THE ASTROPHYSICAL JOURNAL, 178:623–666, 1972 December 15

GALACTIC BRIDGES AND TAILS

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Received 1972 May 19

ABSTRACT

This paper argues that the bridges and tails seen in some multiple galaxies are just tidal relics of close encounters. These consequences of the brief but violent tidal forces are here studied in a deliberately simple-minded fashion: Each encounter is considered to involve only two galaxies and to be roughly parabolic; each galaxy is idealized as just a disk of noninteracting test particles which initially orbit a central mass point.

As shown here, the two-sided distortions provoked by gravity alone in such circumstances can indeed evolve kinematically into some remarkably narrow and elongated features: (i) After a relatively direct passage of a *small* companion, the outer portions of the primary disk often deform both into a near-side spiral arm or "bridge" extending toward this satellite, and into a far-side "counterarm." (ii) A similar encounter with an *equal* or more massive partner results typically in a long and curving "tail" of escaping debris from the far side of the victim disk, and in an avalanche of near-side particles, most of which are captured by the satellite.

Besides extensive pictorial surveys of such tidal damage, this paper offers reconstructions of the orbits and *outer* shapes of four specific interacting pairs: Arp 295, M51 + NGC 5195, NGC 4676, and NGC 4038/9. Those models can be found in the fairly self-explanatory figures 19, 21, 22, and 23.

Also discussed are some closely related issues of eccentric bound orbits, orbital decay, accretion, and forced spiral waves.

I. INTRODUCTION

When galaxies appear to be neighbors and at least one looks peculiar, *tidal* interactions are often suspected and sometimes even blamed. Yet occasionally these temptations have been resisted with considerable vigor. Such exceptions have included (i) various bridgelike situations where a pronounced spiral arm of a large galaxy extends to the vicinity of a lesser companion, and (ii) the long, usually curving, faintly luminous "tails" which accompany certain of the multiple galaxies. To be sure, even those features, first studied extensively by Zwicky (1953, 1956, 1959), have often been described as belonging to systems "in obvious interaction." But judging from the reservations admitted even by Zwicky (1963, 1967) despite his former use of words like "countertide" and "tidal extensions"—and especially from the vehement doubts expressed by Vorontsov-Velyaminov (1958, 1961, 1964), Gold and Hoyle (1959), Burbidge, Burbidge, and Hoyle (1963), Pikel'ner (1965, 1968), Zasov (1967), and most recently by Arp (1966, 1969*a*, *b*, 1971*b*)—it has usually seemed much less obvious that the basis of also such interactions could be simply the old-fashioned gravity.

As stressed in most of those articles, and by the pictures in Vorontsov-Velyaminov's (1959) *Atlas and Catalog of Interacting Galaxies* or in Arp's (1966) *Atlas of Peculiar*

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Galaxies, these filamentary connections and appendages come in a bewildering variety. Any division into categories is far from clear-cut. But surely a classic example of kind (i) is the "bridge" which appears to link M51 with NGC 5195; other prototypes of sorts are NGC 2535/6 (= No. 82 in Arp's Atlas), NGC 3808 (= Arp 87), NGC 7753/2 (= Arp 86), and Zwicky's pair of galaxies pictured as Arp 295. Those pictures remind us also that the bridge phenomenon very often involves a distinct "counterarm" as well. Relatively clean examples of kind (ii) are Arp 100, 101, 173, and 174 showing only single tails, and NGC 4676, 2623, 4038/9, and 2992/3 (= Arp 242–245, respectively) where one tail seems to emanate from each participating galaxy. It is probably because we have grown accustomed to regarding tidal forces as relatively static and spatially gradual that there has been such widespread reluctance to believe that features as *elongated* and as *narrow* as these could have resulted merely from the action of gravity.

In this paper we wish to emphasize that such intuition—that tides cause only broad features—is remarkably mistaken: The outer parts of an initially disk-shaped galaxy can indeed be deformed, by the transient but intense tidal forces arising during the close and roughly parabolic passage of a satellite galaxy, to resemble a narrow bridge and counterarm, or even a long slender tail.

Even so, this paper only expands on certain existing work. For one thing, we ourselves have already published a brief example (Toomre 1970) of such bridge-building by a small satellite and have exhibited in a movie (Toomre and Toomre 1970) both that process and the tail-making that occurs during the passage of an equally massive companion. In a second movie (Toomre and Toomre 1971) we simulated a tidal encounter which results in a pair of thin, crossed tails not unlike those of NGC 4038/9. Some of those findings have now been corroborated by Wright (1972) in an article which also rebuts explicitly many of the reasons why Vorontsov-Velyaminov (1961) held tidal explanations to be untenable. And similarly, Yabushita (1971) has recently shown that certain tidal bridges can result even from moderately hyperbolic encounters.

Prior to all that, Tashpulatov (1969, 1970) reported that gravitational extraction of particles by a passing galaxy from a *single* spot on a prolate ellipsoid representing a second galaxy produces tidal bridges and tails. Although our general conclusions indeed resemble his, our manner of building the bridges seems more realistic, and our tail-forming process differs (cf. § V*a*) categorically from his.

But most important, much of what we are about to describe—except perhaps for the tail-building and the importance of very eccentric *bound* companions, especially ones of small mass—was in principle already appreciated by Pfleiderer and Siedentopf (1961) and by Pfleiderer (1963). It is indeed something of a puzzle why these last two papers were as widely overlooked or underestimated as they seem to have been. Apart from being in German, perhaps it is because they dealt only with test particles, and were addressed chiefly to the question of whether the spiral phenomenon in general could be due to interactions with passing field galaxies (a question which Pfleiderer finally answered in the negative, on the reasonable ground that sufficiently close passages of such unbound galaxies seem much too infrequent).

Our present demonstrations will actually share the foremost of those flaws with Pfleiderer and Siedentopf: Like their examples, ours will be based exclusively on restricted three-body computations performed with massless particles which we pretend constitute the outer disks of pairwise interacting galaxies. By supposing these elements of either disk to move simply under inverse-square forces from the two mass points representing the bulk of each galaxy, we too will ignore all explicit self-gravity of the disk material (except in certain final estimates). Obviously this is an important sin of omission, and it is one which ought soon to be remedied, perhaps via some proper N-body calculations. In the meantime we rationalize it on three grounds: One is computational economy, perhaps by a factor of 100. Another is that the self-attraction of the *outer* parts of any galaxy is by definition small, and hence unlikely to alter our qualitative conclusions. But above all we hope that our continuing reliance on these mere test particles will help make it clear that the development—and the eventual dissolution—of these narrow, filamentary structures is in essence *kinematic*.

Owing to the length of this paper, a first-time reader should probably concentrate on §§ II, V, and VII. Those sections offer four simple but instructive examples, a theoretical summary as well as some comparison with observations, and finally speculations on the broader issues involved. One of those issues is that the very close (and relatively slow) encounters that we are about to postulate may intuitively seem highly *improbable*; anyone strongly of that opinion is urged to turn to § VII*a* immediately. The largely pictorial bridge and tail surveys of the remaining §§ III and IV, and even the object reconstructions in § VI, can await later reading.

II. FOUR ELEMENTARY EXAMPLES

The gist of this story lies in the four examples which we are about to describe. All four refer to passages that are exactly *parabolic*. In each example, one of the two point masses arrives at the scene of the encounter surrounded by a flat, annular disk of 120 test particles; the other arrives bare. More exactly, the unperturbed disk consists of five discrete rings, of 12, 18, 24, 30, and 36 particles apiece. Long before the encounter —to be exact, at $t = -10$ in the time units used below—these particles in the first three examples were set to orbit one mass point with uniform angular spacing at exactly 20, 30, 40, 50, and 60 percent of the eventual pericenter distance $R_{\min}$; in the fourth example, they were placed into circular orbits at 12, 18, 24, 30, and 36 percent of $R_{\min}$. In each of these simple examples, the orbit plane of the second mass coincides with that of the test particles, and hence all the action remains in that plane.

For convenience, we shall throughout this paper reckon the time t from the perigalactic instant $t = 0$ in multiples of exactly 10^8 years, on the assumption that $R_{\min} =$ 25 kpc and that the *heavier* mass in each encounter equals 10^{11} $M_{\odot}$. These adopted values mean that the outermost test particles in our first three examples orbit their central mass initially at 15 kpc, with periods of 5.442 time units or 544.2 million years. But whoever wishes can, of course, simply scale all our results to suit other masses and distances.

On the practical side, all test-particle results in this paper stem from fourth-order Runge-Kutta numerical integrations of the restricted three-body equations of motion (cf. Pfleiderer 1963, eqs. [1] and [2]). Generally, those integrations were here performed *separately* for each such particle, so that the size of every integration step could be optimized to the instantaneous speed of that test particle and its proximity to either of the heavy masses. Despite the bother of many repeated determinations of the positions of those massive bodies themselves, this procedure proved considerably more efficient than the alternative of advancing all particles together with the same time step.

As might be expected, numerical accuracy was not difficult to achieve with these simple calculations; in fact, it distinctly exceeds our plotting ability for all but a few grazing test-particle trajectories. And even graphical errors here have, we hope, been held to a minimum by the fact that much in our figures has been traced directly from computer-drawn diagrams.

a) *A Retrograde Passage*

Our first example merely expands on Pfleiderer's (1963) figure 3. Here the mass of the perturbing body equals that of the perturbed, and its orbit is retrograde with respect to the revolution of the test particles. This entirely flat encounter is pictured face-on in figure 1. The first four frames there have been drawn relative to the center of mass

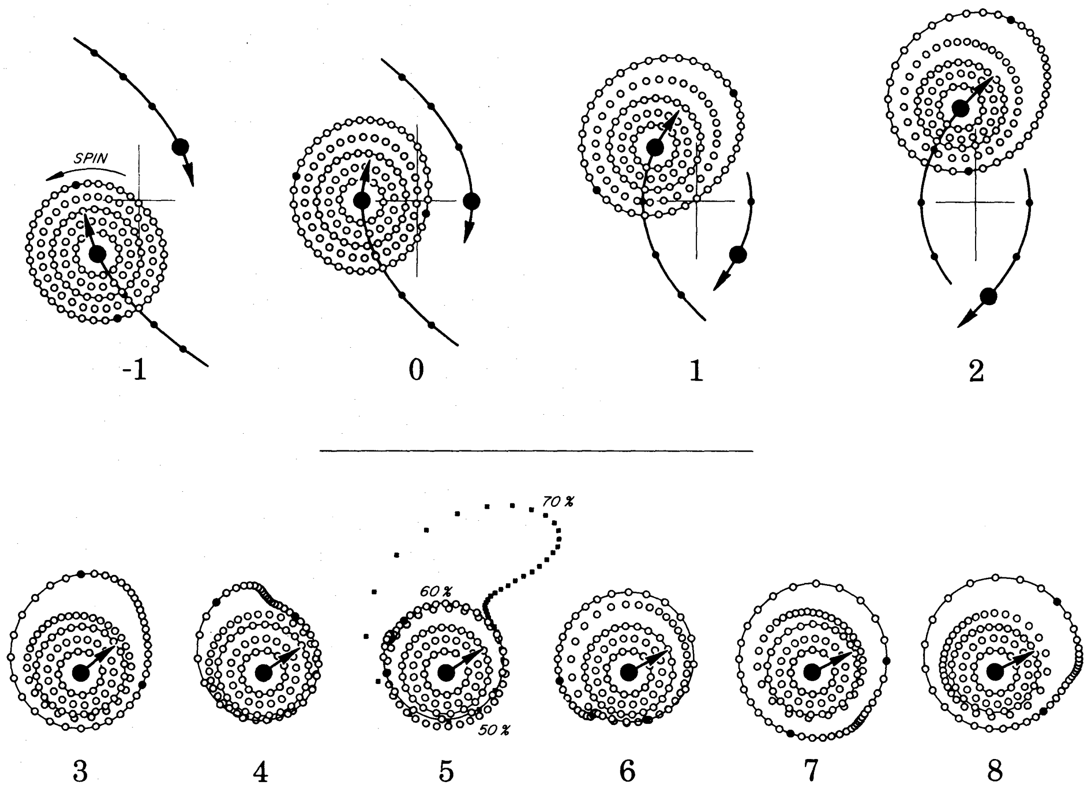

FIG. 1.—A flat retrograde ($i = 180°$) parabolic passage of a companion of equal mass. The two small filled circles denote test particles from the $0.6R_{min}$ ring which, in the absence of the encounter, would have reached positions exactly to the right and left of the victim mass at $t = 0$. The filled squares at $t = 5$ depict additional test particles from $0.7R_{min}$. (Note the partial interpenetrations of the outermost rings at $t = 4$, 5, and 6, and their continuing oscillations thereafter.)

of the system, and the later six relative to the victim galaxy. (Pfleiderer's single diagram corresponds to our $t \simeq 22$, but it extends outward to only the 50 percent ring.)

Had the second body been surrounded by a disk of test particles as extensive (and similarly oriented) as that of the first, the respective disks would obviously have interpenetrated to some extent during this encounter. And yet, excepting only such far-out particles as the additional 42 from $0.7R_{min}$ that have been added to frame $t = 5$, figure 1 indicates that any purely tidal consequences of such a passage would have been remarkably mild.

b) *A Direct Passage*

By contrast, a flat *direct* passage of the same, equally massive companion results in the commotion shown in figures 2 and 3. Those diagrams again only elaborate on one of Pfleiderer (1963), in this case his figure 2 which corresponds to our $t \simeq 0.9$. But there is now more to elaborate!

Perhaps least surprising is the violent tearing away of test particles in figure 2 from the 0.6, 0.5, and even $0.4R_{min}$ rings on the near side of the disk. This arises, of course, from the near-resonance or matching of their orbital speeds with the peak angular motion of the companion. Consequently a particle such as the filled circle halfway between the massive bodies at $t = -1$ experiences the very considerable disturbing force much longer than it did in the retrograde sequence of figure 1.

