Chapter 24
Wave Optics

Answers to Even Numbered Conceptual Questions

2. The wavelength of light is extremely small in comparison to the dimensions of your hand, so the diffraction of light around obstacles the size of your hand is totally negligible. However, sound waves have wavelengths that are comparable to the dimensions of the hand or even larger. Therefore, significant diffraction of sound waves occurs around hand sized obstacles.

6. Every color produces its own interference pattern, and we see them superimposed. The central maximum is white. The first maximum is a full spectrum with violet on the inside and red on the outside. The second maximum is also a full spectrum, with red in it overlapping with violet in the third maximum. At larger angles, the light soon starts mixing to white again.

10. The skin on the tip of a finger has a series of closely spaced ridges and swirls on it. When the finger touches a smooth surface, the oils from the skin will be deposited on the surface in the pattern of the closely spaced ridges. The clear spaces between the lines of deposited oil can serve as the slits in a crude diffraction grating and produce a colored spectrum of the light passing through or reflecting from the glass surface.

20. The first experiment. The separation between maxima is inversely proportional to the slit separation (see Eq. 24.5), so increasing the slit separation causes the distance between the two maxima to decrease.

Problem Solutions

24.1 $$\Delta y_{bright} = y_{m+1} - y_m = \frac{\lambda L}{d}(m+1) - \frac{\lambda L}{d}m = \frac{\lambda L}{d}$$

$$= \frac{\left(632.8\times10^{-9}\text{ m}\right)\left(5.00\text{ m}\right)}{0.200\times10^{-3}\text{ m}} = 1.58\times10^{-2}\text{ m} = \boxed{1.58\text{ cm}}$$

24.6 The position of the first order bright fringe for wavelength λ is $y_1 = \dfrac{\lambda L}{d}$

Thus, $$\Delta y_1 = \frac{(\Delta\lambda)L}{d} = \frac{\left[(700-400)\times10^{-9}\text{ m}\right](1.5\text{ m})}{0.30\times10^{-3}\text{ m}} = 1.5\times10^{-3}\text{ m} = \boxed{1.5\text{ mm}}$$

24.15 Light reflecting from the upper surface undergoes phase reversal while that reflecting from the lower surface does not. The condition for constructive interference in the reflected light is then

$$2t - \frac{\lambda_n}{2} = m\lambda_n\text{, or } t = \left(m+\frac{1}{2}\right)\frac{\lambda_n}{2} = \left(m+\frac{1}{2}\right)\frac{\lambda}{2n_{film}}\text{, } m = 0,1,2,\ldots$$

For minimum thickness, $m = 0$ giving

$$t = \frac{\lambda}{4n_{film}} = \frac{500\text{ nm}}{4(1.36)} = \boxed{91.9\text{ nm}}$$

24.18 Since $n_{air} < n_{oil} < n_{water}$, light reflected from both top and bottom surfaces of the oil film experiences phase reversal, resulting in zero net phase difference due to reflections. Therefore, the condition for constructive interference in reflected light is

$$2t = m\lambda_n = m\frac{\lambda}{n_{film}}\text{, or } t = m\left(\frac{\lambda}{2n_{film}}\right) \text{ where } m = 0,1,2,\ldots$$

Assuming that $m = 1$, the thickness of the oil slick is

$$t = (1)\frac{\lambda}{2n_{film}} = \frac{600\text{ nm}}{2(1.29)} = \boxed{233\text{ nm}}$$

24.20 The transmitted light is brightest when the reflected light is a minimum (that is, the same conditions that produce destructive interference in the reflected light will produce constructive interference in the transmitted light). As light enters the air layer from glass, any light reflected at this surface has zero phase change. Light reflected from the other surface of the air layer (where light is going from air into glass) does have a phase reversal. Thus, the condition for destructive interference in the light reflected from the air film is $2t = m\lambda_n$, $m = 0,1,2,\ldots$

Since $\lambda_n = \dfrac{\lambda}{n_{film}} = \dfrac{\lambda}{1.00} = \lambda$, the minimum non-zero plate separation satisfying this condition is $$d = t = (1)\frac{\lambda}{2} = \frac{580\text{ nm}}{2} = \boxed{290\text{ nm}}$$

