Chapter 23

Mirrors and Lenses

Answers to Even Numbered Conceptual Questions

20. (d). The entire image would appear because any portion of the lens can form the image. The image would be dimmer because the card reduces the light intensity on the screen by 50%.

Problem Solutions

23.3 (1) The first image in the left-hand mirror is 5.00 ft behind the mirror, or $\boxed{10.0 \text{ ft from the person}}$

(2) The first image in the right-hand mirror serves as an object for the left-hand mirror. It is located 10.0 ft behind the right-hand mirror, which is 25.0 ft from the left-hand mirror. Thus, the second image in the left-hand mirror is 25.0 ft behind the mirror, or $\boxed{30.0 \text{ ft from the person}}$

(3) The first image in the left-hand mirror serves as an object for the right-hand mirror. It is located 20.0 ft in front of the right-hand mirror and forms an image 20.0 ft behind that mirror. This image then serves as an object for the left-hand mirror. The distance from this object to the left-hand mirror is 35.0 ft. Thus, the third image in the left-hand mirror is 35.0 ft behind the mirror, or $\boxed{40.0 \text{ ft from the person}}$

23.5 Since the mirror is convex, $R < 0$. Thus, $R = -0.550 \text{ m}$. With a real object, $p > 0$, so $p = +10.0 \text{ m}$. The mirror equation then gives the image distance as

$$\frac{1}{q} = \frac{2}{R} - \frac{1}{p} = \frac{2}{-0.550 \text{ m}} - \frac{1}{10.0 \text{ m}}, \text{ or } q = -0.268 \text{ m}$$

Thus, the image is $\boxed{\text{virtual}}$ and located $\boxed{0.268 \text{ m behind the mirror}}$

The magnification is $M = -\dfrac{q}{p} = -\dfrac{-0.268 \text{ m}}{10.0 \text{ m}} = \boxed{0.0268}$

Therefore, the image is $\boxed{\text{upright}}$ $(\text{since } M > 0)$ and $\boxed{\text{diminished in size}}$ $(\text{since } |M| < 1)$

23.7 The radius of curvature of a concave mirror is positive, so $R = +20.0\text{ cm}$. The mirror equation then gives

$$\frac{1}{q} = \frac{2}{R} - \frac{1}{p} = \frac{1}{10.0\text{ cm}} - \frac{1}{p} = \frac{p - 10.0\text{ cm}}{(10.0\text{ cm})p}, \text{ or } q = \frac{(10.0\text{ cm})p}{p - 10.0\text{ cm}}$$

(a) If $p = 40.0\text{ cm}$, $q = +13.3\text{ cm}$ and $M = -\dfrac{q}{p} = -\dfrac{13.3\text{ cm}}{40.0\text{ cm}} = \boxed{-0.333}$

The image is $\boxed{\text{13.3 cm in front of the mirror, real, and inverted}}$

(b) When $p = 20.0\text{ cm}$, $q = +20.0\text{ cm}$ and $M = -\dfrac{q}{p} = -\dfrac{20.0\text{ cm}}{20.0\text{ cm}} = \boxed{-1.00}$

The image is $\boxed{\text{20.0 cm in front of the mirror, real, and inverted}}$

(c) If $p = 10.0\text{ cm}$, $q = \dfrac{(10.0\text{ cm})(10.0\text{ cm})}{10.0\text{ cm} - 10.0\text{ cm}} \to \infty$

and $\boxed{\text{no image is formed. Parallel rays leave the mirror}}$

23.13 The image is upright, so $M > 0$, and we have

$$M = -\frac{q}{p} = +2.0, \text{ or } q = -2.0p = -2.0(25\text{ cm}) = -50\text{ cm}$$

The radius of curvature is then found to be

$$\frac{2}{R} = \frac{1}{p} + \frac{1}{q} = \frac{1}{25\text{ cm}} - \frac{1}{50\text{ cm}} = \frac{2-1}{50\text{ cm}}, \text{ or } R = 2\left(\frac{0.50\text{ m}}{+1}\right) = \boxed{1.0\text{ m}}$$

23.16 A convex mirror $(R < 0)$ produces upright, virtual images of real objects.

Thus, $M > 0$ giving $M = -\dfrac{q}{p} = +\dfrac{1}{3}$, or $q = -\dfrac{p}{3}$

Then, $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{2}{R}$ becomes $\dfrac{1}{p} - \dfrac{3}{p} = -\dfrac{2}{10.0\text{ cm}}$, and yields $p = +10.0\text{ cm}$

The object is $\boxed{\text{10.0 cm in front of the mirror}}$

23.21 From $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$, with $R\to\infty$, the image position is found to be

$$q=-\frac{n_2}{n_1}p=-\left(\frac{1.00}{1.309}\right)(50.0\text{ cm})=-38.2\text{ cm}$$

or the virtual image is $\boxed{\text{38.2 cm below the upper surface of the ice}}$

23.29 From the thin lens equation, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, the image distance is found to be

$$q=\frac{fp}{p-f}=\frac{(20.0\text{ cm})p}{p-20.0\text{ cm}}$$

(a) If $p=40.0\text{ cm}$, then $q=40.0\text{ cm}$ and $M=-\frac{q}{p}=-\frac{40.0\text{ cm}}{40.0\text{ cm}}=\boxed{-1.00}$

