ANSWER KEY

Physics 1B(b) Final Exam

Winter 2010

Version A

Problem 1: A parallel plate capacitor (dimensions 1 cm × 1 m) is designed to store energy. Putting a dielectric into the capacitor makes a huge difference to the amount of energy I can store. If I have a dielectric with a dielectric constant of k = 100 and the plates are spaced only 1 micron (10^{-6} m), how much energy can I store with 100 Volts across the capacitor?

(a) 4.43×10^{-2} J

(b) 8.85×10^{-2} J

(c) 4.43×10^{-4} J

(d) 4.43 J

$$C = k\frac{\varepsilon_0 A}{d} = 100\,\frac{(8.85\times10^{-12}\ C^2/N\cdot m^2)(10^{-2}\,m \times 1\,m)}{10^{-6}\ m}$$

$$= 8.85\times10^{-6}\ F$$

$$E = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(8.85\times10^{-6}\ F)(10^2\ V)^2$$

$$= \boxed{4.43\times10^{-2}\ J}$$

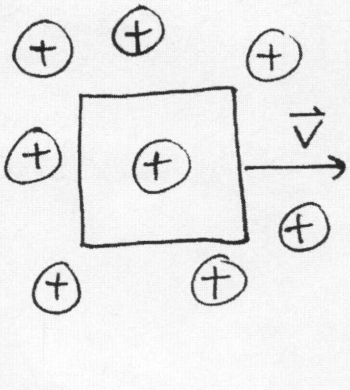
Problem 2: I have a square copper loop that is moving in an applied magnetic field B with velocity v. Which direction is the current in the loop?

(a) clockwise

(b) counterclockwise

(c) zero

No change in B or A. Therefore, $\frac{d\phi}{dt} = 0$ and EMF = 0.



Version A

Problem 3: This is a hollow metal sphere. The sphere is charged with a charge of 1.0×10^{-9} Coulombs. What is the electric field just inside the inner surface (assume I mean at 0.9 cm.)?

(a) 1.11×10^{3} N/C

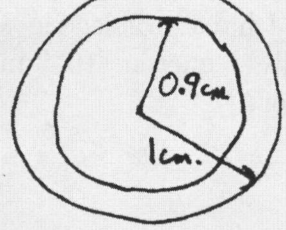
(b) 8.99×10^{4} N/C

(c) 1.11×10^{5} N/C

(e) 0 N/C

By Gauss' Law, $EA = \frac{Q_{enc}}{\varepsilon_0}$

$Q_{enc} = 0 \Rightarrow E = 0$.



Problem 4: I have a charge of 1.0×10^{-6} Coulombs inside a cube of one meter on each side. What is the electric field flux coming out of each of the faces of the cube?

(a) $1.13 \times 10^{5}\ Nm^2/C$

(b) $1.88 \times 10^{4}\ Nm^2/C$

(c) $2.26 \times 10^{5}\ Nm^2/C$

(d) None of these

$$\phi_E = \frac{Q_{enc}}{\varepsilon_0} = \frac{10^{-6}C}{8.85\times10^{-12}\frac{C^2}{N\cdot m^2}}$$

$$= 1.13\times10^{5}\ \frac{Nm^2}{C}$$

This flux is divided equally among the six faces of the cube.

$\Rightarrow \phi_E$ per face $= 1.88\times10^{4}\ \frac{Nm^2}{C}$

Version A

Problem 5: I have stored energy in a capacitor of $10^{-}6$ F that has 100 V applied across it. How much current do I need in a 1 Henry inductor to store the same ~~about~~ amount of energy?

