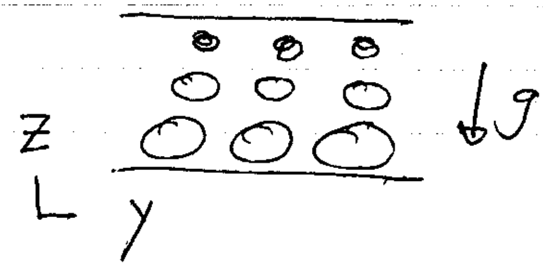
The Physics:

a.) Structure



$$\frac{dB_{0x}}{dz} < 0, \quad \frac{d\rho_0}{dz} < 0$$

→ stratified magnetic field (vertically)

→ $\underline{B}_0 \perp \tilde{\underline{V}}$ → interchanges (rolls in y, z; exchanging filled tubes)

$\underline{B}_0 \parallel \tilde{\underline{V}}$ → undular instability
(i.e. $B_{0y} \parallel \tilde{V}_y$) (i.e. buoyancy coupled to Alfven wave)

nominally expect undular modes more stable than interchanges, but ...

b.) Buoyancy Coupling

– Recall for Rayleigh Benard: $\begin{cases} \omega < k c_s \\ \lambda_z / H_\rho < 1 \end{cases}$

$$\Rightarrow \frac{\delta p}{p_0} \cong 0 \Rightarrow \frac{\hat{\rho}}{\rho_0} = -\frac{\hat{T}}{T_0}$$

$\delta p = 0$ as $\omega < k c_s$

expand here

$$\rho \frac{dU}{dt} = -\nabla(\hat{p}) + \frac{B_0 \cdot \nabla B}{4\pi} - \frac{\hat{\rho}}{\rho_0} g$$

– with B-field: $\dfrac{\delta P_{total}}{P_0} \cong 0 \iff \omega < k V_{magnetosonic}$

$\omega < k V_{MS} \to \delta p_{tot} = 0$

$$\Rightarrow R(\hat{\rho} T_0 + \hat{T} \rho_0) + \frac{B_0 \cdot \tilde{B}}{4\pi} \cong 0$$

$$\frac{\hat{\rho}}{\rho_0} = -\frac{\hat{T}}{T_0} - \frac{B_0 \cdot \tilde{B}}{4\pi \rho_0} = -\frac{\hat{T}}{T_0} - \frac{\tilde{p}_m}{p_0} \leftarrow \text{magnetic pressure}$$

This obviously suggests that an equation for magnetic pressure would be useful.

c.) B - Pressure evolution

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{V} \times \underline{B})$$ induction

$$\Rightarrow \frac{\partial \underline{B}}{\partial t} + \underline{V}\cdot\underline{\nabla}\,\underline{B} = \frac{d\underline{B}}{dt} = \underline{B}\cdot\underline{\nabla}\,\underline{V} - \underline{B}\,\underline{\nabla}\cdot\underline{V}$$

① Now, anelastic approximation $\quad \underline{\nabla}\cdot(\rho\underline{V}) = 0$

i.e. compressible, but eliminate sound wave

$$\cancel{\frac{\partial \rho}{\partial t}} + \hat{V}_z \frac{\partial \rho_0}{\partial z} = -\rho_0\, \underline{\nabla}\cdot\underline{\hat{V}}$$

$\omega < k c_{ms}$

$$\underline{\nabla}\cdot\underline{V} = +\frac{\hat{V}_z}{L_\rho} \qquad \left(1/L_\rho = \frac{-1}{\rho}\frac{d\rho}{dz}\right)$$