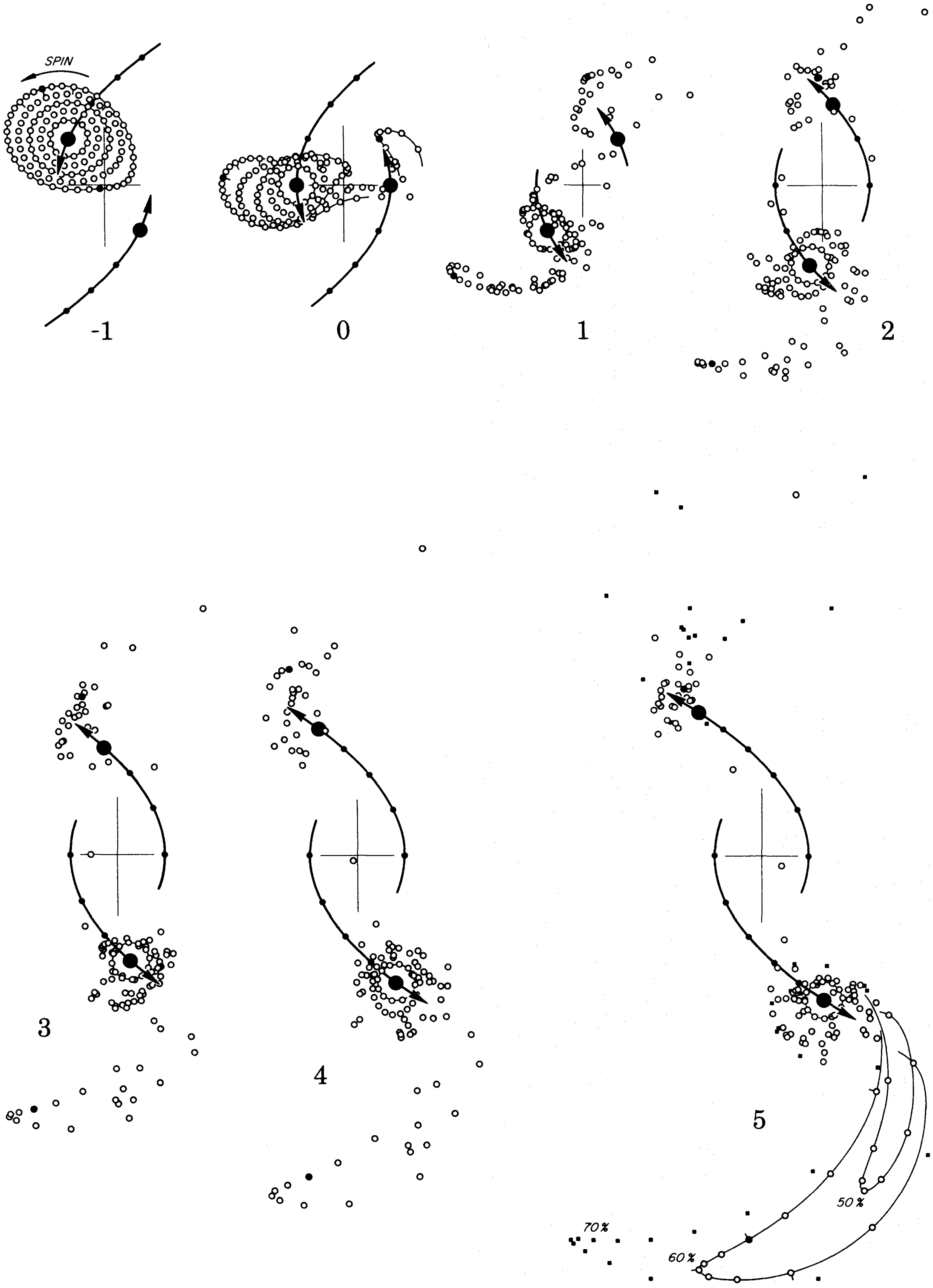

FIG. 2.—A flat direct ($i = 0°$) parabolic passage of a companion of equal mass

Somewhat more remarkable, in view of this violence, is that the resulting "bridge" between the two masses manages to become as narrow as we find it at $t = 1$—and this despite the fact that it then comprises particles from no fewer than three distinct rings. As noted already by Pfleiderer, however, that bridge is so transient that it has become almost unrecognizable at twice the time.[1]

Thick or thin, where does the bridge disappear to? Some of it, as can just be discerned from the present diagrams, falls back into the parent galaxy. However, of the properly torn-off material, a surprisingly large fraction is not flung away into distant space but gets captured by the companion: For instance, of the 13 near-side particles yanked loose by the companion from the $0.6R_{\text{min}}$ ring, only one escapes eventually from both masses, and (in slight disagreement with Pfleiderer's diagram) none of our 10 torn-off particles from $0.5R_{\text{min}}$, or of the six from $0.4R_{\text{min}}$, escapes in that sense at all.

The above is an important reminder that one galaxy can steal material from another without any of the dissipation required for mass accretion by a single body. But it seems especially intriguing that the orbits of the accreted material in this three-body situation become quite deep-plunging: That depth of accretion may not at once be apparent from figure 2 where most of the accreted material from $t = 2$ onward seems simply to cruise in front of the companion at modest distances of order $0.5R_{\text{min}}$. Yet the free-fall time from rest at such a distance is only 0.73 of our time units; hence the actual situation can hardly be so serene. In fact, those captured particles are found mostly in that vicinity only because that is roughly where their apocenters are. Their pericenters remain much closer, as may be gauged from the statistics that the median eccentricity of those 28 particle orbits relative to the companion tends to $e \simeq 0.91$ as the heavy masses draw apart from each other, and that as many as 17 of the particles continue to plunge repeatedly within $0.04R_{\text{min}}$ (or 1 kpc in the present units) of that second mass. Or to put it yet another way: If the eventual angular momenta of those 28 captives were shared equally among them, and if their orbits were also circularized (as perhaps by mutual impacts), they would all revolve counterclockwise around their captor at a distance of only $0.046R_{\text{min}}$. However, we must caution that the depth of these orbits is here exaggerated not only by the sharpness of the mass point but also by this most favorable inclination of passage.

Turning to the far side of the disk, the reader should examine carefully the initial development of the countertidal feature in figure 3. Notice especially how a lagging particle such as that marked 120° manages almost to overtake the one marked 210°. In one way or another, this largely kinematic process of catching up by going faster on the inside is responsible for every tidal "counterarm" or "tail" that we exhibit in this paper. Naturally, the process was already known to Pfleiderer, whose figure 2 without the $0.6R_{\text{min}}$ ring displays a counterarm that is only moderately shorter than the embryonic tail in the $t = 1$ frame of our figure 2. Nor can these early phases of the tail-making come as a real surprise to anyone else who accepts the simultaneous, if relatively abortive, efforts at bridge-building on the other side, and who remembers that tides are notoriously two-sided.

But then what is one to think of the distinctly *one*-sided and comma-like appearance of the severely perturbed disk and its tail from $t = 3$ onward?

This rhetorical question simply reemphasizes that much of our intuition about tidal effects—such as presumably that of Vorontsov-Velyaminov (1961) when he doubted the tidal origin of tails, among other reasons because "tails are much more frequent than the connecting filaments"—derives from experiences in which the tidal forces are both gentler and more persistent. By contrast, a picture such as the $t = 5$ frame of

[1] To be sure, figure 3—which resembles Yabushita's (1971) figures 3 and 4—reminds us that such links are never broken in a strict sense if the original disk can be regarded as continuous. But there obviously comes a time in any given case when the density of particles still remaining in a bridge falls below any reasonable minimum. And that time here comes quite soon.

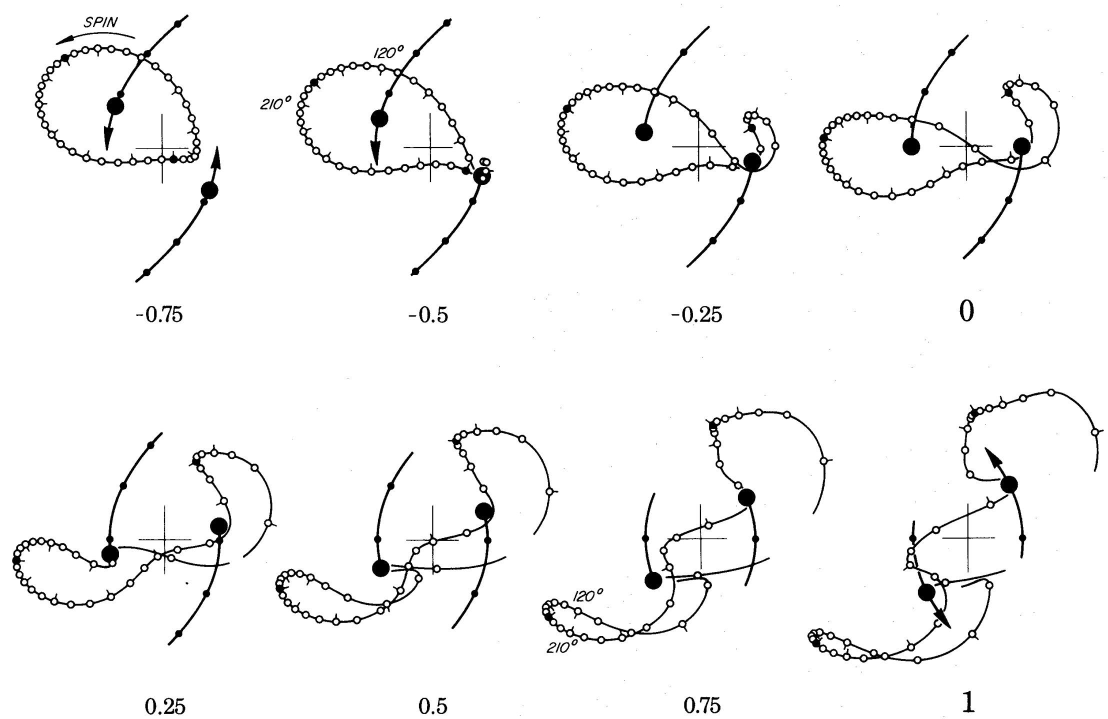

FIG. 3.—A slow-motion study of the distortion of the $0.6R_{\min}$ ring during the direct passage of the equal-mass companion. Originally spaced 30° apart in longitude, the tick marks on every third test particle help identify the "inside" of the curve in the later frames.

figure 2 can only remotely be said to depict galaxies that are still interacting; it seems much more correct to regard such systems instead as ones which *interacted*, and to view the tail as only a relic.

That relic grows longer and longer with time because this particular encounter happened to endow nine of the far-side particles from $0.6R_{\min}$ (though none from $0.5R_{\min}$) with more than enough relative speed to escape from both bodies. It indeed represents the kind of occasional return of galactic material to intergalactic space that was envisaged by Zwicky (1953, 1956).

Needless to say, the tail would also have been longer if the passage had been yet closer. That amount can be judged from the debris of the additional outside ring that we have superposed on the $t = 5$ frame of this (and out of fairness every other) elementary example. But its *width* remains nothing to boast about. Admittedly, it can be argued in this particular instance that any interstellar matter from the far side of the disk could not plausibly have crossed itself with the ease of the test particles in the early tail between $t = 0.5$ and $t = 2$; such "snowplowed" material and any consequent young stars should now constitute a much narrower ridge or spine within the final tail. However, this special argument falters when the encounter is truly three-dimensional, and such gas impacts cannot always be relied upon. Fortunately, we shall instead find in § IV that projection effects alone can already make some tails appear remarkably narrow.

c) *Passage of a Small Companion*

For our third example, in figure 4, we have reduced the mass of the companion to one-quarter of that of the primary. By coincidence, this mass ratio is the same as that used by Wright (1972) in most of his examples. The sense of the passage remains direct.

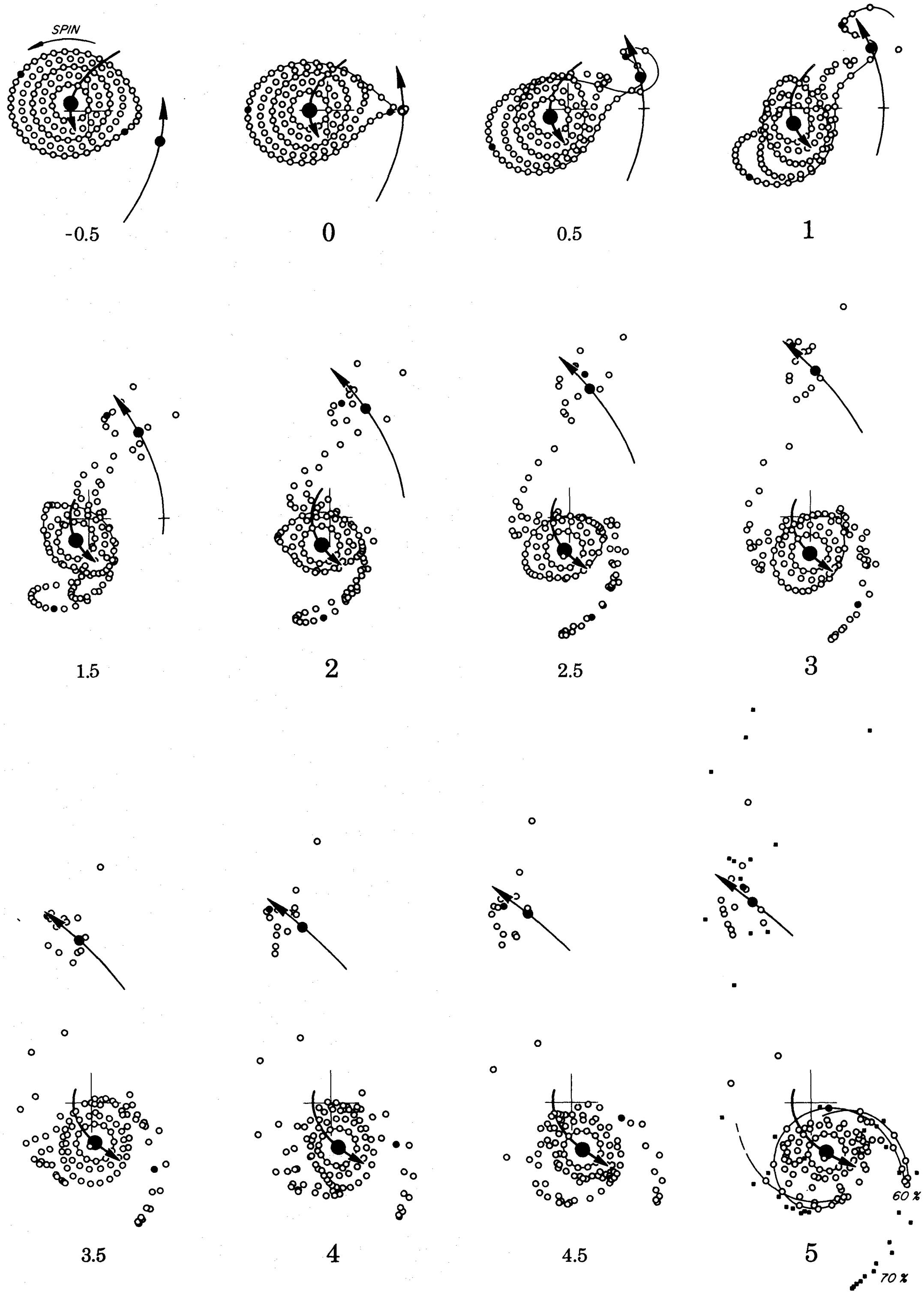

FIG. 4.—A flat direct ($i = 0°$) parabolic passage of a quarter-mass companion

This example was chosen to indicate that such more modest culprits tend to build better bridges; indeed, we could have illustrated this theme for slightly closer approaches with mass ratios even as small as one-tenth (e.g., Toomre and Toomre 1970). Compared with figure 2, of course, the present bridge takes longer to develop. But then also—owing to the inability of the companion to "gobble it up" quite as fully or as quickly—it lasts rather longer.

As it is, the companion finally captures 0, 4, 10, and 9 particles from the 0.4, 0.5, 0.6, and $0.7R_{\rm min}$ rings, respectively, or altogether about half as many as in the preceding example. The relative orbits of such accreted material again remain highly eccentric, as corroborated by the successive positions in figure 4 of one of the two small filled circles: this test particle first grazes the companion mass at about time 0.1, later at 2.3, and then again at 4.8. The only material truly destined to escape into "intergalactic space" in this example consists of four near-side particles from the $0.7R_{\rm min}$ ring.