(a) 0.707 A
(b) 0.10 A
(c) 1.41 A
(d) 0.01 A

$$E_{cap} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(10^{-6}\,F)(10^2\,V)^2$$
$$= \tfrac{1}{2}\times 10^{-2}\,J$$

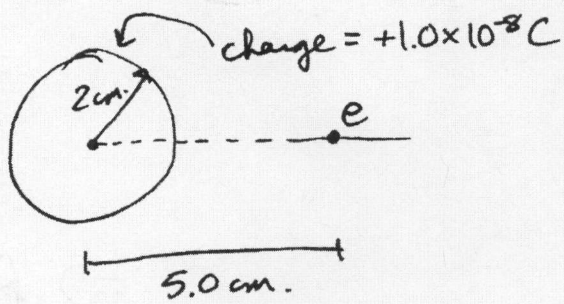
$$E_{cap} = E_{ind} = \tfrac{1}{2}LI^2 = \tfrac{1}{2}I^2$$

$$\Rightarrow \cancel{\tfrac{1}{2}}I^2 = \cancel{\tfrac{1}{2}}\times 10^{-2}$$

$$\Rightarrow \boxed{I = 0.10\,A}$$

Problem 6: I have a metallic sphere of radius 2.0 cm. I charge the sphere with 1.0×10^{-8} C. I release an electron 5.0 cm. away from the center of the sphere. What is the velocity of the electron when it hits the sphere?

(a) $9.48 \times 10^{14}\ m/s$
(b) $2.18 \times 10^{7}\ m/s$
(c) $3.97 \times 10^{7}\ m/s$
(d) $3.08 \times 10^{7}\ m/s$



$$\Delta PE = -q\Delta V = \Delta KE = \tfrac{1}{2}mv^2$$

$$\Rightarrow q_1\left[-kq_2\left(\frac{1}{.05m} - \frac{1}{.02m}\right)\right] = \tfrac{1}{2}mv^2$$

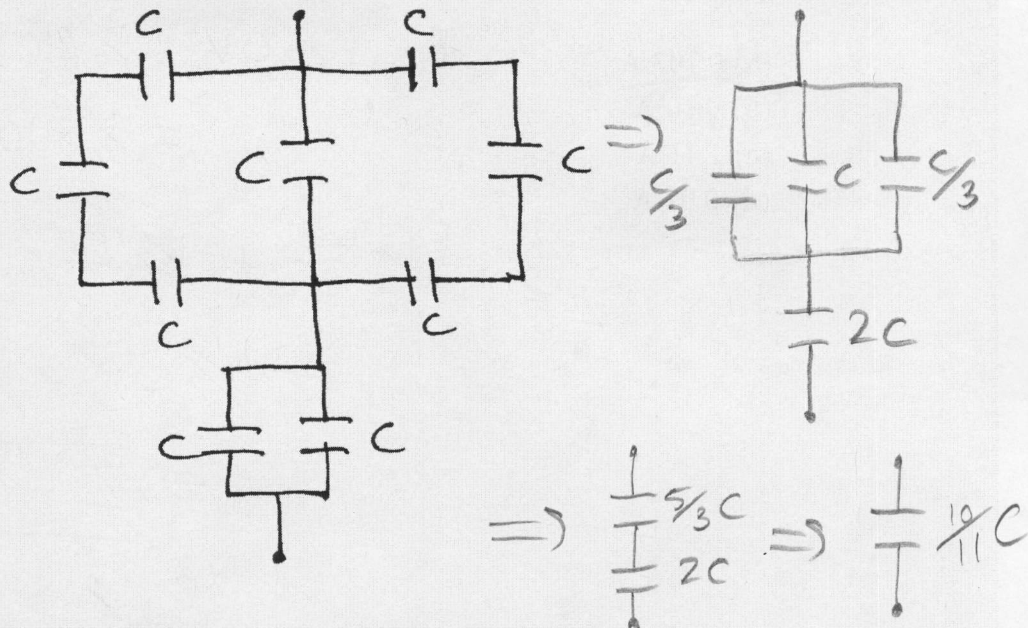
$$\Rightarrow v = \sqrt{\frac{2(8.99\times10^9\,\frac{Nm^2}{C^2})(10^{-8}\,C)(1.6\times10^{-19}\,C)(30\,\tfrac{1}{m})}{9.11\times10^{-31}\,kg.}}$$

$$= 3.08\times10^7\ m/s$$

Version A

Problem 7: What is the equivalent capacitance of the following circuit? The capacitors are all have the same capacitance "C"?