② $\underline{B}\cdot\underline{\nabla}\,\underline{V} = 0$ (interchange limit)

$$\therefore \frac{\partial \tilde{B}}{\partial t} + \tilde{V}_z \frac{\partial B_0}{\partial z} = -B_0 \frac{\tilde{V}_z}{L_\rho}$$

$$\frac{\partial}{\partial t} B_0\cdot\tilde{B} + \frac{B_0^2 \tilde{V}_z}{B_0}\frac{\partial}{\partial z} B_0 = +\frac{B_0^2}{\rho_0}\frac{\partial \rho_0}{\partial z}\tilde{V}_z$$

so, can write:

$$\frac{\partial}{\partial t}\tilde{P}_m + \left[P_{m,0}\frac{\partial}{\partial z}\ln\left(B_0/\rho_0\right)\right]\hat{V}_z = 0$$

if include resistive dissipation:

$$\frac{\partial}{\partial t}\tilde{P}_m \quad \underset{\substack{\text{can dissipate}\\ \text{magnetic energy response.}}}{-\eta}\nabla^2\tilde{P}_m = -\hat{V}_z P_{m,0}\frac{\partial}{\partial z}\left[\ln B_0/\rho_0\right]$$

Now, can proceed with basic equations:

(before)

$$\begin{cases} \dfrac{\partial}{\partial t}(-\nabla^2\phi) = |g_z|\dfrac{\partial}{\partial y}\left(\dfrac{\hat{T}}{T_0} + \dfrac{\hat{P}_m}{P_0}\right) \\[2ex] \dfrac{\partial}{\partial t}\tilde{P}_m + \left[P_{m,0}\dfrac{\partial}{\partial z}\ln\left(B_0/\rho_0\right)\right]\hat{V}_z = 0 \quad ; \quad \hat{V}_z = -\nabla_y\hat{\phi} \end{cases}$$

$$\frac{\partial}{\partial t}\left(\frac{\hat{T}}{T_0} - (\gamma-1)\frac{\hat{\rho}}{\rho_0}\right) + \hat{V}_z\frac{dS_0}{dz} = 0$$

but: $\dfrac{\hat{\rho}}{\rho_0} = -\dfrac{\hat{T}}{T_0} - \dfrac{P_m}{P_0}$

$$S = \ln\left(T\rho^{-(\gamma-1)}\right)$$

$$= \ln\left(\underset{p}{(\rho T)}\,\rho^{-\gamma}\right)$$

$$\frac{\partial}{\partial t}\left(\frac{\hat{T}}{T_0} + (\gamma-1)\left(\frac{\hat{T}}{T_0} + \frac{\hat{P}_m}{P_0}\right)\right) + \hat{V}_z \frac{dS_0}{dz} = 0$$

$$\left\{ \frac{\partial}{\partial t}\left(\cancel{\gamma}\,\frac{\hat{T}}{T_0} + \frac{(\gamma-1)}{\gamma}\frac{\hat{P}_m}{P_0}\right) + \frac{\hat{V}_z}{\gamma}\frac{dS_0}{dz} = 0 \right.$$

So can proceed:

$$\Rightarrow \frac{\partial^2}{\partial t^2}\left(-\nabla^2\phi\right) = |g_z|\frac{\partial}{\partial y}\left(\frac{\partial}{\partial t}\frac{\hat{T}}{T_0} + \frac{\partial}{\partial t}\frac{\hat{P}_m}{P_0}\right)$$

$$\frac{\partial}{\partial t}\frac{\hat{T}}{T_0} = -\frac{\left(1-\frac{1}{\gamma}\right)}{P_0}\frac{\partial \hat{P}_m}{\partial t} - \frac{\hat{V}_z}{\gamma}\frac{dS_0}{dz}$$

$$\frac{\partial}{\partial t}\frac{\hat{P}_m}{P_0} = -\frac{P_{m,0}\hat{V}_z}{P_0}\frac{\partial}{\partial z}\ln\left(B_0/P_0\right)$$

simple limit → total driving gradient

$$\frac{\partial}{\partial t}\left(\frac{\hat{P}_m}{P_0} + \frac{\hat{T}}{T_0}\right) = -\hat{V}_z\left\{\left[\frac{P_{m,0}}{P_0}\frac{d}{dz}\ln\left(B_0/P_0\right) + \frac{1}{\gamma}\frac{dS_0}{dz}\right] - \left(1-\frac{1}{\gamma}\right)\left(\frac{P_{m,0}}{P_0}\frac{\partial}{\partial z}\ln\left(B_0/P_0\right)\right)\right\}$$