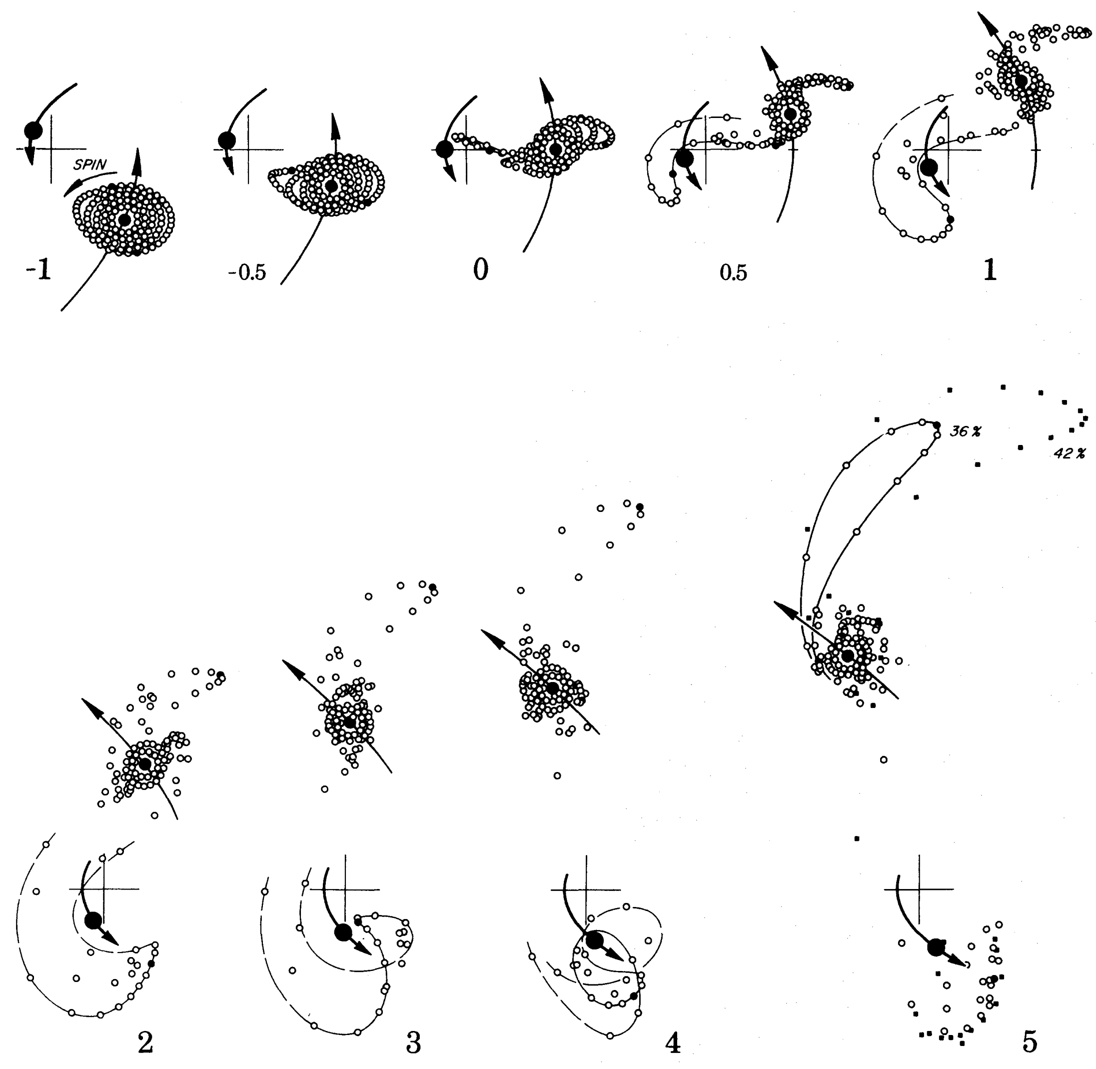

FIG. 5.—A flat direct ($i = 0°$) parabolic passage of a companion four times as massive as the "victim." (Note that these pictures can simply be superposed on the respective frames of fig. 4 for a valid composite sequence.)

We would also like to draw attention to the counterarm:[2] Note again its interesting manner of construction between about $t = 0$ and $t = 2.5$. Notice its *thinness* in all the remaining frames, especially with the $0.7R_{\min}$ particles included. And finally imagine how lopsided that surviving disk would appear at yet somewhat later times.

d) *Passage of a Heavy Companion*

We chose our fourth example mainly for logical balance. It seeks to answer the question: What if the "satellite" from figure 4 had itself been a disk, also spinning in the sense of the passage?

Our choice of the unperturbed ring radii in the resulting figure 5 deserves a brief explanation: Even though the orbits of our massive bodies about each other are far from circular, there exists, on the line connecting them at any instant, a Lagrangian point at which a test particle could be placed and would remain in a dynamically consistent (if unstable) parabolic orbit of its own. With the present 4-to-1 ratio of the masses, the distance of this point from the heavier mass is always a fraction 0.638, and that from the lighter mass a fraction 0.362, of the instantaneous separation of those masses themselves. Thus $0.362/0.638 \simeq 0.57$ seems at least a rough ratio between "tidally equivalent" radii in the disks surrounding the two masses; we merely rounded this estimate to 0.6 in placing the outermost (standard) ring in figure 5 at a radius of $0.36R_{\min}$ instead of the former $0.6R_{\min}$.

Otherwise, we have only the following comments on figure 5: (i) Even less than our second example, it is evidently *not* the way to build thin, enduring bridges. (ii) This figure shows especially well the entangling of the accreted material around the heavy mass. (The curve there which connects the torn-off particles from the $0.36R_{\min}$ ring was drawn accurately with the aid of numerous intermediate particles from the same.) (iii) Already by $t = 3$, a profound asymmetry has again come to characterize the combined appearance of the severely perturbed disk and its rather broad tail.

III. SURVEY OF BRIDGES—REAL AND APPARENT

In this section, and in the next one on tails, we turn to orbits and projections that are truly three-dimensional. Each section surveys those diverse possibilities at some length. We deal first with the bridges because their geometrical complexities seem a little easier to visualize.

One of those complexities is the fact that basically *two* angles are required to describe an orbit of the companion mass relative to the victim disk—even ignoring time dependence, and given the center-to-center distance $R_{\min}$ of closest approach, the orbital eccentricity e, and the ratio of the masses of the partners. As shown in figure 6*a*, our choice of those angles is fairly conventional: The angle i herewith denotes the *inclination* of that orbit to the spin plane of the victim; by our convention, i can be of either sign, but its absolute value $|i|$ must not exceed 180° (that corresponding to the flat retrograde passage of fig. 1). The other angle ω, reckoned in the orbit plane itself from one of the two nodes, measures the delay between that crossing of the spin plane and the actual closest approach to the center of the disk being perturbed. In keeping with the standard usage of celestial mechanics, we shall call ω the *argument* of the pericenter, but unlike there we shall limit it to values such that $|\omega| \leq 90°$, with no loss of generality.

Two other angles, such as a *longitude* λ and (what would normally be called the "inclination" of an observed galaxy but which we shall here call) a *tilt* β, are needed to specify a particular direction of viewing. Our λ and β are defined in figure 6*b*. Even

[2] Here and in the rest of this paper, we shall limit our use of the word "counterarm" to those narrow countertidal features which remain obviously *bound* to the perturbed disk, and the word "tail" to those which do not. Just as obviously, this distinction remains somewhat fuzzy.

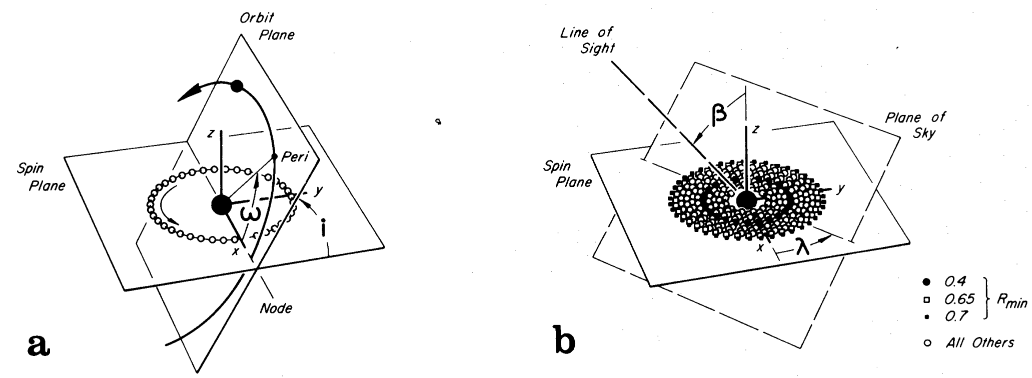

FIG. 6.—(*a*) Orbit geometry. (*b*) Viewing geometry.

so, the "north" or some such direction in the projective "plane of the sky" remains to be designated. However, instead of defining such a further angle, we shall just include the projected x, y, z spin axes in all the diagrams that follow.

Figure 6*b* also depicts an unperturbed disk consisting not only of the previous 162 test particles from rings $0.2(0.1)0.7R_{\min}$, but an additional 15, 21, 27, 33, and 39 particles from the intermediate radii 0.25, 0.35, 0.45, 0.55, and $0.65R_{\min}$, respectively. (Here, that disk and the axes have been drawn as if viewed from $\lambda = 75°$, $\beta = 60°$, and the orbital positions of the test particles are our standard ones which they would have reached at $t = 0$ in the absence of an encounter.) This diagram is to the same uniform scale as all others in this paper, excepting figures 15, 19, 20*b*, 21, and 23. It defines special codes to be used to distinguish the 0.4, 0.65, and $0.7R_{\min}$ particles in most of the ensuing diagrams. It also illustrates the considerable care that we have taken, especially in the perspective views, to give some "depth" to the pictures by *drawing* these infinitesimal, collisionless particles correctly behind one another *as if* they were opaque and of finite size.

a) *Inclined Orbits*

A comprehensive survey of that six- or eight-dimensional parameter space is obviously out of the question. Instead, figures 7 to 13 below only generalize slightly on figure 4: Like that example, they will be limited to (i) exactly parabolic passages of (ii) a companion[3] of one-quarter mass. Moreover, each victim disk here will again have been (iii) loaded at $t = -10$ with test particles out to a radius of either 0.6 or $0.7R_{\min}$, but the resulting commotion will now be (iv) viewed only at the instant $t \simeq 3.143$ ($t \simeq \pi$ but $t \neq \pi$) that corresponds to exactly 90° of orbital travel since the pericenter, or to a moment when the true separation of the two receding masses has reached exactly $2R_{\min}$. Throughout this section, the only orbits shown will be those of the companion *relative to* the heavier mass.

Probably the simplest three-dimensional passages are the ones explored in figure 7. That set of ten face-on views compares with each other the spiral structures provoked by those orbits of inclination $i = 0°, 15°, \ldots, 135°$ for which the pericenter occurs in the spin plane. The first member of that set is just another snapshot of the consequences of the flat direct passage studied already in figure 4; among other things, that duplicate

[3] It may seem inept to keep referring to a parabolic passerby as a companion or a satellite. However, we persist with this slight misusage because we envisage the partners in several actual "interacting" pairs to be in very eccentric *bound* orbits around one another, and also because the bridges and tails from parabolic encounters seem in our experience rather similar to those obtained with assumed eccentricities e equal to, say, 0.6 or 0.8.

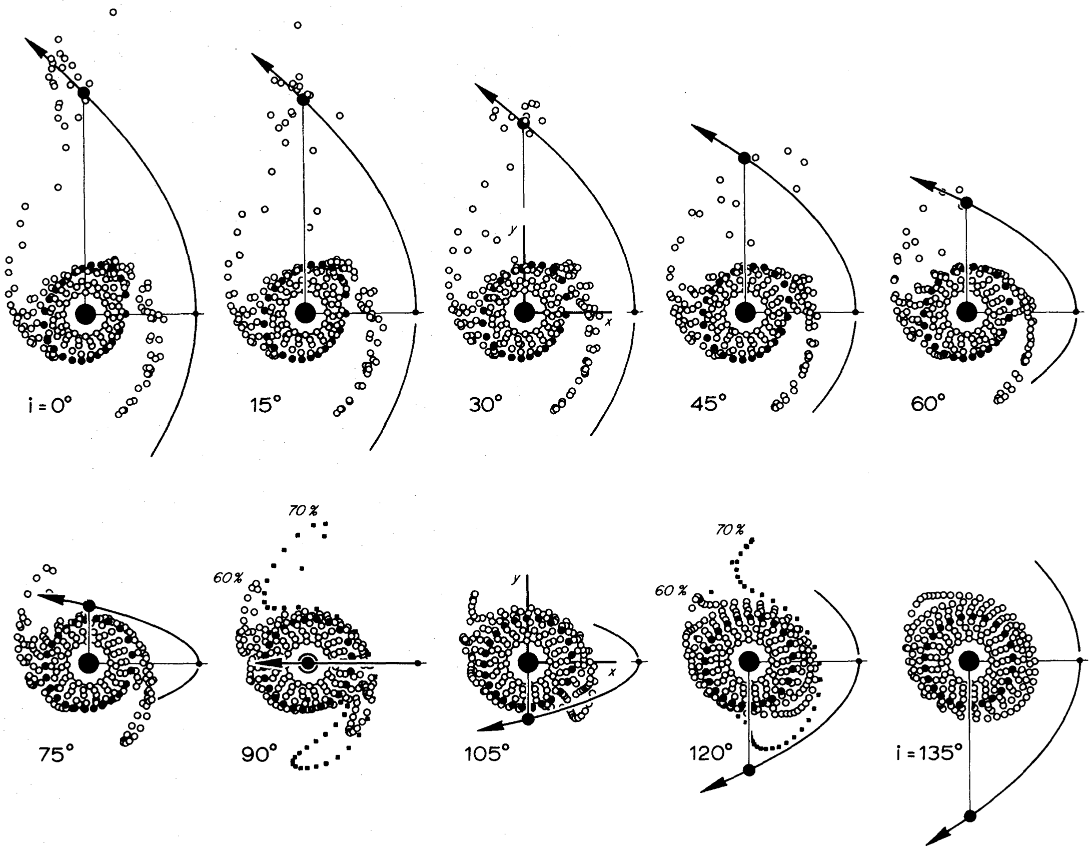

FIG. 7.—Face-on ($\beta = 0°$) views at $t = 3.143$ of disks perturbed by a quarter-mass companion during variously inclined parabolic passages of fixed argument $\omega = 0°$.

