Chapter 30
Nuclear Energy and Elementary Particles

Problem Solutions

30.6 At 40.0% efficiency, the useful energy obtained per fission event is

$$E_{event} = 0.400(200\ \text{MeV/event})(1.60\times10^{-13}\ \text{J/MeV}) = 1.28\times10^{-11}\ \text{J/event}$$

The number of fission events required each day is then

$$N = \frac{\mathcal{P}\cdot t}{E_{event}} = \frac{(1.00\times10^{9}\ \text{J/s})(8.64\times10^{4}\ \text{s/d})}{1.28\times10^{-11}\ \text{J/event}} = 6.75\times10^{24}\ \text{events/d}$$

Each fission event consumes one ^{235}U atom. The mass of this number of ^{235}U atoms is

$$m = N m_{atom}$$

$$= (6.75\times10^{24}\ \text{events/d})\left[(235.044\ \text{u})(1.66\times10^{-27}\ \text{kg/u})\right] = \boxed{2.63\ \text{kg/d}}$$

In contrast, a coal-burning steam plant producing the same electrical power uses more than $6\times10^{6}\ \text{kg/d}$ of coal.

t 1800 miles)

30.8 The estimated *weight* of the naturally occurring uranium is

$$w = \left(1.0\times 10^{9}\text{ tons}\right)\left(\frac{2\,000\text{ lbs}}{1\text{ ton}}\right)\left(\frac{1\text{ N}}{0.224\,8\text{ lbs}}\right) = 8.9\times 10^{12}\text{ N}$$

and its mass is $m = \dfrac{w}{g} = \dfrac{8.9\times 10^{12}\text{ N}}{9.80\text{ m/s}^2} = 9.1\times 10^{11}\text{ kg}$

The total number of uranium nuclei contained in this mass of uranium is

$$N_{total} = \frac{m}{\text{average mass of uranium atom}} = \frac{9.1\times 10^{11}\text{ kg}}{(238.03\text{ u})\left(1.66\times 10^{-27}\text{ kg/u}\right)} = 2.3\times 10^{36}$$

Of this total, 0.720% is the fissionable ^{235}U isotope (see percentage abundance in Appendix B). Assuming all will fission, releasing 208 MeV per event (see statement of Problem 1), the total energy potentially available is

$$E = (208\text{ MeV})N_{235} = (208\text{ MeV})\left(0.720\times 10^{-2}\right)N_{total}$$

$$= (208\text{ MeV})\left(0.720\times 10^{-2}\right)\left(2.3\times 10^{36}\right)\left(1.60\times 10^{-13}\text{ J/MeV}\right) = 5.5\times 10^{23}\text{ J}$$

At a rate of $P = 7.0\times 10^{12}\text{ J/s}$, the time that this energy could supply the world's energy needs is

$$\Delta t = \frac{E}{P} = \frac{5.5\times 10^{23}\text{ J}}{7.0\times 10^{12}\text{ J/s}}\left(\frac{1\text{ yr}}{3.156\times 10^{7}\text{ s}}\right) = \boxed{2.5\times 10^{3}\text{ yr}}$$

30.9 The total energy required for one year is

$$E = \left(2\,000\text{ kWh/month}\right)\left(3.60\times 10^{6}\text{ J/kWh}\right)(12.0\text{ months}) = 8.64\times 10^{10}\text{ J}$$

The number of fission events needed will be

$$N = \frac{E}{E_{event}} = \frac{8.64\times 10^{10}\text{ J}}{(208\text{ MeV})\left(1.60\times 10^{-13}\text{ J/MeV}\right)} = 2.60\times 10^{21}$$

and the mass of this number of ^{235}U atoms is

$$m = N m_{atom} = \left(2.60\times 10^{21}\right)\left[(235.044\text{ u})\left(1.66\times 10^{-27}\text{ kg/u}\right)\right]$$

$$= 1.01\times 10^{-3}\text{ kg} = \boxed{1.01\text{ g}}$$

30.13 The energy released in the reaction ${}_1^2\text{H} + {}_1^3\text{H} \rightarrow {}_2^4\text{He} + {}_0^1\text{n}$ is

$$Q = (\Delta m)c^2 = \left[m_{^2\text{H}} + m_{^3\text{H}} - m_{^4\text{He}} - m_n\right]c^2$$

$$= \left[2.014\,102\text{ u} + 3.016\,049\text{ u} - 4.002\,602\text{ u} - 1.008\,665\text{ u}\right](931.5\text{ MeV/u})$$

$$= 17.6\text{ MeV}\left(1.60\times10^{-13}\text{ J/MeV}\right) = 2.81\times10^{-12}\text{ J}$$

The total energy required for the year is

$$E = (2\,000\text{ kWh/month})(12.0\text{ months/yr})\left(3.60\times10^{6}\text{ J/kWh}\right) = 8.64\times10^{10}\text{ J/yr}$$

so the number of fusion events needed for the year is

$$N = \frac{E}{Q} = \frac{8.64\times10^{10}\text{ J/yr}}{2.81\times10^{-12}\text{ J/event}} = \boxed{3.07\times10^{22}\text{ events/yr}}$$