$$= -\frac{\hat{V}_z}{\gamma}\left[\frac{P_{m,0}}{P_0}\frac{d}{dz}\ln\left(\frac{B_0}{P_0}\right) + \frac{dS_0}{dz}\right]$$

comment → double diffusion →
→ opposite grads
→ η ν s χ

and

$$-\frac{\partial^2}{\partial t^2}(+\nabla^2\phi) = |g_z| + \frac{\partial^2}{\partial y^2}\phi\left[\frac{1}{\gamma}\left(\frac{P_{M0}}{P_0}\frac{d}{dz}\ln\left(\frac{B}{\rho_0}\right) + \frac{dS_0}{dz}\right)\right]$$

$$\Rightarrow \boxed{\omega^2 = +\frac{k_y^2}{k^2}|g_z|\left[\frac{1}{\gamma}\left(\frac{dS_0}{dz} + \frac{P_{M0}}{P_0}\frac{d}{dz}\ln\left(\frac{B_0}{\rho_0}\right)\right)\right]}$$

→ magnetic buoyancy criterion (magnetic Schwarzschild criterion):

$$\boxed{\omega^2 = \frac{k_y^2}{k_\perp^2}|g_z|\left[\frac{1}{\gamma}\left(\frac{dS_0}{dz} + \frac{P_{M0}}{P_0}\frac{d}{dz}\ln\left(\frac{B_0}{\rho_0}\right)\right)\right]}$$

note: for $S_0' = 0$, instability $\Rightarrow \frac{d}{dz}\left(B_0/\rho_0\right) < 0$

for buoyancy instability

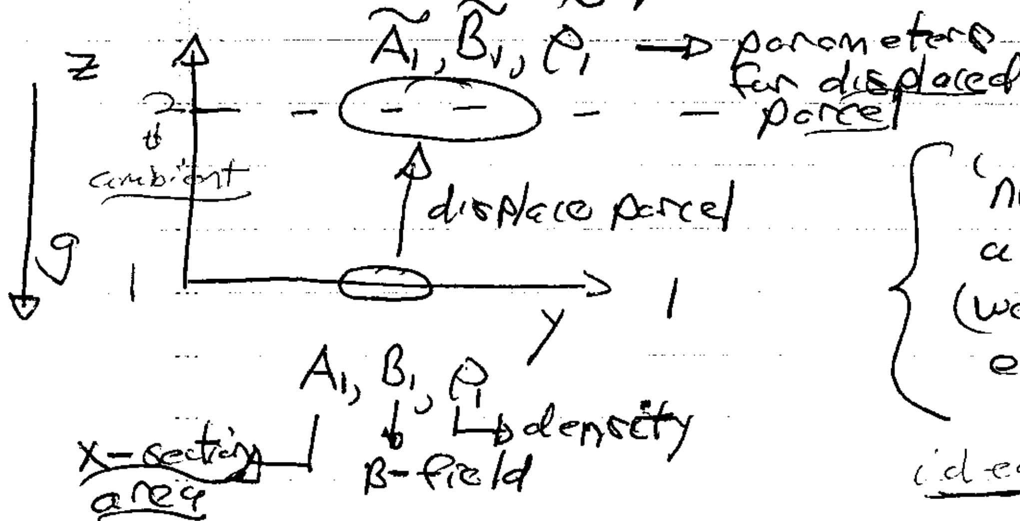
→ instabilities are flute / interchanges, if $\underline{B}_0 \cdot \underline{\nabla}\,\underline{\tilde{V}} \neq 0 \Rightarrow$ 'undular instability'

⇕

Alfvenic coupling → Parker.

→ origin of $(B_0/\rho_0)' < 0$ criterion.

⇒ Reconsider basic story: (ideal interchange)



ρ_2, A_2 (ambient)

'new wrinkle' → A as parameter ⇔ (weak) compressibility essential

ideal same as Schwarz
⇒ compare with surroundings.

so now; in ideal displacement:

mass conserved → $\rho_1 A_1 = \tilde{\rho}_1 \tilde{A}_1$

magnetic flux conserved → $A_1 B_1 = \tilde{A}_1 \tilde{B}_1$ (rec)

$$\frac{\tilde{A}_1}{A_1} = \frac{B_1}{\tilde{B}_1} = \frac{\rho_1}{\tilde{\rho}_1}$$

$$\Rightarrow B_1/\rho_1 = \tilde{B}_1/\tilde{\rho}_1 \quad \text{(freezing in)}$$

Now, obviously: stability ⇒ $\tilde{\rho}_1 > \rho_2$ (rise)

instability ⇒ $\tilde{\rho}_1 < \rho_2$ (sink)

<u>total</u> pressure equilib.