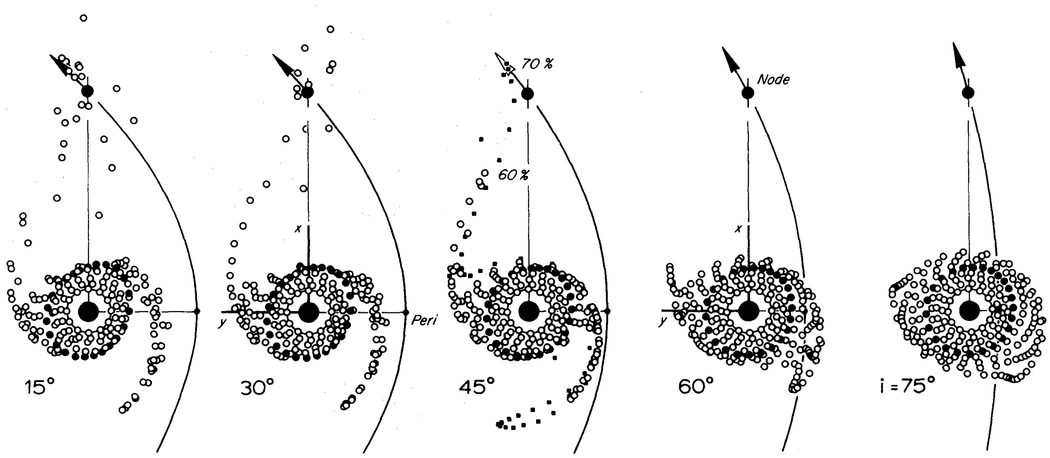

FIG. 8.—Face-on views of disks perturbed during passages of fixed argument $\omega = -90°$

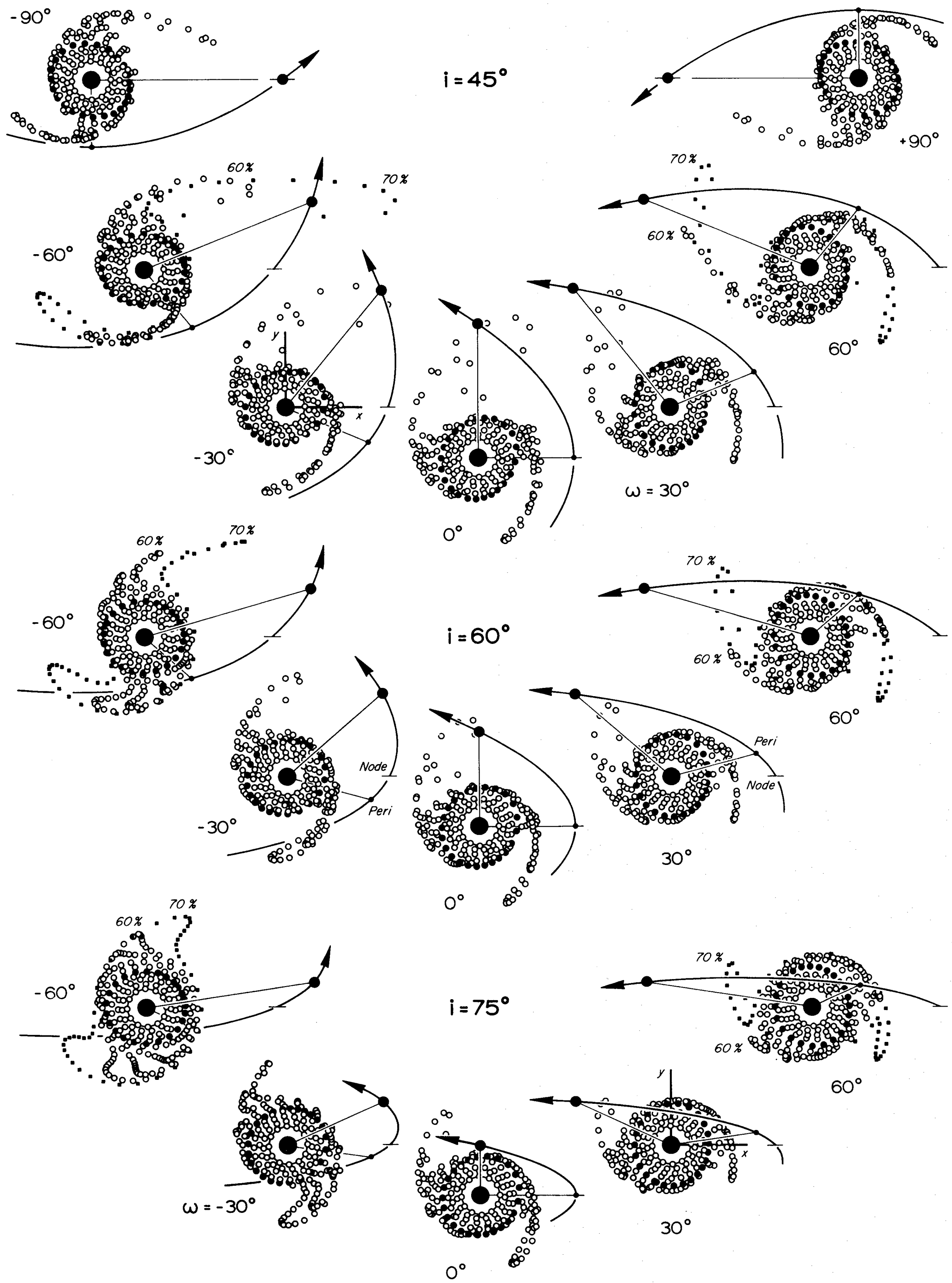

FIG. 9.—A survey of face-on views of disks perturbed during passages of inclination $i = 45°$, 60°, and 75°, and a range of values of ω. (Note that the line of nodes has been drawn in the same direction in all 17 of these views.)

shows how much the continuity both of the bridge and of the surviving disk appears to have been enhanced here by the mere inclusion of the four intermediate rings of particles.

The main lesson to be drawn from figure 7 is that, although the violence is indeed greatest for $i = 0°$, such spiral-making from outside is not confined to passages of extremely low inclination. This is important because one might have suspected from § II that the process was greatly indebted to the near-resonance between the maximum angular motion of the companion and the speeds of revolution of the outermost test particles. Instead, we now see that if such transient forcing can indeed be thought of as a resonance, that resonance must be quite broad. Otherwise one would be hard put to comprehend the still impressive two-armed spiral structure that is provoked by the vertical $i = 90°$ passage, for instance.

This seems a good time to reemphasize, however, that much of this explicit tidal violence is (understandably) confined to the *outer* parts of a disk. We shall not repeat these views for disks of yet larger relative dimensions; as Wright (1972) shows incidentally in his related article, their debris would generally look more splattered. But the reader himself can accomplish something of the opposite simply by recalling that the small filled circles in figures 7–13 represent test particles originally from $0.4R_{\min}$: By imagining absent every particle currently outside those darkened rings, he will quickly obtain an impression of the much more subdued and merely *oval* distortions that would have resulted among these noninteracting test particles if any given passage had occurred at a distance of 5/2 (rather than 5/3 or 10/7) times the outer radius of the disk.

The aftereffects of another five relatively simple passages are shown in figure 8. For these orbits, $\omega = -90°$. Hence the companions there are only now recrossing the spin plane, and their respective pericenters occurred as far from that plane as possible, given the fixed $R_{\min}$ and the inclinations $i = 15°, \ldots, 75°$. To no great surprise, the tides from such passages seem much less effective than before for inclinations $i \geqslant 60°$, whereas the ensuing spiral shapes for $i \leqslant 30°$ can hardly be distinguished face-on from those in figure 7.

Figure 9 presents a third collection of face-on views. Most of these pictures refer to passages yet more complex than those for the previous special values of the argument ω: For twelve of the cases here, $i = 45°$, $60°$, or $75°$ as before, but $\omega = \pm 30°$ or $\pm 60°$; for a thirteenth, $i = 45°$ and $\omega = +90°$. In addition, figure 9 repeats the $\omega = 0°$ shapes from figure 7 for each of these three inclinations. It likewise repeats the $i = 45°$, $\omega = -90°$ picture from figure 8, omitting only the $0.7R_{\min}$ particles; incidentally, those two versions again illustrate how much one's impressions of such a perturbed disk depend on the choice of the outermost ring. We are also reminded by the latter version, when compared with the picture for $\omega = +90°$ in figure 9, that any two orbits whose ω's differ by 180° are logically equivalent, except for a trivial reversal of the z-coordinate (and a half-notch discrepancy here in the initial longitudes of particles in four out of our nine rings). Thus, if one were to borrow also the $i = 60°$ and 75° views from figure 8, each of the three crescents in figure 9 could readily be extended into a full circle of views representing in 30° increments of ω every conceivable parabolic passage confined to the given orbit plane.

We did not think it worthwhile to exhibit in figure 9 any of the cases with $i \leq 30°$, for they, as hinted before, seem too self-similar when projected onto the spin plane. On the other hand, the five $i = 75°$ examples were here included mainly to extend this single diagram to inclinations high enough to serve as a rough search pattern for M51.

b) *Tilted Views*

Thus far, our discussion of figures 7–9 has touched on making spirals but not upon bridge-building as such. This omission was deliberate: As we are about to see, the latter is distinctly more a low-inclination phenomenon than the former. And of course also, those face-on views alone could not distinguish between mere projected links and the bridges that really deserve the name, i.e., those which include some particles that have already been, or will yet become, *captured* by the companion.

This distinction between real and apparent bridges becomes much more evident in figure 10. That diagram views five of the objects of figure 7 from a direction such that both the spin and the orbit planes appear edge-on. It leaves hardly any doubt that, at least for the present $\omega = 0°$, the passages with $i \leq 30°$ produce real bridges and those with $i \geq 60°$ do not. That impression is confirmed, and extended to some other ω, by the fact that for inclination $i = 30°$ an almost constant seven or eight out of our 36 test particles from the $0.6R_{\min}$ ring are accreted by the companion during passages with $\omega = -90°(30°)90°$, whereas for $i = 60°$ no captures whatever occur from among the 0.6 and even the $0.7R_{\min}$ particles shown in figures 8 and 9.

Only the case $i = 45°$ remains ambiguous in figure 10. Again we could simply cite the statistic that four of its particles from $0.6R_{\min}$—and another four from $0.7R_{\min}$, if that ring were included also—will eventually be caught by the companion. Thus in a technical sense this example, too, produces a bridge. But if we said only that, then we would not do justice to the remarkable nature of the present object.

As shown in figure 11, the bridge-like debris from that $i = 45°$, $\omega = 0°$ passage looks splattered indeed when particles from $0.7R_{\min}$ are included in its previous face-on and edge-on views. And yet, edge-on to the original disk at longitude $\lambda = 0°$, as well as at a tilt $\beta = 60°$ at $\lambda = 45°$, we now find that the same extensive debris projects as an astonishingly thin and linear feature!

This thinness means, of course, that all those "bridge" particles form a tilted—and in this instance almost plane—*surface* in space. That their locus should be a well-defined surface is not itself surprising; after all, our flat cold disk of test particles began as a surface (albeit one that was sampled only at a small number of locations), and topologically it must always remain one. The surprise is merely that such a surface has not yet become more curved or convoluted.

The three remaining views in figure 11 indicate how the same object looks when viewed again at a 60° tilt but from longitudes $\pm 90°$ and 180° removed from the previous $\lambda = 45°$. Even those additional views only begin to convey the rich variety of projected appearances of this single three-dimensional object: Just imagine one's perplexity if confronted with only the $\lambda = 135°$ view of this simulated galaxy.

A lesser surprise in figure 11 is that the thin tongue in the $\lambda = 45°$ perspective view clearly misses the satellite and thus does not suggest an actual bridge, contrary to the

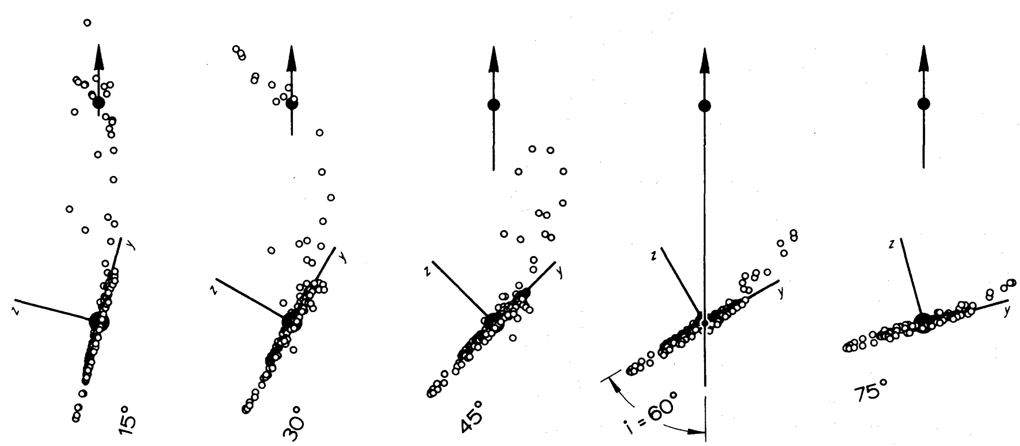

FIG. 10.—Edge-on ($\lambda = 90°$, $\beta = 90°$) views from the direction of the line of nodes, of five perturbed disks from fig. 7.

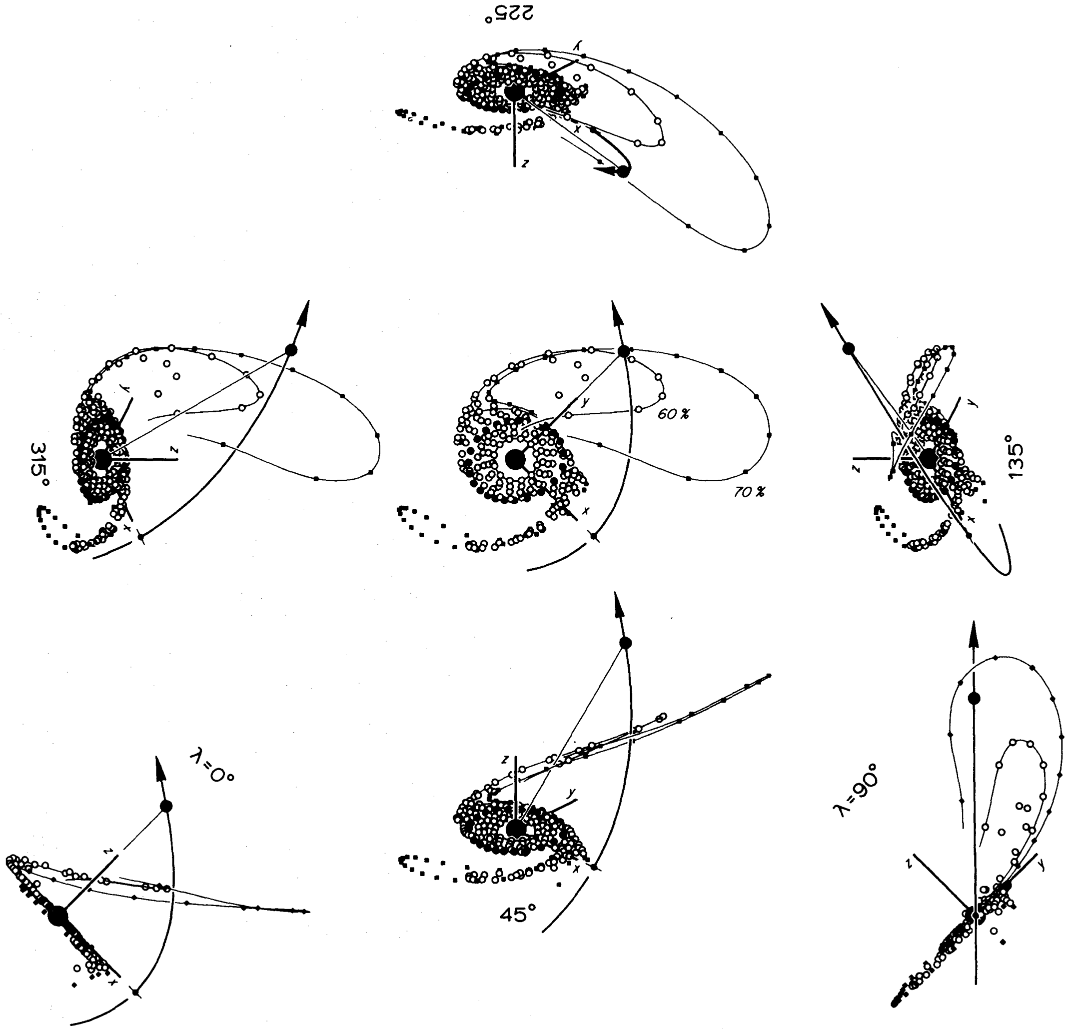

FIG. 11.—Geometric study of the $i = 45°$, $\omega = 0°$ object. Besides the face-on view in the center, and two edge-on ($\beta = 90°$; $\lambda = 0°, 90°$) views in the bottom corners, the diagram shows how this object would appear at time $t = 3.143$ when viewed from longitudes $\lambda = 45°, 135°, 225°$, and $315°$ at tilt $\beta = 60°$.

statistics. This paradox is resolved only by the fact that captures by the satellite here will become evident from $t \simeq 6$ onward; only about then will the companion manage to extract for itself a bunch of particles essentially from the middle of the present tongue.[4]

To show that the above example is not unique in its varied appearance and occasional thinness, figure 12 presents an entire hexagon of $\beta = 60°$ views of the disk and debris from the $i = 45°$, $\omega = -60°$ passage that has already been pictured face-on in figure 9 without the 39 additional particles from $0.65R_{\min}$. Figure 12 again offers one view ($\lambda = 210°$) in which the long arm seems remarkably slender, and another ($\lambda = 90°$) that suggests an altogether different object. Also notice, both here and in figure 11, how even the counterarm changes in thickness with the longitude of viewing.