The image is $\boxed{\text{real, inverted, and 40.0 cm beyond the lens}}$

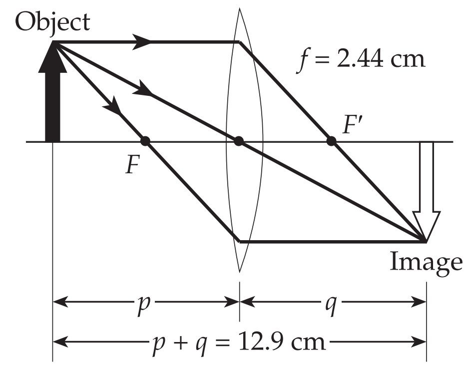
(b) If $p=20.0\text{ cm}$, $q\to\infty$ $\boxed{\text{No image formed. Parallel rays leave the lens.}}$

(c) When $p=10.0\text{ cm}$, $q=-20.0\text{ cm}$ and

$$M=-\frac{q}{p}=-\frac{(-20.0\text{ cm})}{10.0\text{ cm}}=\boxed{+2.00}$$

The image is $\boxed{\text{virtual, upright, and 20.0 cm in front of the lens}}$

23.33 (a) The real image case is shown in the ray diagram. Notice that $p+q=12.9\text{ cm}$, or $q=12.9\text{ cm}-p$. The thin lens equation, with $f=2.44\text{ cm}$, then gives



$$\frac{1}{p}+\frac{1}{12.9\text{ cm}-p}=\frac{1}{2.44\text{ cm}}$$

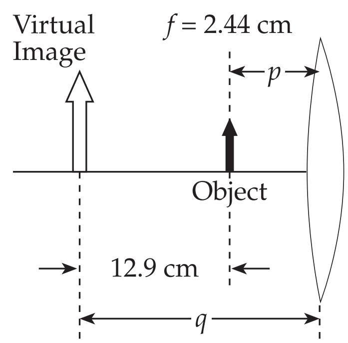
or $p^2-(12.9\text{ cm})p+31.5\text{ cm}^2=0$

Using the quadratic formula to solve gives

$$\boxed{p=9.63\text{ cm} \text{ or } p=3.27\text{ cm}}$$

Both are valid solutions for the real image case.

(b) The virtual image case is shown in the second diagram. Note that in this case, $q=-(12.9\text{ cm}+p)$, so the thin lens equation gives



$$\frac{1}{p}-\frac{1}{12.9\text{ cm}+p}=\frac{1}{2.44\text{ cm}}$$

or $p^2+(12.9\text{ cm})p-31.5\text{ cm}^2=0$

The quadratic formula then gives $p=2.10\text{ cm}$ or $p=-15.0\text{ cm}$

Since the object is real, the negative solution must be rejected leaving $\boxed{p=2.10\text{ cm}}$.

23.35 It is desired to form a magnified, real image on the screen using a single thin lens. To do this, a converging lens must be used and the image will be inverted. The magnification then gives

$$M=\frac{h'}{h}=\frac{-1.80\text{ m}}{24.0\times10^{-3}\text{ m}}=-\frac{q}{p}\text{, or } q=75.0p$$

Also, we know that $p+q=3.00\text{ m}$. Therefore, $p+75.0p=3.00\text{ m}$ giving

(b) $p=\dfrac{3.00\text{ m}}{76.0}=3.95\times10^{-2}\text{ m}=\boxed{39.5\text{ mm}}$

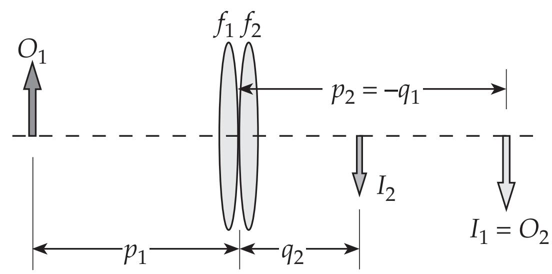
(a) The thin lens equation then gives $\frac{1}{p}+\frac{1}{75.0p}=\frac{76.0}{75.0p}=\frac{1}{f}$

or $f=\left(\frac{75.0}{76.0}\right)p=\left(\frac{75.0}{76.0}\right)(39.5\text{ mm})=\boxed{39.0\text{ mm}}$

23.39 Since the light rays incident to the first lens are parallel, $p_1=\infty$ and the thin lens equation gives $q_1=f_1=-10.0\text{ cm}$.

The virtual image formed by the first lens serves as the object for the second lens, so $p_2=30.0\text{ cm}+|q_1|=40.0\text{ cm}$. If the light rays leaving the second lens are parallel, then $q_2=\infty$ and the thin lens equation gives $f_2=p_2=\boxed{40.0\text{ cm}}$.

23.46 Consider an object O_1 at distance p_1 in front of the first lens. The thin lens equation gives the image position for this lens as $\frac{1}{q_1}=\frac{1}{f_1}-\frac{1}{p_1}$.



The image, I_1, formed by the first lens serves as the object, O_2, for the second lens. With the lenses in contact, this will be a virtual object if I_1 is real and will be a real object if I_1 is virtual. In either case, if the thicknesses of the lenses may be ignored,

$$p_2=-q_1 \text{ and } \frac{1}{p_2}=-\frac{1}{q_1}=-\frac{1}{f_1}+\frac{1}{p_1}$$

Applying the thin lens equation to the second lens, $\frac{1}{p_2}+\frac{1}{q_2}=\frac{1}{f_2}$ becomes

$$-\frac{1}{f_1}+\frac{1}{p_1}+\frac{1}{q_2}=\frac{1}{f_2} \text{ or } \frac{1}{p_1}+\frac{1}{q_2}=\frac{1}{f_1}+\frac{1}{f_2}$$

Observe that this result is a thin lens type equation relating the position of the original object O_1 and the position of the final image I_2 formed by this two lens combination. Thus, we see that we may treat two thin lenses in contact as a single lens having a focal length, f, given by $\boxed{\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}}$