Now $\tilde{P}_1 + \frac{\tilde{B}_1^2}{8\pi} = P_2 + \frac{B_2^2}{8\pi}$

i.e. $P'_{tot} = P_{tot}$ (i.e.) → below.

<u>for neutral stability</u> $\tilde{\rho}_1 = \rho_2 \Rightarrow \tilde{P}_1 = P_2$, by eqn. state

$\therefore \tilde{B}_1 = B_2$ by pressure balance

$\therefore$ neutral stability $\Rightarrow \tilde{B}_1/\tilde{\rho}_1 = B_2/\rho_2$

<u>so</u> $\boxed{B_1/\rho_1 = B_2/\rho_2}$ <u>i.e.</u> <u>neutral</u> if $\nabla(B/\rho) = 0$

$\Rightarrow$ <u>instability</u> requires: $\frac{d}{dz}\left(B/\rho\right) < 0$

<u>i.e.</u> to solve for $\tilde{\rho}_1$ as Schwarz. ?

$$\tilde{P}_1 + \frac{\tilde{B}_1^2}{8\pi} = P_2 + \frac{B_2^2}{8\pi}$$

$$\tilde{P}_1 = P_2 + \frac{B_2^2}{8\pi} - \left(\frac{B_1}{\rho_1}\right)^2 \frac{\tilde{\rho}_1^2}{8\pi}$$

$$\tilde{P}_1 = P_1 (\tilde{\rho}_1/\rho_1)^\gamma$$

$$\Rightarrow \left[P_1 (\tilde{\rho}_1/\rho_1)^\gamma + \frac{B_1^2}{8\pi}\left(\frac{\tilde{\rho}_1}{\rho_1}\right)^2\right] = P_2 + \frac{B_2^2}{8\pi} = P_1 + \frac{dP}{dz}\Delta z + \frac{B^2}{8\pi} + \frac{d}{dz}\left(\frac{B^2}{8\pi}\right)\Delta z$$

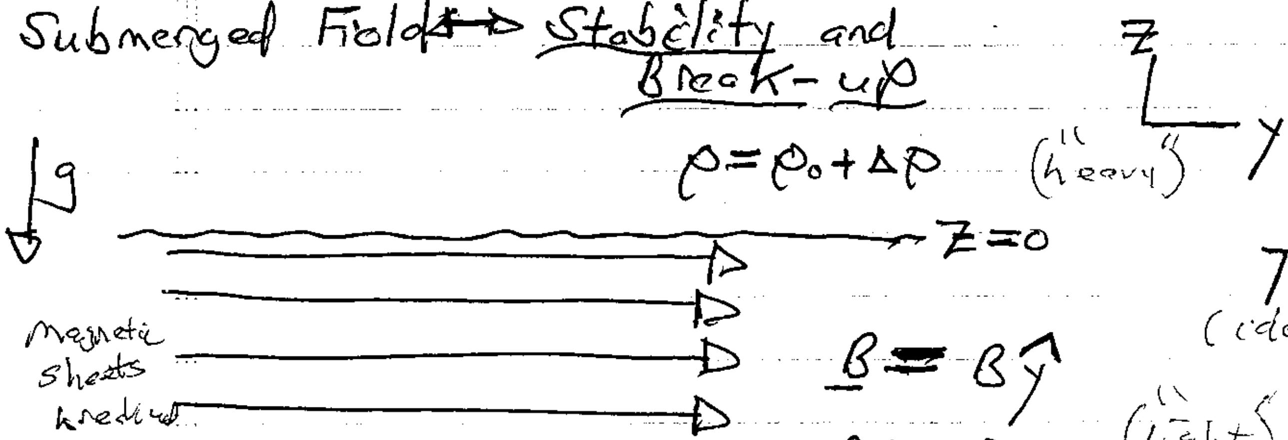
$\Rightarrow \tilde{\rho}_1$ and compare to ρ_2