[4] Such late captures notwithstanding, some of that "bridge" will remain as an ever-lengthening swath of debris long after the tidal counterarm has wrapped back into the body of the victim galaxy. This sense of bridge/tail asymmetry is comparatively rare: at least for parabolic passages, it seems characteristic only of an *intermediate* class where the inclination is sufficiently low to provide an appreciable tidal impulse, and yet too high for the small "satellite" to accrete efficiently.

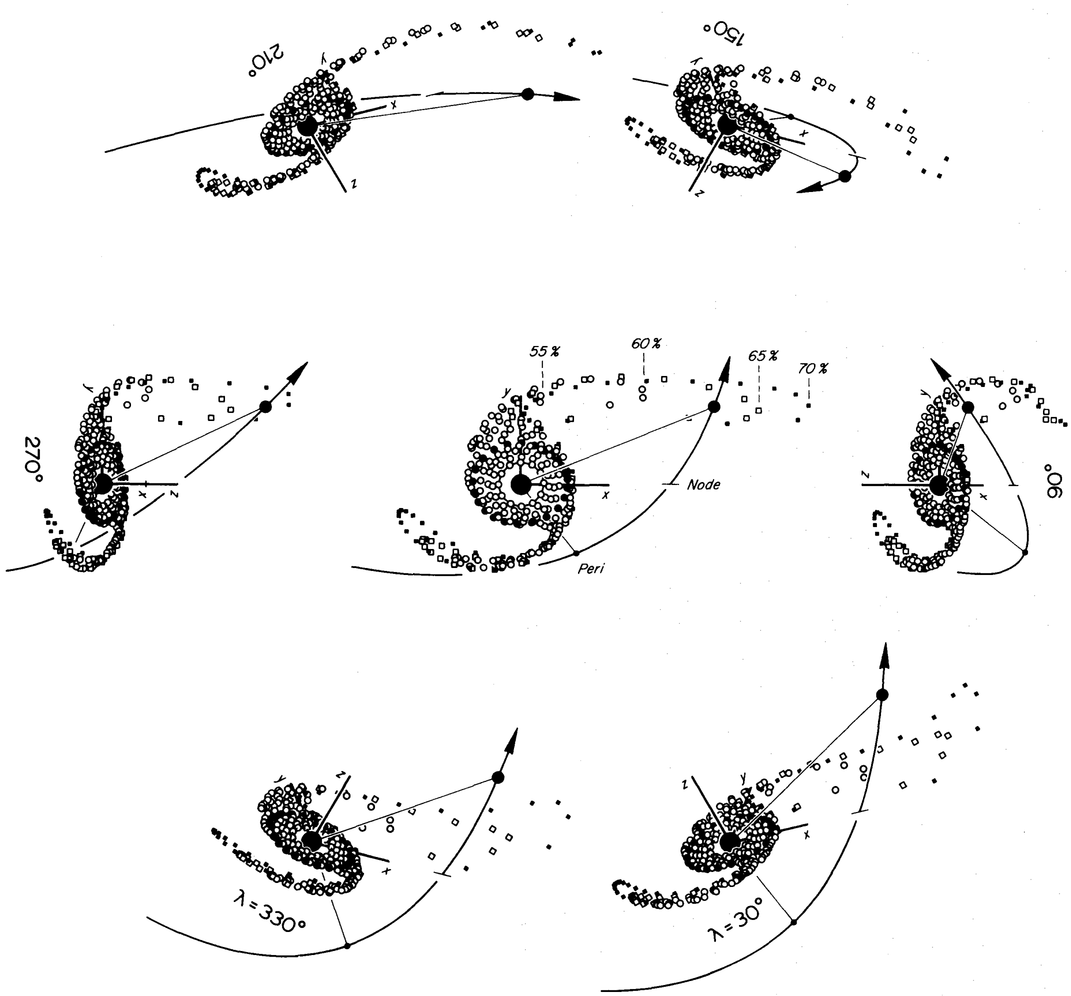

FIG. 12.—Geometric study of the $i = 45°$, $\omega = -60°$ object. Here six $\beta = 60°$ views of this object from equally spaced longitudes λ encircle its face-on view at $t = 3.143$.

The bent shape of the long arm in figure 12 makes it easier to believe that six particles from its $0.65R_{\rm min}$ ring and five from $0.7R_{\rm min}$ (though none from $0.6R_{\rm min}$) are currently *in the process* of being caught by the companion. Thus, like the example in figure 11, that arm is perhaps best described as a bridge-to-be. However, unlike the deep accretion that followed the $i = 0°$ passages of figures 2, 4, and 5, the captures here (and in fig. 11) will be quite distant: the median pericentric and apocentric distances (relative to the companion) of these 11 particles seem to approach about 0.4 and $1.1R_{\rm min}$, respectively, as the time $t \to \infty$.

We conclude this incomplete survey of possible orbits and views with another comparison of passages that differ only in inclination. For this set of examples, in figure 13, we have chosen the argument $\omega = +60°$, to be logically equidistant from the earlier choices in figures 11 and 12.

Figure 13 shows more explicitly than any other the gradual nature of the transition from the bridges that are to those that are not. Again the capture statistics complement the pictures: a total of 28, 21, 10, and 0 of the present 297 test particles ultimately become satellites of the intruder for $i = 30°$, 40°, 50°, and 60°, respectively (whereas 5, 0, 0, and 0 escape from both masses). Figure 13 also emphasizes that the thinnest

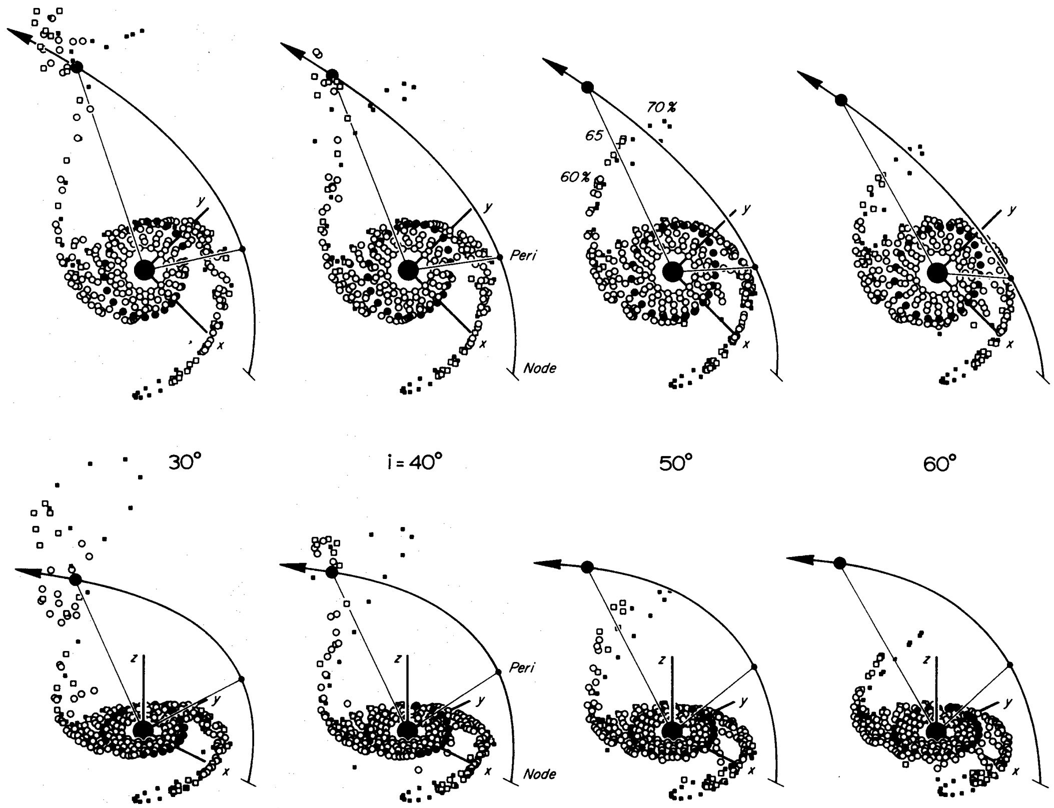

FIG. 13.—Results of four differently inclined passages of fixed argument $\omega = +60°$. The top row depicts the $\beta = 0°$ face-on appearances of these four severely perturbed disks at $t = 3.143$, whereas the bottom row shows how each object would look if viewed at tilt $\beta = 60°$ from the same longitude $\lambda = 45°$.

features are not invariably found in perspective; on the contrary, these $\omega = +60°$ tidal arms at the instant $t = 3.143$ appear just about as thin as possible when projected onto the original spin plane. Incidentally, the same is true also of the $i = 45°$, $\omega = -90°$ center frame in figure 8, which depicts another real bridge.

IV. SURVEY OF TAILS

Proper tail-making in the sense of escape to infinity of particles from the "antitidal" side of a victim disk requires the perturbing mass to be at least comparable to the perturbed. For instance, even for the most favorably inclined $i = 0°$ passages of the parabolic sort, one finds no such escapers whatever among test particles from initial radii 0.8, 0.7, 0.6, 0.5, and 0.4 times the pericenter distance $R_{\min}$ unless that satellite-to-primary mass ratio exceeds 0.38, 0.48, 0.67, 1.08, and 2.10, respectively. Logically, this means that if major tidal tails are to protrude from *both* of two "interacting" disks of roughly equal dimensions, as they do in NGC 4676 or 4038/9, then the masses involved must themselves be about equal and the passage must have been close indeed.

With this excuse and motivation, we have assumed the mass ratio to be exactly unity in our survey of genuinely three-dimensional tails in figures 14–18 below. This is not to disparage figure 5, which showed that the tidal shredding of a small but light disk during its near-encounter with a relatively heavy galaxy can also produce a tail. Rather, it is only for brevity that such tail-making will receive no further attention in this paper, apart from one important example in figure 21.

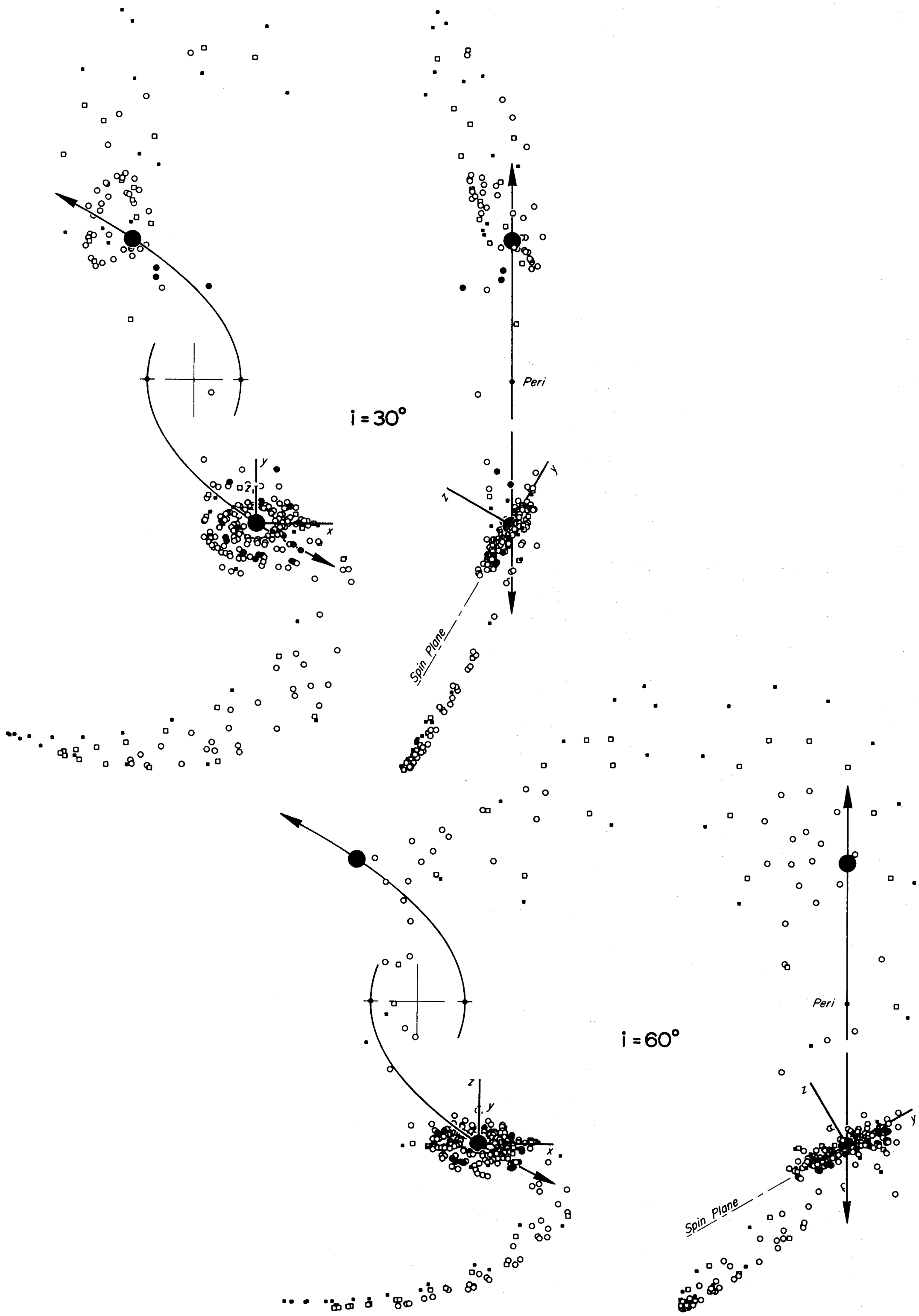

FIG. 14.—Tails and debris from two distinct inclined (i = 30° and 60°; ω = 0°) parabolic (e = 1) passages of a companion just as massive as the victim. The two outcomes are here viewed at time t = 5 both normal (λ = 0°, β = i) and edge-on (λ = 90°, β = 90°) to the respective orbit planes.

We begin this equal-mass exploration with two views apiece in figure 14 of the aftereffects of two simply inclined parabolic passages. These views are to the same scale and refer to the same final instant $t = 5$ as figure 2. However, the unperturbed disks here consisted of a total of 297 test particles, again coded as in figure 6*b*. They had been placed at $t = -10$ into eleven concentric rings of radii $0.2(0.05)0.7R_{\min}$. Hence the present pictures appear almost twice as dense—though they involve the same outermost particles—as the last frame in figure 2.

The close resemblance of the present $i = 30°$ remnant to the one from the elementary $i = 0°$ passage extends even to the statistics: It turns out, for instance, that a total of 23 near-side particles solely from rings 0.4, 0.5, and $0.6R_{\min}$, rather than the 28 particles from before, are here captured by the companion, and that none rather than one truly escapes. Also, the farthest $0.7R_{\min}$ particle in the tail now lies at a true distance of about $3.6R_{\min}$, instead of the former $3.7R_{\min}$, from its parent mass.