24.27 There is a phase reversal upon reflection at each surface of the film and hence zero net phase difference due to reflections. The requirement for constructive interference in the reflected light is then

$$2t = m\lambda_n = m\frac{\lambda}{n_{film}}, \text{ where } m = 1, 2, 3, \ldots$$

With $t = 1.00 \times 10^{-5}\text{ cm} = 100\text{ nm}$, and $n_{film} = 1.38$, the wavelengths intensified in the reflected light are

$$\lambda = \frac{2n_{film}t}{m} = \frac{2(1.38)(100\text{ nm})}{m}, \text{ with } m = 1, 2, 3, \ldots$$

Thus, $\lambda = \boxed{276\text{ nm}, 138\text{ nm}, 92.0\text{ nm} \ldots}$

and $\boxed{\text{none of these wavelengths are in the visible spectrum}}$

24.29 The distance on the screen from the center to either edge of the central maximum is

$$y = L\tan\theta \approx L\sin\theta = L\left(\frac{\lambda}{a}\right)$$

$$= (1.00\text{ m})\left(\frac{632.8 \times 10^{-9}\text{ m}}{0.300 \times 10^{-3}\text{ m}}\right) = 2.11 \times 10^{-3}\text{ m} = 2.11\text{ mm}$$

The full width of the central maximum on the screen is then

$$2y = \boxed{4.22\text{ mm}}$$

24.36 (a) The longest wavelength in the visible spectrum is 700 nm, and the grating spacing is $d = \frac{1\text{ mm}}{600} = 1.67 \times 10^{-3}\text{ mm} = 1.67 \times 10^{-6}\text{ m}$

Thus, $m_{max} = \frac{d\sin 90.0°}{\lambda_{red}} = \frac{(1.67 \times 10^{-6}\text{ m})\sin 90.0°}{700 \times 10^{-9}\text{ m}} = 2.38$

so $\boxed{\text{2 complete orders}}$ will be observed.

(b) From $\lambda = d\sin\theta$, the angular separation of the red and violet edges in the first order will be

$$\Delta\theta = \sin^{-1}\left[\frac{\lambda_{red}}{d}\right] - \sin^{-1}\left[\frac{\lambda_{violet}}{d}\right] = \sin^{-1}\left[\frac{700\times10^{-9}\text{ m}}{1.67\times10^{-6}\text{ m}}\right] - \sin^{-1}\left[\frac{400\times10^{-9}\text{ m}}{1.67\times10^{-6}\text{ m}}\right]$$

or $\Delta\theta = \boxed{10.9°}$

24.40 With 2 000 lines per centimeter, the grating spacing is

$$d = \frac{1}{2\,000}\text{ cm} = 5.00\times10^{-4}\text{ cm} = 5.00\times10^{-6}\text{ m}$$

Then, from $d\sin\theta = m\lambda$, the location of the first order for the red light is

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{(1)\left(640\times10^{-9}\text{ m}\right)}{5.00\times10^{-6}\text{ m}}\right] = \boxed{7.35°}$$

24.45 (a) From Brewster's law, the index of refraction is

$$n_2 = \tan\theta_p = \tan(48.0°) = \boxed{1.11}$$

(b) From Snell's law, $n_2\sin\theta_2 = n_1\sin\theta_1$, we obtain when $\theta_1 = \theta_p$

$$\theta_2 = \sin^{-1}\left(\frac{n_1\sin\theta_p}{n_2}\right) = \sin^{-1}\left(\frac{(1.00)\sin 48.0°}{1.11}\right) = \boxed{42.0°}$$

Note that when $\theta_1 = \theta_p$, $\theta_2 = 90.0° - \theta_p$ as it should.

24.46 Unpolarized light incident on a polarizer contains electric field vectors at all angles to the transmission axis of the polarizer. Malus's law then gives the intensity of the transmitted light as $I = I_0\left(\cos^2\theta\right)_{av}$. Since the average value of $\cos^2\theta$ is $1/2$, the intensity of the light passed by the first polarizer is $I_1 = I_0/2$, where I_0 is the incident intensity.

Then, from Malus's law, the intensity passed by the second polarizer is

$$I_2 = I_1\cos^2(30.0°) = \left(\frac{I_0}{2}\right)\left(\frac{3}{4}\right), \text{ or } \frac{I_2}{I_0} = \boxed{\frac{3}{8}}$$