Moral: Magnetic field, like hot air, rises .... **145.**

$\underline{B}$ can't stop R-T

Related Phenomena / Problem: **Interface**

Submerged Field $\leftrightarrow$ Stability and Break-up



$\rho = \rho_0 + \Delta\rho$ ("heavy")

$z = 0$

$T = \text{const.}$ (idealized)

$\underline{B} = B\hat{y}$

$\rho = \rho_0$ ("light" + field)

$$\underbrace{P_{tot}}_{z<0} = \underbrace{P_{tot}}_{z>0}$$

ρ mass-less here

$$\Rightarrow \quad k_B \rho_0 T + \frac{B^2}{8\pi} = k_B(\rho_0 + \Delta\rho)T$$

$$\boxed{\frac{\Delta\rho}{\rho_0} = \frac{B^2/8\pi}{k_B T \rho_0}} = 1/\beta$$

What happens?

→ Rayleigh-Taylor like instability should occur (modified by bending)

→ bubbles of light fluid (and field) will rise. ⇒ bubble scale $\leftrightarrow$ eruption scale

Linear Theory:

→ old Rayleigh-Taylor analysis, i.e.

- $\underline{V} = -\underline{\nabla}\phi$ ⇒ excludes Alfven waves (rotational)
  $\underline{\nabla}\cdot\underline{V} = 0$ ⇒ $\nabla^2\phi = 0$
- $\tilde{V}_z$ and δp_{tot} continuous at interface

i.e.

unperturbed interface:

$$\frac{B_0^2}{8\pi} + P_1 = P_2$$

perturbed interface:

$$\tilde{P}_1 - |g|\rho_0\tilde{\eta} + \frac{\underline{B}_0\cdot\underline{\tilde{B}}}{4\pi} = \tilde{P}_2 - |g|(\rho_0+\Delta\rho)\tilde{\eta}$$

but: $\tilde{P} = \rho\dfrac{\partial\tilde{\phi}}{\partial t}$ (Bernoulli)

$$\Rightarrow \boxed{\rho_0\frac{\partial\tilde{\phi}_1}{\partial t} - |g|\rho_0\tilde{\eta}_1 + \frac{B_0\cdot\tilde{B}}{4\pi} = (\rho_0+\Delta\rho)\frac{\partial\tilde{\phi}_2}{\partial t} - |g|(\rho_0+\Delta\rho)\tilde{\eta}_2}$$

where: $\dfrac{\partial \tilde{B}_y}{\partial t} = \underline{B}_0 \cdot \underline{\nabla}\, \tilde{V}_y$

$\gamma \tilde{B}_y = i k_y B_0 (-i k_y \hat{\phi}_1)$

$\tilde{B}_y = \dfrac{k_y^2 B_0 \hat{\phi}_1}{\gamma}$

$\underline{V} = -\nabla \phi$

and $\gamma \eta = V_z$

for ①: $\gamma \tilde{\eta}_1 = -k \hat{\phi}_1$

②: $\gamma \tilde{\eta}_2 = +k \hat{\phi}_2$

$\phi_1 \sim e^{kz}$ $(\to 0,\ z = -\infty)$

$k = (k_x^2 + k_y^2)^{1/2}$

$\phi_2 \sim e^{-kz}$

$\phi_{1,2} = \hat{\phi}_{1,2}\, e^{i \underline{k}_\perp \cdot \underline{x}}\, e^{\pm kz}$