Indeed, the only serious difference between the $i = 0°$ and 30° results may be that—contrary to what one might have supposed from the direction of the greatest tidal stress during the encounter—the relatively flat $i = 30°$ tail lies not nearly in the orbit plane. Instead, as we find also with the $i = 60°$ tail, its mean plane seems considerably more parallel to the spin plane. Evidently even those torn-off particles "remember" more of their "spin" around the parent mass than their orbital motion relative to the companion. This is interesting because it would be geometrically impossible to obtain *crossed* tails, as in NGC 4038/9, in any viewing if all tail-like debris remained in the orbit plane.

The $i = 60°$ diagram in figure 14 emphasizes that even tail-making in the strict sense persists to quite high inclinations. This is corroborated by the report that at least some far-side particles escape from the $0.6R_{\min}$ ring provided $i \leq 69°$, and from $0.7R_{\min}$ provided $i \leq 85°$. Roughly the same story is told by the measured distances 3.4, 3.2, 2.8, and $2.3R_{\min}$ to the farthest $0.7R_{\min}$ tail particles at $t = 5$, assuming $i =$ 45°, 60°, 75°, and 90°.

However, just as in § III, here again the severe accretion by the companion ceases at relatively modest inclinations. Indeed, in this respect the current $i = 60°$ example, with its ugly and extensive spray near the companion, seems as marginal as the $i = 45°$ quarter-mass passage and its flaplike "bridge" from figure 11. Again it is mildly surprising that as many as about three-quarters—here 29 out of 38—of those extracted near-side particles still become technically bound to the intruder; yet also somewhat as before, 10 of these 29 captives will never again come within $0.5R_{\min}$ of that second mass point, and only 10 will hereafter ever approach to within $0.3R_{\min}$ of the same.

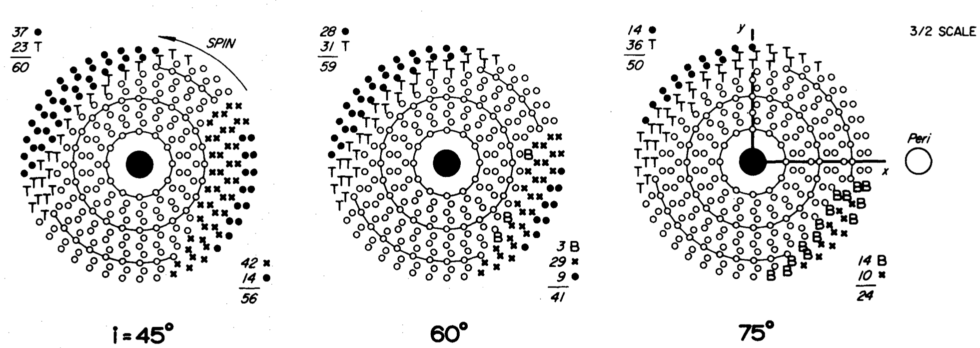

FIG. 15.—Scorecards of tail-making and accretion for three ($i = 45°$, 60°, and 75°) inclined $\omega = 0°$ parabolic passages of a companion of equal mass. The open symbols represent test particles retained by the primary mass point, crosses are those captured by the intruder, *T*'s are nonescaping tail particles which at $t = 5$ lie farther than $1.0R_{\min}$ from their parent mass, *B*'s are similar bridge-like particles, and the filled symbols denote particles that escape from both systems. The initial radii of the three connected rings were 0.2, 0.4, and $0.6R_{\min}$.

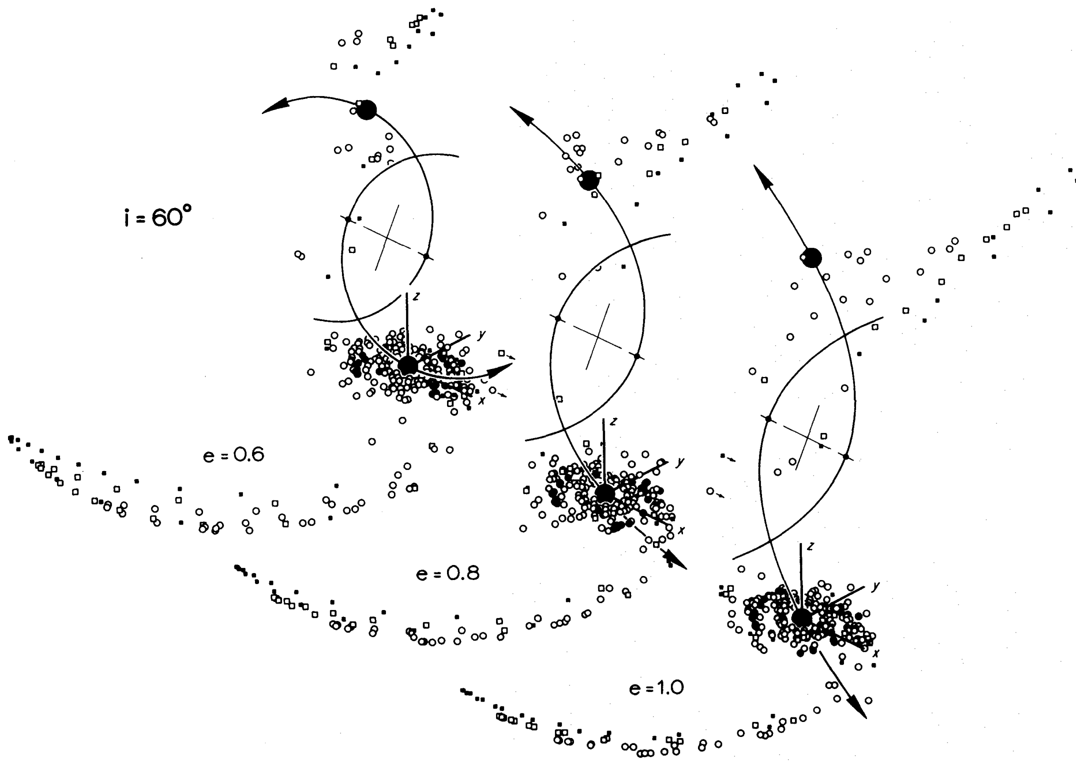

FIG. 16.—Tilted ($\lambda = 45°$, $\beta = 60°$) view of the $i = 60°$, $\omega = 0°$ object from fig. 14, and the same for two slower elliptic passages of eccentricity $e = 0.8$ and 0.6. All three pictures refer to the instant $t = 5$.

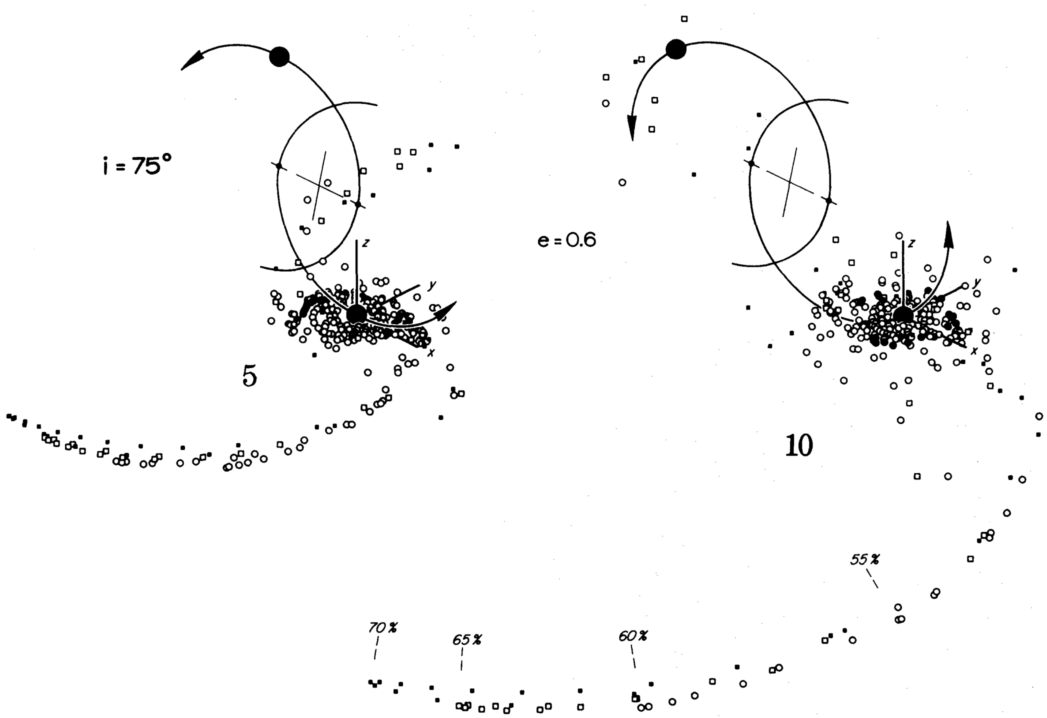

FIG. 17.—Tilted ($\lambda = 45°$, $\beta = 60°$) views at times $t = 5$ and 10, of the tails and debris from an elliptic $e = 0.6$ passage of inclination $i = 75°$ and argument $\omega = 0°$.

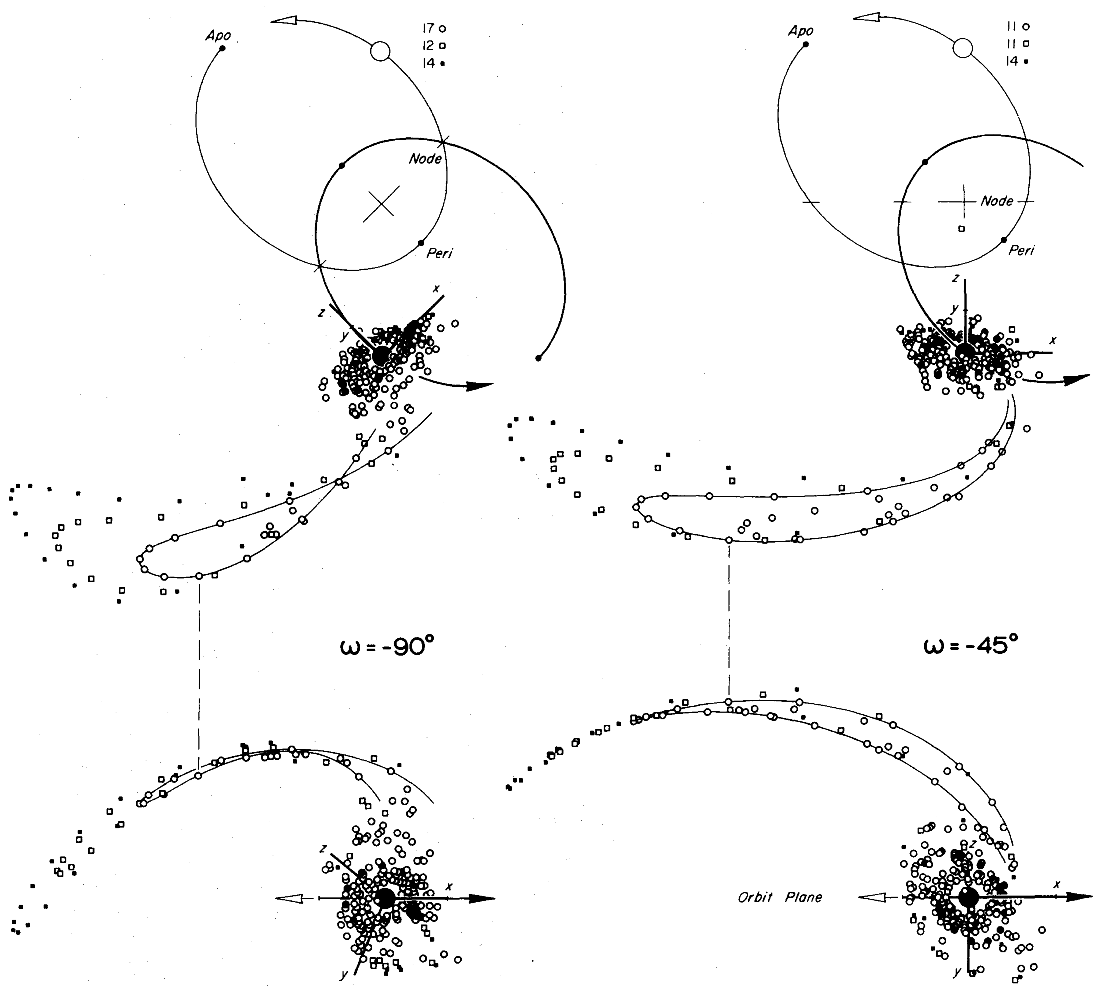

FIG. 18.—A survey of tails produced by elliptic $e = 0.6$ equal-mass passages of fixed inclination $i = 60°$ but of different orbital arguments $\omega = -90°, -45°, 0°$, and $45°$. These configurations are here shown at time $t = 6.086$, which corresponds to exactly 135° of orbital travel since pericenter.

Figure 15 summarizes the specific fate of our various disk particles not only in the above $i = 60°$ parabolic encounter but also during two otherwise identical passages of inclinations $i = 45°$ and 75°. Each of these three (1.5 × enlarged) diagrams shows the 297 particles at positions they would have reached at time $t = 0$ in the absence of forcing. Together, they indicate far better than any table the two-sided but not altogether symmetric bites taken from the disk by the tides. They also demarcate the gradual disappearance of first the accretion and then the tail-making at these moderate to high inclinations.

Our figure 16 serves three purposes. The first is to provide a perspective view of the $i = 60°$ object; the others are to exhibit the dependences of the tails and the spray on the assumed eccentricity e. (In these as all other explicitly *elliptic* examples of this paper, it was our practice to presume the disks unperturbed at the previous apocenter, and to integrate forward from that instant.)

First, that perspective makes it perhaps clearer that not only the tail particles but even our seemingly randomly splattered test particles near the companion still lie on relatively simple *surfaces* in space.