(a) 3.67 C
(b) 9.0 C
(c) 0.91 C
(d) 1.0 C



Problem 8: I have two light bulbs that are 100 W and 75 W. To get the most light out of the pair of them, I should

(a) connect them in series
(b) connect them in parallel
(c) doesn't matter

$P = I^2 R$

Because P is fixed, the smaller R is, the greater I and the light emitted by the bulbs will be.

$\Rightarrow R_{eq, \text{parallel}} < R_{eq, \text{series}}$

Version A

① $I_1 + I_2 = I_3 \Rightarrow 2A + I_2 = I_3$

② $\mathcal{E} - (2A)(1\Omega) - (2A)(3\Omega) - (5\Omega)(I_3) - (1\Omega)(I_3) + 8V = 0$

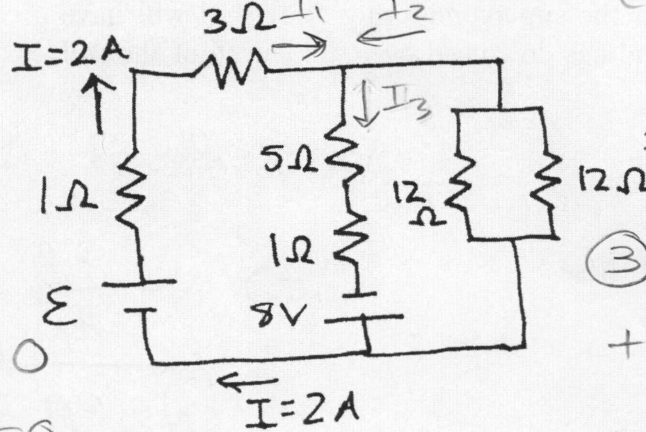
$\Rightarrow \mathcal{E} - (6\Omega)I_3 = 0$

③ $\mathcal{E} - (2A)(1\Omega) - (2A)(3\Omega) + I_2\left(\frac{144\Omega^2}{24\Omega}\right) = 0$

$\Rightarrow \mathcal{E} - 8V + (6\Omega)I_2 = 0$

Problem 9: The emf of the battery is

(a) 10.0 V
(b) 10.7 V
(c) 8.3 V
(d) 12.0 V



$\Rightarrow \mathcal{E} - (6\Omega)(2A + I_2) = 0$

$\Rightarrow \mathcal{E} - 12V - (6\Omega)\cancel{I_2} = 0$

$+ (\mathcal{E} - 8V + \cancel{6\Omega I_2}) = 0$

$\Rightarrow \boxed{\mathcal{E} = 10\,V}$

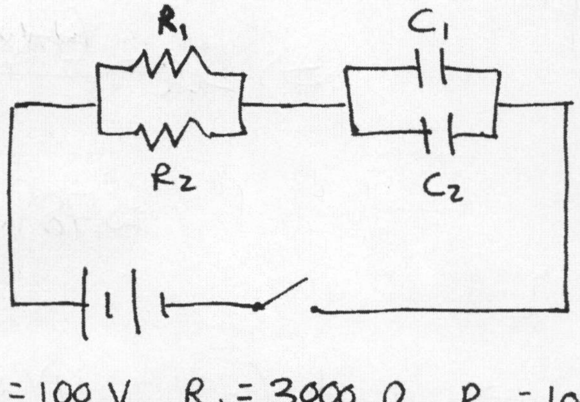
Problem 10: What is the time constant for this circuit, and what is the final charge on each capacitor after a long time?

(a) $\tau = 3.0 \times 10^{-6}$ s, $q_1 = 4.0 \times 10^{-7}$ C, $q_2 = 4.0 \times 10^{-7}$ C
(b) $\tau = 3.0 \times 10^{-6}$ s, $q_1 = 3.0 \times 10^{-7}$ C, $q_2 = 1.0 \times 10^{-7}$ C
(c) $\tau = 1.0 \times 10^{-6}$ s, $q_1 = 3.0 \times 10^{-7}$ C, $q_2 = 1.0 \times 10^{-7}$ C
(d) $\tau = 3.0 \times 10^{-6}$ s, $q_1 = 4.0 \times 10^{-7}$ C, $q_2 = 4.0 \times 10^{-7}$ C

$C_{eq} = 4 \times 10^{-9}$ F

$R_{eq} = \frac{3 \times 10^6 \Omega^2}{4000\,\Omega} = \frac{3}{4} \times 10^3\,\Omega$

$\Rightarrow \tau = 3 \times 10^{-6}$ s $= RC$



V = 100 V, $R_1 = 3000\,\Omega$, $R_2 = 1000\,\Omega$

$C_1 = 3 \times 10^{-9}$ F, $C_2 = 1 \times 10^{-9}$ F.