$+ \to ①$
$- \to ②$

so $\hat{\phi}_1 + \hat{\phi}_2 = 0$

$$\gamma \rho_0 \hat{\phi}_1 + |g| \rho_0 \frac{k}{\gamma} \hat{\phi}_1 + \frac{k_y^2 B_0^2}{4\pi\gamma} \hat{\phi}_1 = \gamma(\rho_0 + \Delta\rho) \hat{\phi}_2 - |g| \frac{(\rho_0 + \Delta\rho)}{\gamma} k \hat{\phi}_2$$

$$\Rightarrow \gamma (2\rho_0 + \Delta\rho) \hat{\phi}_1 = \frac{|g| \Delta\rho}{\gamma} k \hat{\phi}_1 - \frac{k_y^2 B_0^2}{4\pi\gamma} \hat{\phi}_1$$

$$\gamma^2 = \frac{g k_\perp \Delta\rho}{2\rho_0 + \Delta\rho} - \frac{k_y^2 B_0^2}{4\pi\rho_0\left(1 + \frac{\Delta\rho}{\rho}\right)}$$

$$\boxed{\gamma^2 = \frac{\Delta\rho\, g k_\perp}{2\rho_0 + \Delta\rho} - \frac{\rho_0 V_A^2 k_y^2}{2\rho_0 + \Delta\rho}}$$

B-field ↔ "surface tension with direction"
only k_y matters

Note:

$$\rightarrow \gamma^2 = \overbrace{A g k}^{\text{R-T growth}} - \underbrace{\frac{\rho_0}{2\rho_0 + \Delta\rho}\left(k_y^2 V_A^2\right)}_{\text{bending stabilization}}$$

A → Atwood #

bending stabilization
(~ magnetized mass fraction)

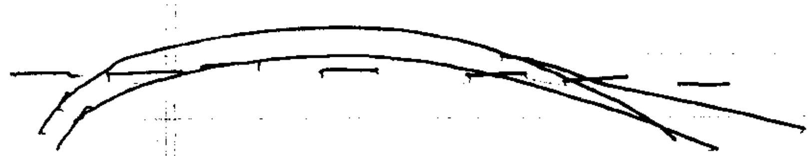
→ message is that field and magnetized fluid buoyant
⇒ rising bubbles

$$\Rightarrow \quad \underbrace{1/L_\parallel}_{} \sim \frac{\Delta\rho}{\rho}\frac{g}{V_A^2} = \frac{k_y^2}{k} \sim \frac{1}{L}$$

↓

minimum bubble scale, along field

← $L_\parallel$ →



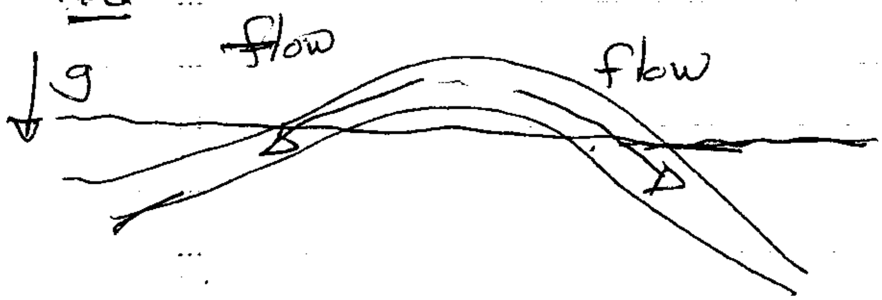
→ easily seen that field 'breaks-up', into structures, even for $k_y \equiv 0$.

⇔ magnetic buoyancy instability generic ⇒ idea underpinning ubiquitous formation of structure in magnetic field.

but → different from Rayleigh Taylor: Field Connection

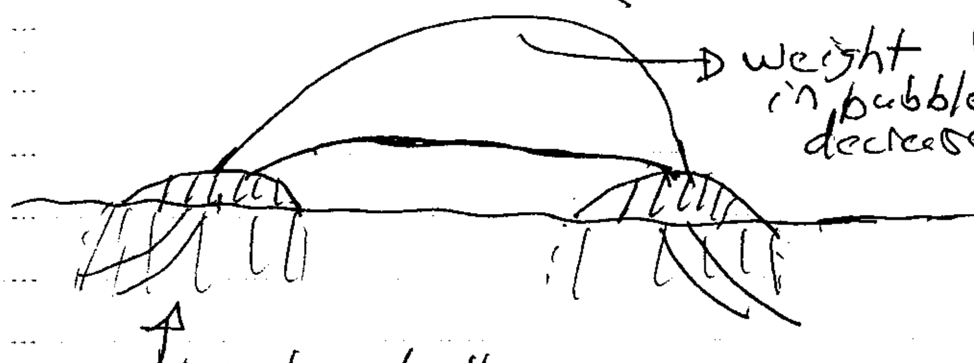
→ furthermore: matter/mass can slide along magnetic field → Parker instability → a/c/ compressible R-T.

i.e.