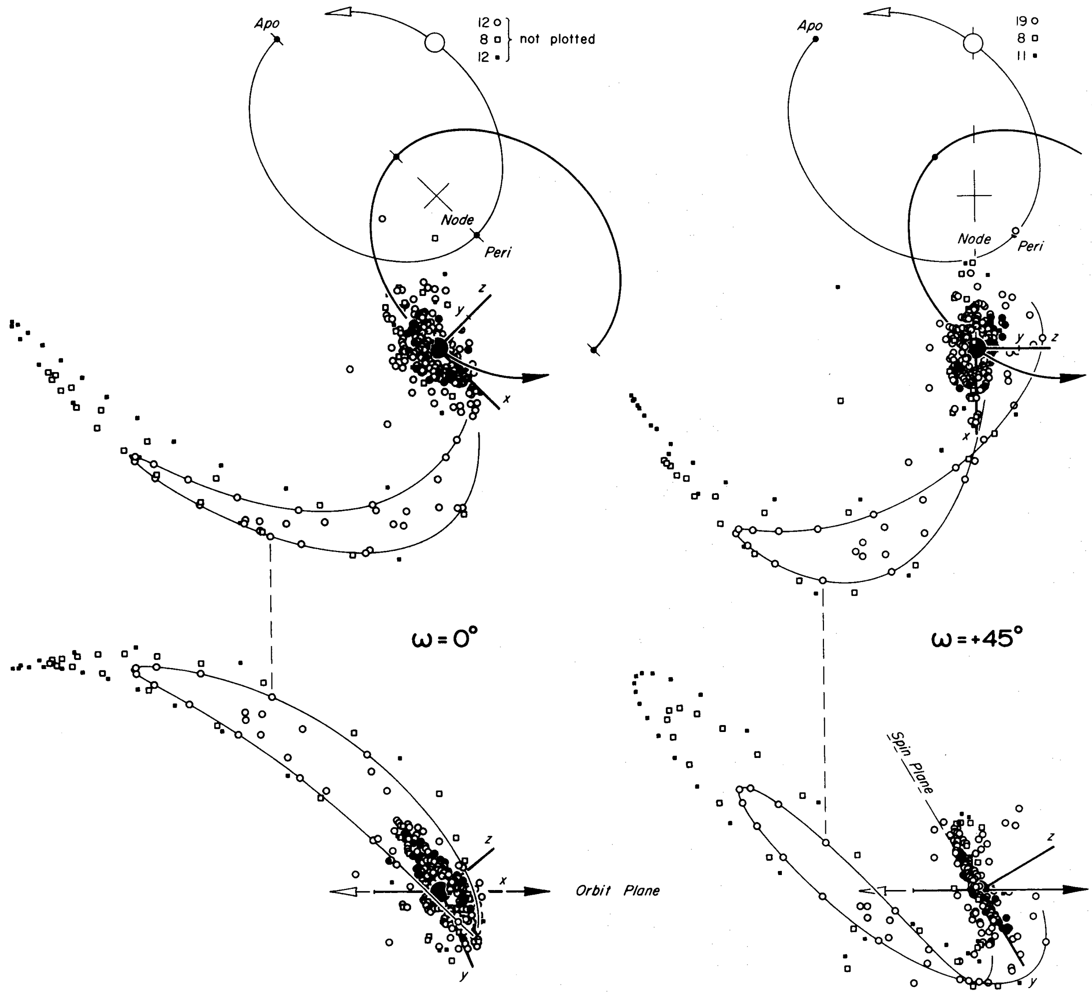

FIG. 18.—*Continued.* Each tail is again viewed normal and edge-on to the orbit plane; however, the edge-on views now are exactly from directions which superpose the centers of the victim and the satellite. For clarity, these diagrams omit the accreted material and all other near-side "spray."

Second, figure 16 also shows that the general character of the tails hardly changes with decreasing orbital eccentricity. To be sure, tails formed by slower passages tend to be a little longer: the center-to-tip distance in the present examples is 3.2, 3.4, and $3.7R_{\min}$ when $e = 1.0$, 0.8, and 0.6. But this increase seems hardly significant, except when compared to the $t = 5$ separations 3.3, 2.9, and $2.5R_{\min}$ of the heavy masses themselves. Similarly, one should not be misled by the greater apparent width of the $e = 0.6$ tail here: owing to its different warp, that tail, if viewed at tilt $\beta \simeq 85°$, would seem about as narrow as the $e = 1.0$ tail currently seen from $\beta = 60°$.

The third and perhaps most interesting indication of figure 16 concerns the spray. Evidently these slower passages enable the companion to *retain* proportionately more of the test particles it extracts than in the parabolic—let alone a hyperbolic—case. And more paradoxically, figure 16 even shows that the total number of near-side extractions itself *decreases* as the speed of this inclined passage is reduced.

Figure 17 continues this theme of diminished clutter near the companion. It shows that, as during the parabolic $i = 75°$ passage already reported in figure 15, only about 10 near-side test particles were torn loose during this $e = 0.6$ passage of the same high inclination; by contrast, roughly four times as many particles still end up in the tail! This degree of asymmetry—where *all* near-side damage seems so much feebler than the "antitidal"—is quite remarkable. After all, following a low-inclination passage such as that in figure 2, the tail particles were at least balanced by a comparable number of accreted particles near the companion. Moreover, we recall no tidal spirals stemming from excessively inclined passages of the quarter mass in § III that were nearly so asymmetric.

The two time frames in figure 17 also remind us vividly that one thing suffices to increase all tail lengths: patience. The true diagonal length of the present tail grows from $3.4R_{\min}$ at $t = 5$ to $5.9R_{\min}$ at $t = 10$ and again to $8.5R_{\min}$ at $t = 15$. Obviously this increase is almost linear with time.

We must confess that the computed $t = 10$ tail in figure 17 appears about as slender as any that we have encountered. Why this astonishing thinness? Is it merely an artifact stemming from the discrete rings into which our particles had been loaded? No, that cannot be (though doubtless the tail would have been thicker if the original particles had been endowed with some random motion): the tail here comprises particles from no fewer than four separate rings. Rather, much of this thinness again stems—as in the bridge figures 11 and 12—from the fact that this particular projection happens to view *nearly edge-on* a gently curving *ribbon* (whose maximum true width here is about $1.1R_{\min}$ at $t = 10$). If all this seems deceptive, let one at least beware that a similar sharpening of otherwise faint and diffuse features might occasionally occur also in nature.

Thus far, our tail diagrams have dealt only with passages for which the argument $\omega = 0°$. To allay fears that such results may be unrepresentative, and to provide even a minimal catalog of shapes that may help reconstruct real objects, we have assembled in figure 18 two views apiece of the distinct tails that resulted from four 60° inclined $e = 0.6$ passages with the logically equidistant $\omega = -90°, -45°, 0°$, and 45°. In this abbreviated equal-mass counterpart to figure 9 we have arbitrarily omitted all particles that are currently closer to the companion than to the primary mass. Such accreted and similar particles would have confused the edge-on views; their numbers are indicated in each diagram. Each set of views refers deliberately to the orbit plane rather than the spin plane because the former seems the more useful intermediary in envisaging *pairs* of disturbed disks.

Apart possibly from those omitted particles, figure 18 is a fair comparison that speaks largely for itself. We feel obliged to remark only that (i) tail-making evidently occurs for all ω even at this rather high inclination, (ii) some of the tail particles have risen far above the orbit plane, and (iii) as especially the connected $0.6R_{\min}$ particles help one to judge, each tail is again some sort of a twisted ribbon or filament in space.

V. COMPARISONS

We have now reached a point where it seems appropriate to ask how these conclusions relate not only to other work but especially to various actual objects in the sky.

a) *Theory*

As stressed in § I, our main theoretical progenitor is not in doubt: The restricted three-body computations which Pfleiderer (1963) performed while a student and 1961 coauthor of Siedentopf showed quite explicitly that some remarkable two-armed (though temporary) spiral structures can result from the tidal action of a passing galaxy, provided the passage is *close* and of a *direct* sense. In his solo paper, Pfleiderer even demonstrated *bridge*-building of a sort during a parabolic encounter.

To be sure, such bridge- and spiral-making had previously been suspected at least by Zwicky (1956; see especially his imaginative fig. 7). But it is also clear that Lindblad

(1960), in his own test-particle study of the effects of a *circling* companion, did not anticipate Pfleiderer's essential ingredient of a transient but not too rapid—i.e., a roughly *parabolic*—passage. And slightly later, Contopoulos and Bozis (1964) missed that payoff in the opposite sense when they reaffirmed that very *hyperbolic* encounters likewise build no bridges; ironically their last remark, made unaware of Pfleiderer's work, was that perhaps "much slower collisions. . .may. . .eventually explain the formation of bridges."

In our view, progress since Pfleiderer has essentially been threefold:

1. At least for parabolic passages, it is now clear that *good bridges*—i.e., links that are reasonably dense and narrow and survive for appreciable time—do *not* arise if the perturbed and perturbing masses are roughly equal. They do arise if the satellite mass is fairly *small* compared to the primary. In addition, it helps to have the orbit plane somewhat *inclined* to the spin plane of the victim disk.

2. To obtain really *long tails*, the mass ratio cannot be much less than unity. Otherwise the passage simply needs to be yet *closer*, and preferably also somewhat *slower*, than any that Pfleiderer dared to contemplate—and then one must *wait*.

3. Cases of *exceptional thinness* of either the bridges or tails may simply be accidents of *viewing* these three-dimensionally curving ribbons or *surfaces* of tidal debris from especially favorable directions.

To the best of our knowledge, both positive ingredients of item 1 were first stumbled upon by Toomre (1970) for what we would today call a slightly softened 3/16-mass, $i = 36°$, $\omega = -7°$, $e \simeq 0.9$ passage. They were repeated for $i = 45°$, $\omega = 0°$, $e = 1.0$, and a mass ratio of one-tenth in our 1970 A.A.S. movie. In this respect the work of Yabushita (1971)—which otherwise stresses to a much greater (and deserved) extent than did Pfleiderer the actual *capture* of near-side particles—represents progress only insofar as its figures 3 and 4 suggest that the "bridge" from a mildly hyperbolic flat direct encounter with an equal mass is perhaps not quite so fleeting as in the parabolic case (cf. our figs. 2 and 3). More significant is Wright's (1972) quick survey, mainly of seven parabolic quarter-mass passages for which the initial disks had been loaded at random, and distinctly farther than ours. It is reassuring that his best bridge examples —case 10 ($i = 45°$, $\omega = \pm 90°$ in the present notation) and especially case 12 ($i = 45°$, $\omega = 0°$)—also involve moderately inclined orbits.

The first explicit example of proper tail-making as in item 2 appears also to have been one of ours, though today we would hesitate to call "remarkably narrow" that ever-lengthening crescent of a tail which we reported (Toomre and Toomre 1970) for a likewise inclined $i = 45°$, $\omega = 0°$ parabolic equal-mass passage. Its actual width can be imagined from the two neighboring cases pictured in the present figure 14; the aforesaid words should have been reserved for the tails in our subsequent NGC 4038/9 movie (cf. fig. 23 below). To avoid a different confusion, notice that all but one of the far-side features termed "tails" by Wright (1972) are in fact of the nonescaping sort which generally accompany attempts at bridge-building by a small companion. We would have merely called them "counterarms" (cf. n. 2). Wright's enthusiasm notwithstanding, they are quite insufficient to explain such really long *pairs* of tails as found in NGC 2623 or 4038/9.

Item 3, which seems obvious enough in retrospect, appears to have escaped serious notice altogether until this paper.

Only the work of Tashpulatov (1969, 1970) does not fit neatly into this summary. The difficulty is scarcely his general view that "the observed shapes of the bridges and tails [result] from the tidal interaction between galaxies." Nor is it that, like ourselves, he considers direct parabolic equal-mass passages, stresses mass exchange with the companion, and notes that observed "tails often occur even if bridges are absent." Rather, it is only our idealizations of the victim galaxy that differ severely: From the outset, Tashpulatov postulates that a stream of test particles leaves the perturbed or prolate "ellipsoidal" galaxy from only a *single* point or "vertex" during an encounter. Hence it is not exactly surprising that all such extracted particles lie subsequently on a thin curve. Also, in our opinion Tashpulatov's model underestimates the real violence of a direct close passage. And most important, we feel that Tashpulatov completely misjudges the tails as bridgelike material that has flown *past* the companion: in decided contrast to the far-side tidal features which Pfleiderer, Yabushita, Wright and ourselves produce in such abundance, he concludes outright that "the antitide is very weak . . . and no antitidal tails have been found to form."

b) Observations

By and large, the degree of resemblance of our computed diagrams to the various "extensions," "filaments," "plumes," and "streamers" in such photographic collections as Zwicky's (1956) article or in Vorontsov-Velyaminov's (1959) or Arp's (1966) Atlases is something each reader must judge for himself. We ourselves obviously *suspect* most such peculiarities to be the result of tides. Yet even to us that resemblance seems unmistakable in only about 30 of the 300-plus examples contained in Arp's Atlas.

Prominent among these select few are the eight objects with tails already cited in § I. Two of them, Arp 242 and 244, will shortly receive closer scrutiny in § VI. Not included in our "obvious" 30 are Zwicky's (1952, 1956) apparent triplet involving IC 3481, IC 3483, and an anonymous galaxy, nor those half-dozen or so Arp objects in which extensive and often tangled tails seem very evident, and yet where only a single major hulk of a galaxy can now be recognized. We postpone our speculations on the latter intriguing objects until § VII*b*. On the other hand, the Zwicky triplet (= Arp 175) has often been cited (e.g., Hoyle and Narlikar 1971; Arp 1971*b*) as a possible example of a bridge connecting galaxies with very disparate redshifts. Yet now even the shape of that "bridge" warns of an impostor: Such a long, broad, curving crescent seems much more characteristic of a *tail* emanating from the anonymous galaxy solely as the result of an encounter with IC 3481. This makes IC 3483 presumably just a foreground galaxy.

Of the five bridge examples mentioned in § I, M51 and Arp 295 will likewise merit special attention in § VI. In addition, the *inner* spiral structures of both M51 and NGC 7753 will raise some quite unresolved questions in § VII*d*. But NGC 2535/6 and 3808, both discussed by Arp (1969*b*), now seem almost rudimentary. In each of these two pairs of galaxies the larger member strongly resembles certain of the views in our figures 4, 7, 8, 9, 12, or 13. Neither case involves a startling redshift difference. And in both instances the appearance of the smaller galaxy, too, seems incriminating: Though not as strikingly as NGC 5394 in Arp 84, the wide-open S shape of the outer parts of NGC 2536 at least suggests tidal damage. In NGC 3808, to use Arp's words, it is remarkable that "the spiral arm appears actually to approach the companion in a helical fashion." We cannot claim to have duplicated such exquisite wrapping in any of our calculations. Yet such examples as we have explored (cf. fig. 5, though not directly applicable) certainly encourage the belief that the observed "helix" there is transient and indeed resulted from recent tidal accretion and entanglement of material from NGC 3808 by its companion in a significantly inclined, perhaps $i \simeq 45°$ passage.

The only embarrassment to the tidal hypothesis in the last two galaxy pairs may be the exceptionally *long* and kinked faint counterarm (or tail?) of NGC 2535, to which Arp quite properly draws attention. Our counterarms from parabolic quarter-mass passages tend to remain quite a bit shorter. To be sure, that discrepancy can be lessened with a larger relative mass of the companion and/or a slower passage and/or a larger victim disk. Still, the total extent of that counterarc leaves us slightly uneasy.

Of course, in one sense our figures 7–13 are quite biased. Like perhaps some of the real objects just discussed, they refer mostly to an instant when the induced spiral structure has temporarily become very photogenic. Do there exist galaxies that represent other, less distinctive phases of the same purported evolution? We believe so.

The two best examples of *earlier phases* in the evolution of a large disk subsequent to a tidal impulse are probably NGC 2798 and 5566, pictured in Arp's Atlas as Nos. 283 and 286, respectively. In both those cases, a counterarm-like feature, which we presume to be developing more or less kinematically in the manner of figures 4 and 20*a*, is still very broad. In each case also, an obviously disturbed companion lies close by.

Of necessity more ambiguous are any *later phases* where a bridge is sparser, or has altogether fallen down. Even so, one such favorite of ours is Wild's (1953) triple system (= Arp 248; see also Zwicky 1956, and fig. 6 of Zwicky and Humason 1961). Another is Arp 54, though this might be an instance where a real bridge never formed. A third is Arp 285, where a curious "narrow tail [that] leads away from northern nucleus" is perhaps explicable as formerly bridgelike material from NGC 2854 which has by now flown beyond the companion NGC 2856, somewhat as envisaged in general by Tashpulatov.

In a related vein, recall that the late time frames in both figures 4 and 5 showed the surviving disks to be severely *one*-armed. We have been struck by this tendency among inclined passages as well: The far-side arm usually contains more particles, remains thinner, and survives longer than the bridge arm itself. To be sure, especially these remarks suffer from our deliberate neglect of all mutual attraction between disk particles. Nonetheless, it is perhaps time to ask seriously: Could it be that some comma-like, one-armed spirals like NGC 4618/25 (= Arp 23) are themselves tidal relics?