Voltage across capacitors is the same.

$C = Q/V$

$\Rightarrow$

$Q_1 = C_1 V = 3 \times 10^{-7}$ C

$Q_2 = C_2 V = 1 \times 10^{-7}$ C

Version A

Problem 11: We will design a magnet (solenoid) to do Functional Magnetic Resonance Imaging (FMRI). We want 2.5 Tesla with a length of 2 meters. The wire will carry 100 A in the superconducting state and will have a core diameter of 1 meter. How many windings do I need over the length of the coil?

(a) 39,789

(b) 19,894

(c) 79,577

(d) none of these

For solenoid, $B = \mu_0 n I = \mu_0 \frac{N}{\ell} I$

$$\Rightarrow N = \frac{B\ell}{\mu_0 I} = \frac{(2.5\,T)(2\,m)}{\mu_0 (100\,A)}$$

$$\boxed{N \approx 39{,}789}$$

Problem 12: What is the energy stored in the magnet in the previous question when it is carrying is full current of 100 A?

(a) 3.91×10^4 J

(b) 7.81×10^4 J

(c) 3.91×10^6 J

(d) 1.95×10^4 J

$W = \frac{1}{2} L I^2 \qquad L_{solenoid} = \frac{\mu_0 N^2 A}{\ell}$

$$\Rightarrow L_{sol} = \frac{(4\pi \times 10^{-7}\,\frac{Wb}{Am})(39{,}789)^2(\pi(.05\,m)^2)}{2\,m}$$

$$\approx 781.26\,H$$

$$\Rightarrow W = \tfrac{1}{2}(781.26\,H)(100\,A)^2$$

$$\Rightarrow W \approx 3.91 \times 10^6\,J$$

Version A

Problem 13: An alpha particle (a helium atom missing two electrons, $^4He^{++}$) is accelerated in a Van de Graaff accelerator up to an energy of 2 MeV (2×10^6 eV). It is fired into a region with a magnetic field B = 10 T. The alpha particle has a mass of 2 protons and 2 neutrons. What is the radius of the cyclotron orbit, and which way will the particle rotate?

$E = \frac{1}{2}mv^2 \Rightarrow 2mE = m^2v^2$

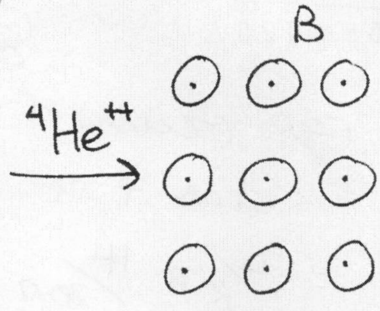
$\Rightarrow mv = \sqrt{2mE}$

(a) 8.38×10^{-4} m, clockwise

Sorry! (b) 1.44×10^{-2} m, clockwise

(c) 1.44×10^{-2} m, counter-clockwise

off by $\sqrt{2}$ (d) 1.70×10^{-1} m, counter-clockwise



$r = \frac{mv}{qB} = \frac{\sqrt{2mE}}{qB}$

$\Rightarrow r = \frac{\sqrt{2 \cdot 4 \cdot (1.67 \times 10^{-27}\ kg)(2 \times 10^6\ eV)(1.6 \times 10^{-19}\ J/eV)}}{2 \cdot 1.6 \times 10^{-19} C \cdot 10\ T}$