⇒ gas slides from peaks → valleys of magnetic undulations

⇒



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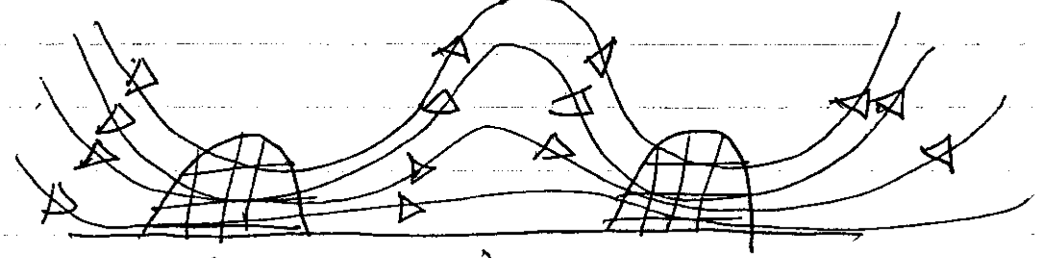
∴ – process is self-reinforcing (i.e. relieveing weight at bubble tip allows further rise of tip)

Parker instability → compressible

Key: modes with $k_{\parallel} \neq 0$, ~~more~~ line bending penalty offset by reduction in grav. pot. energy.

- matter will form / coagulate in dense clumps, with field attached, but bowed upward



clump/ mass concentration due sliding

upward buoyant undulations

Galaxy as fluid of clumps, threaded by field

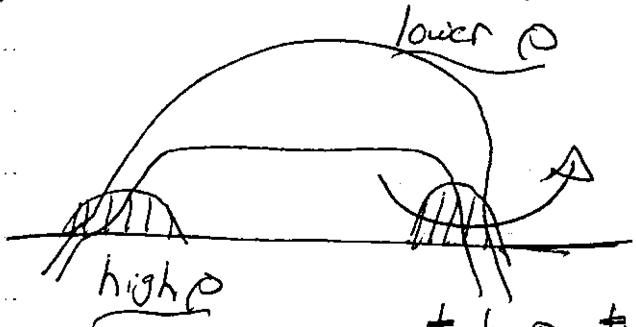
- B/ρ freezing (i.e. 'sliding' ⟺ refers to field line; i.e. - slide along a given field line)

⇒ B increases (bundle converges) at/in undulation valleys (clumps), as ρ increases.

⇒ B decreases (bundle diverges) in undulation peaks (bubble tips) as ρ decreases.

∴ ⇒ reinforces trend toward energy minimization.

→ Implications for Sunspots and Prominences (Parker mechanism)



tubes twist → granulation motion

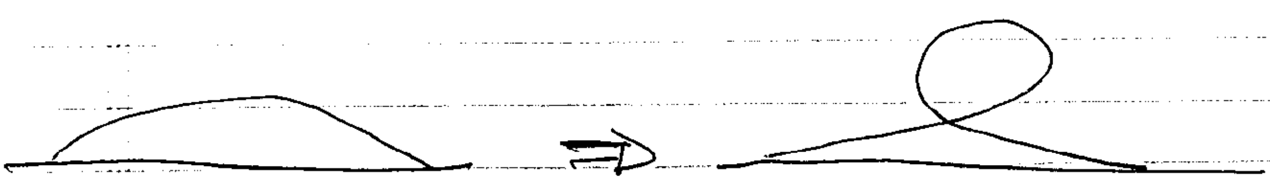
B field anchored in high-density valleys. B/c freezing ⇒ high ρ ⇒ high B. Thus,

- Parker mechanism will strengthen magnetic field and raise ("overload") density in sunspots

⇒

- furthers cooling/darkening due to convection inhibition and mass increase (→ radiation)

- i.e. twist ⇒ (kink process)



⇒ reconnection, prominences, etc.

by whatever means,

→ B-fields rise, buoyed upward

→ sunspot formation → Butterfly Diagram

→ coronal loop formation

→ coronal heating ⇒ solar wind.

Key component of story:

① ⇒ why do {heat / fields} rise?

⇒ magnetic buoyancy!

② why sunspots dark? → B field inhibits convection

③ How form — expulsion

~~[illegible]~~ why → fields in vortices.

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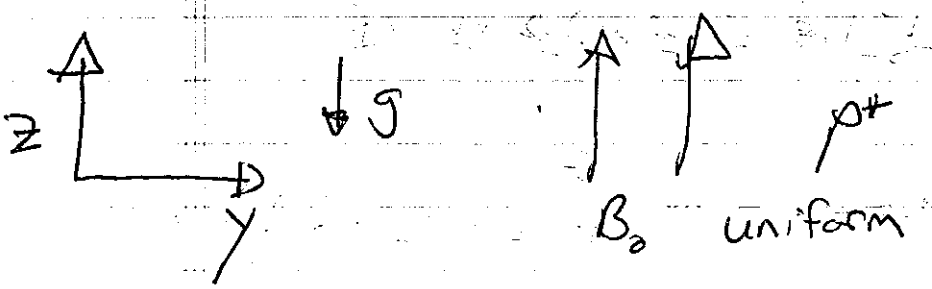
→ Effect of Magnetic Field ($\underline{B}_0 \parallel \underline{g}$) → why sunspots are dark → $\underline{B}$-fields kill convection



$$\frac{\partial V_y}{\partial t} = -\frac{\nabla_y \hat{p}^*}{\rho_0} + \frac{B_0}{4\pi\rho_0}\frac{\partial}{\partial z}\tilde{B}_y$$

$$\frac{\partial V_z}{\partial t} = -\frac{\nabla_z p^*}{\rho_0} + \frac{B_0}{4\pi\rho_0}\frac{\partial}{\partial z}\tilde{B}_z - |g_z|\frac{\tilde{\rho}}{\rho_0}$$

$$\frac{\partial \underline{B}}{\partial t} = B_0 \frac{\partial}{\partial z}\underline{V}$$

others as before

$$\Rightarrow \frac{\partial}{\partial t}\tilde{\omega}_x = |g_z|\frac{\partial}{\partial y}\left(\frac{\hat{T}}{T_0}\right) + \frac{c}{4\pi}\frac{B_0}{4\pi\rho_0}\frac{\partial}{\partial z}\tilde{J}_x$$

$$\frac{c}{4\pi}\frac{\partial}{\partial t}\tilde{J}_x = B_0\frac{\partial}{\partial z}\tilde{\omega}_x$$

$$\frac{\partial}{\partial t}\frac{\hat{T}}{T_0} = \frac{\nabla_y \phi}{\gamma}\frac{ds}{dz}$$

$$\frac{\partial^2}{\partial t^2}(-\nabla^2\phi) = \frac{|g_z|}{\gamma}\frac{\partial^2\phi}{\partial y^2}\frac{dS_0}{dz} + V_A^2\frac{\partial^2}{\partial z^2}(-\nabla^2\phi)$$

$$-\omega^2 k^2 = -k_y^2 N^2 - k_z^2 V_A^2 k^2$$

$$\boxed{\omega^2 = +\frac{k_y^2}{k^2}N^2 + k_z^2 V_A^2}$$

$N^2 < 0$
$\rightarrow$ inst.

$\rightarrow$ usual R-B. / interchange drive vs. Alfvenic bending criterion.

$\Rightarrow$ here – finite vertical λ_z

$\Rightarrow$ – field line bending $k_z^2 V_A^2$

i.e. magnetic field stabilizing as vertical cell dimension $\Rightarrow k_z \Rightarrow$ bending

$$\rightarrow \underset{\text{B-field}}{V_A^2 k_z^2} \leftrightarrow \underset{\text{rotation}}{\frac{4\Omega^2 k_z^2}{k^2}}$$

(HW)

$V_A^2 \rightarrow \infty \Rightarrow \partial_z = 0$ a/a Taylor-Proudman Thm.