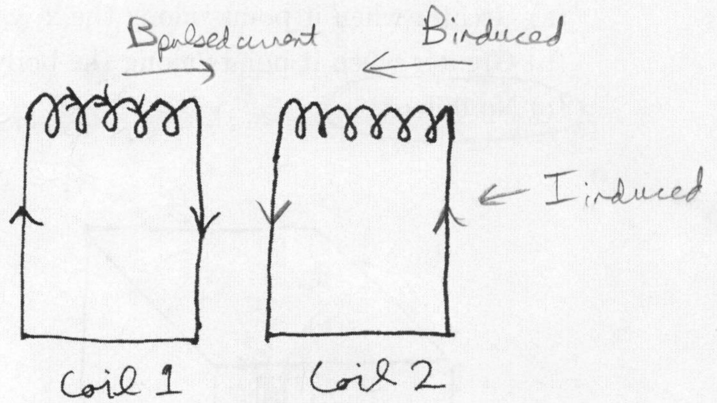
$\Rightarrow \boxed{r = 2.04 \times 10^{-2}\ m,\ \text{clockwise}}$

Problem 14: I pulse a current in the direction indicated in coil 1. Which direction will the current in coil 2 be induced?

(a) clockwise

(b) counter-clockwise

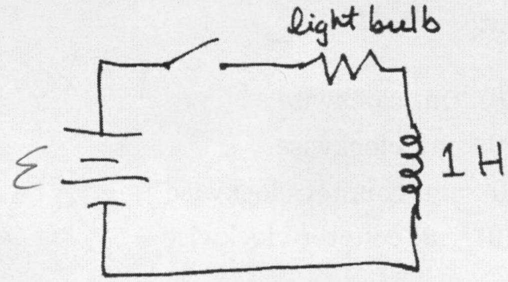
(c) no current in coils



Version A

Problem 15: The switch is closed, and the light bulb glows. The light bulb resistance is 20 Ohms and the battery is 50 V. What is the time for the current to increase 63.2 percent of its maximum value? What is the maximum value?

(a) 0.05 s and 0.4 A
(b) 20 s and 4 A
(c) 0.05 s and 2.5 A
(d) 0.5 s and 2.5 A



$\frac{I}{I_{max}} = .632$ occurs when $t = \tau$

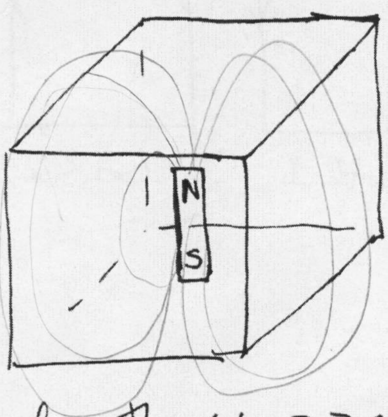
$\tau = L/R = 1H/20\Omega = 0.05s.$

Voltage drop after a long time : $V - L\frac{\Delta I}{\Delta t}^{\to 0} - IR = 0 \Rightarrow V = IR$
$\Rightarrow I = 2.5A$

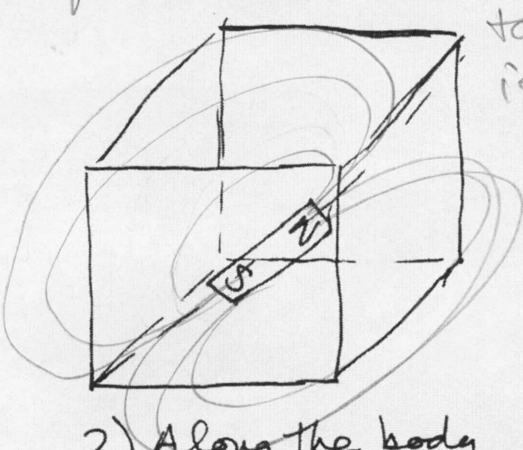
Problem 16: I have a bar magnet enclosed in a cubic box. I hold it in two different orientations in the center of the box as shown. The magnetic flux Φ_B emerging from the entire box

(a) Greater when it points along the x, y, or z direction
(b) Greater when it points along the body diagonal
(c) No difference

In both cases, all magnetic lines that leave the surface enter back in. Therefore, total flux is zero is both cases.



1) along the x, y, or z axis.



2) Along the body diagonal.