Not to be overlooked, finally, is a major selection effect that applies to all *faint*, barely detectable bridges and tails if indeed such structures are genuinely ribbon-like surfaces in space. As figures 11, 12, 17, and 18 again remind us, any such "filament" that is, say, six times thinner in one direction than in another will obviously be more striking and noticeable—not to mention almost 2 mag arcsec^{-2} *brighter*—when viewed locally edge-on rather than face-on. Moreover, as a very rough rule of thumb, such features then will probably appear *straighter* as well.

VI. FOUR SPECIFIC OBJECTS

To complement the general surveys and remarks, this section presents our specific, but still fairly schematic, reconstructions of four particularly well-known pairs of "interacting" galaxies. Each of the first two examples, Arp 295 and M51, involves a companion that appears to lie at the end of an arm or bridge. The other two, NGC 4676 and 4038/9, exhibit two long tails apiece in configurations which obviously challenge us to show that any pair of our single tails can be so compounded.

a) *Arp 295*

No. 295 in Arp's Atlas (= VV 34; see also Zwicky 1953, 1956; Arp 1962) is the fine pair of interconnected galaxies near IC 1505 that was discovered by A. G. Wilson and Zwicky in about 1950. In Zwicky's (1956) words, "the taffy-like filament" there which connects the large, almost edge-on member to its smaller companion "is perhaps the most striking example among intergalactic structures of this type." This remarkably straight and narrow bridge appears tilted about 10° from the mean plane of the surviving large disk; it evidently begins on the far side of that disk, for there is also a pronounced counterarm, tilted only about 3°, which causes some foreground obscuration. The sheer size of Arp 295 is also astonishing: at a nominal distance of 100 Mpc consistent with its mean redshift of almost 7000 km s^{-1}, the projected 4'.5 separation of its two members translates into 130 kpc, and the 2'.5 counterarm into another 70 kpc. Hence the "small" companion has almost the dimensions of our Galaxy!

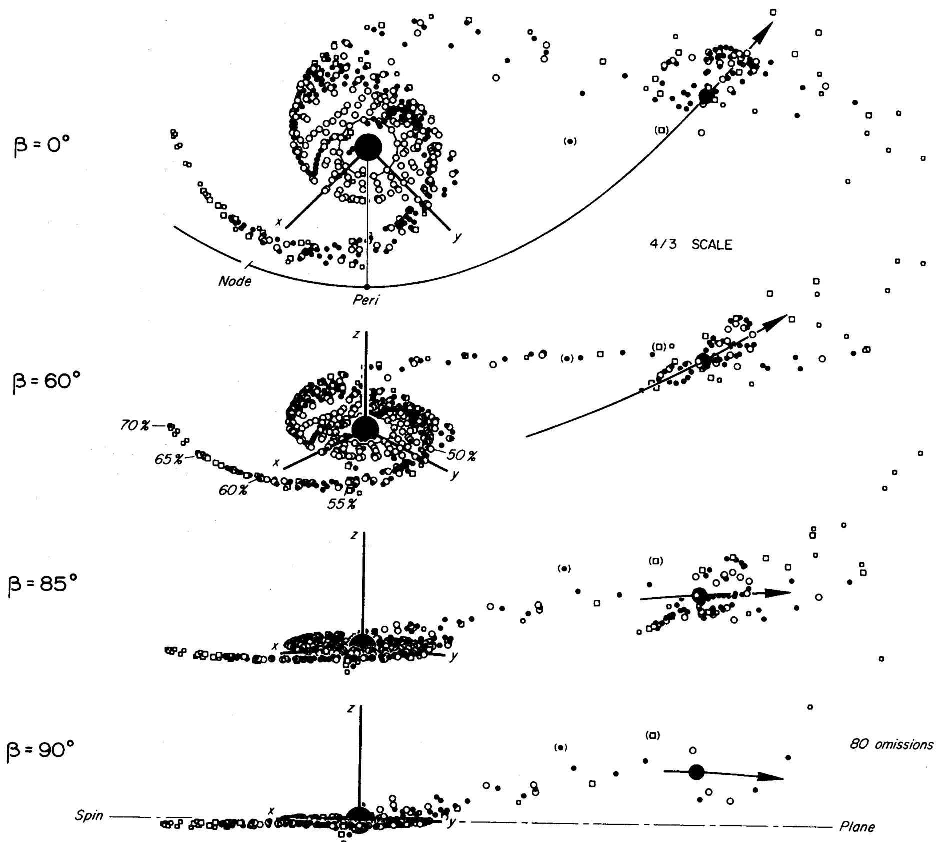

FIG. 19.—Model of Arp 295. This construction supposes a parabolic $i = 15°$, $\omega = 45°$ passage of a quarter-mass companion. Its consequences are here viewed at time $t = 4$ from longitude $\lambda = 135°$ and four distinct latitudes. Except for the heavy "spray" near the companion, the $\beta = 85°$ picture resembles our actual view of this galaxy pair; much of that clutter has been suppressed in the $\beta = 90°$ view by omitting all particles which ever passed within $0.2R_{\min}$ of the companion.

Our crude reconstruction of Arp 295 appears in figure 19. Despite its relatively simple parameters, it is actually the result of a fairly extensive search to make sure that no other choice yields a much better product.

Basically, to explain the bridge and counterarm of Arp 295 by tides, we find that the following conditions must prevail: (i) The two galaxies must recently have approached within about one-third to one-half of their present separation. (ii) The inclination i of the orbit to the larger disk cannot much exceed the geometric minimum of 10°—or else the bridge, if any, would appear much thicker and more curved. (iii) The argument ω of the pericenter must be positive (cf. the $i = 15°$, $\omega = 0°$ edge-on view in fig. 10), and probably between 30° and 60°, to give a straight bridge that tilts above the original spin plane, and a counterarm which ends up slightly below. (iv) At this epoch of viewing, the two galaxies must have traveled roughly 90°–120° since pericenter. (v) The mass ratio could plausibly range between one-third and one-tenth, but clearly the longitude λ of viewing may here be altered 20° or 30° with little detriment.

The construction in figure 19 seems fundamentally correct for the following reasons: (i) Our bridge indeed tilts more than the counterarm. (ii) The counterarm is denser, and it is slightly more than half as long as the bridge. (iii) Like ours (with a nominal speed of 102 km s^{-1} at a projected distance of $2.3R_{\min}$), the actual companion is known to be receding—though the 239 km s^{-1} difference reported by Zwicky seems excessive unless the major member of Arp 295 is exceptionally massive as well as large. (iv) On his original Atlas plate which Arp showed us recently, there is a strong hint of faintly luminous debris in about the same east-to-northeast direction from the companion as our *distant* tidal spray.

Our frustrations with the model remain threefold. (1) The surviving disk is proportionately too large. (2) The bridge in the intended $\beta \simeq 85°$ view appears slightly too thick. (3) Our tidal spray near the companion dominates over the bridge itself.

Conceivably all three reflect just the limitations of the test particles, as discussed below.

1. Neither of the two obvious ploys to lengthen the bridge relative to the body size succeeds adequately here. (i) Of course, one could simply wait longer than our "dimensionless" $t = 4$ (which scales to an almost identical 3.9×10^8 years using the observed separation and relative speed cited above). But at least for parabolic orbits, the *density* of the remaining bridge now decreases rapidly with time. [As it is, to enhance the visual continuity of even the present bridge, we felt obliged to add four extra rings of initial radii $0.475(0.05)0.625R_{\min}$ and 72 test particles apiece to our standard $0.7R_{\min}$ loading. Those added particles are shown as the small filled circles.] (ii) The alternative of a moderately hyperbolic passage indeed results in a proportionately longer (though not narrower) bridge. However, in the process the counterarm shortens markedly compared to the bridge length. To preserve this balance, we estimate that the orbital eccentricity e cannot appreciably exceed 1.5. (iii) In short, we suspect for these reasons among others that test particles systematically *underestimate* the depth of tidal damage to any given victim disk. As a small comfort, note that the ratio of bridge length to body diameter in the central member of Wild's triple system also much exceeds any that we have managed in this paper.

2. Less worrisome is our inability to make the computed bridge as strikingly narrow in the $\beta = 85°$ view as it now appears in the $\beta = 60°$ projection. That bridge is clearly ribbon-like, as prophesied by Zwicky (1953) when he cautioned that such "spiral arms may actually be very much more curved in the plane of sight and may, in fact, be broad sheets in this plane." One imagines that even a small self-gravity could have significantly altered the thinnest orientation of such a ribbon.

3. Finally, the ugly swarm of (largely) captured particles near the lesser body in figure 19 may be an indirect indication that a good deal of the light of the present bridge stems from stars younger than the estimated 390 million years since the encounter. We reason that, whereas any old stars torn off from the major galaxy should indeed have splattered roughly like the indicated particles near the companion, any "stolen" interstellar material should by now have suffered ample impacts with itself or any resident gas to bind much more tightly to the companion. At any rate, that was our rationale for omitting no fewer than 80 particles from the lowest view in figure 19.

Besides needless advice to repeat all this with proper dissipation and self-gravity, we urge the following specific questions upon future workers: (1) Is the velocity difference between the partners in Arp 295 really as great as reported by Zwicky? If so, do the northeast and southwest ends of the larger hulk in fact have relative velocities of the order of $+500$ and -500 km s^{-1}, respectively, as then demanded by our model? (2) Is there indeed some faint but extensive spray to the east or northeast of the companion? And none toward IC 1505, contrary to Zwicky's (1956) tentative impression? (3) Is the light from the bridge in fact polarized, as deemed by Arp (1962)? If so, why? (4) Is it fair to claim that even the smaller galaxy appears "disturbed"?

b) *M51 and NGC 5195*

Unlike the above, the most famous "bridge" linking two galaxies now seems definitely a fraud. Certainly both M51 (= NGC 5194) and its companion NGC 5195 show major signs of tidal damage. But as we are about to demonstrate, that damage only completes the proof that NGC 5195 lies at present well behind M51.

To begin, we would like it clearly understood here that our immediate concern is *not* with the magnificent spiral structure seen in M51 within, say, 2′.0 of its center. Rather, in our view much of the *explicit* tidal damage to M51 itself only commences

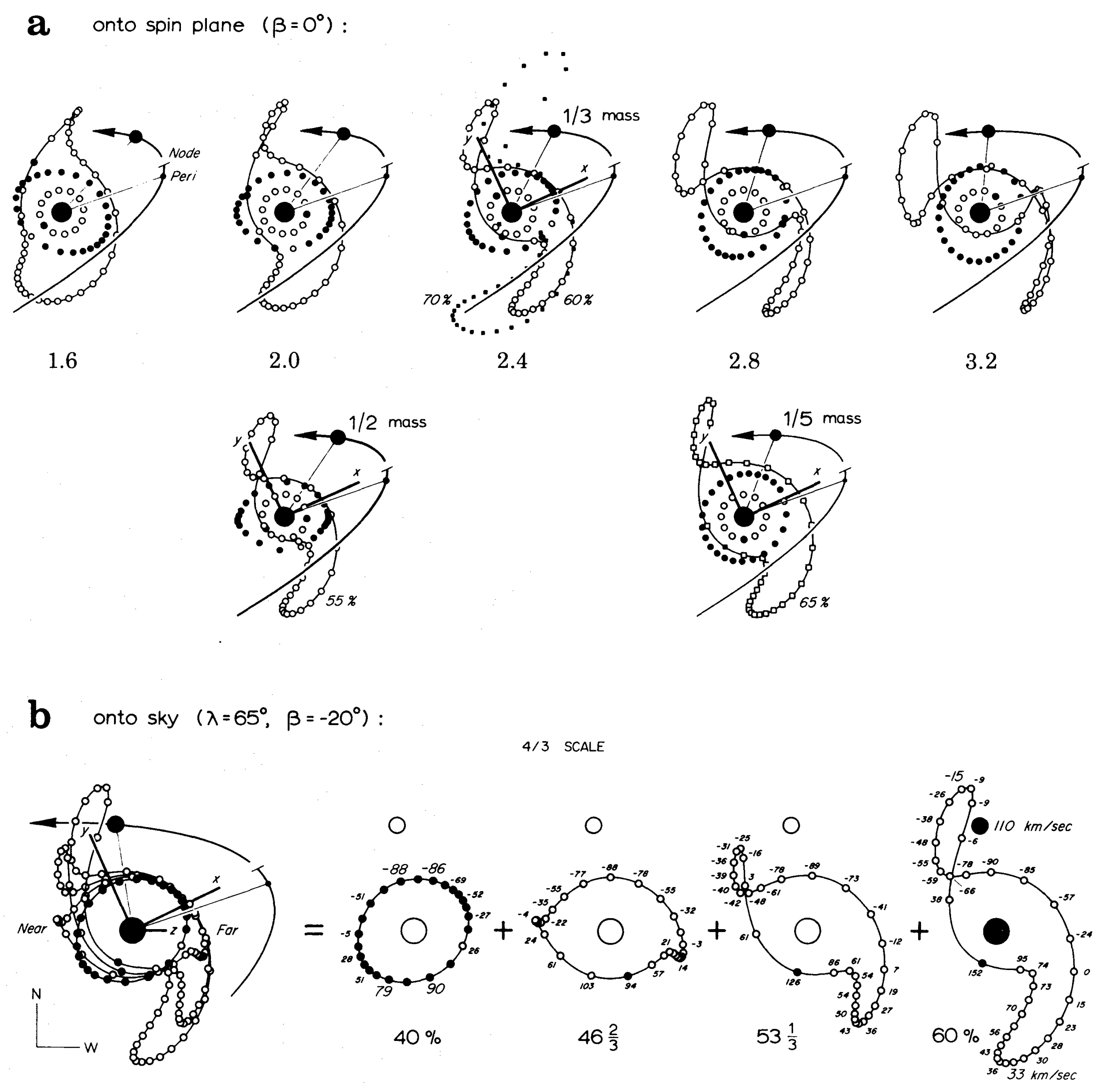

FIG. 20.—Auxiliary studies of M51. (*a*) The top row offers five face-on ($\beta = 0°$) views of the evolving shapes of three test-particle rings of initial radii 0.2, 0.4, and $0.6R_{\rm min}$, after being perturbed by an inclined $i = -70°$, $\omega = -15°$, elliptic $e = 0.8$ passage of a companion of one-third mass. Also shown, at times $t = 2.0$ and 2.8, are the corresponding shapes from instances where the mass ratio of the satellite to the primary was assumed one-half and one-fifth; in those cases, the original radius of the outermost ring was altered to 0.55 and $0.65R_{\rm min}$, respectively. (*b*) Slightly tilted ($\lambda = 65°$, $\beta = -20°$) and $\frac{4}{3}$-enlarged view of the above $t = 2.4$ configuration. It excludes the 0.2 and $0.7R_{\rm min}$ particles, but it includes two additional rings from 7/15 and 8/15 $R_{\rm min}$. The left-hand picture has been decomposed on the right into its four constituent rings; the italicized numbers there are the Doppler velocities of various particles, relative to the primary, obtained after scaling the similar speed of the satellite to $+110$ km s^{-1}.

at about that radius. It consists of two parts: One, of course, is the bridgelike northern arm which a number of observers (cf. Roberts and Warren 1970) have already felt partly obscures the companion. The other is the *broad*, curving, fainter counterarm to the south and southwest of the main disk. Though this second major clue is scarcely visible in the *Hubble Atlas* (Sandage 1961), it is very evident in the deeper *Sky Survey* and Arp 85 photographs and unmistakable in the IIIaJ exposure by van den Bergh (1969). Together with the projected position and excess line-of-sight velocity of the companion, it is these two features—and they alone—that comprised the prime goals of our reconstruction of the encounter shown in figures 20 and 21.

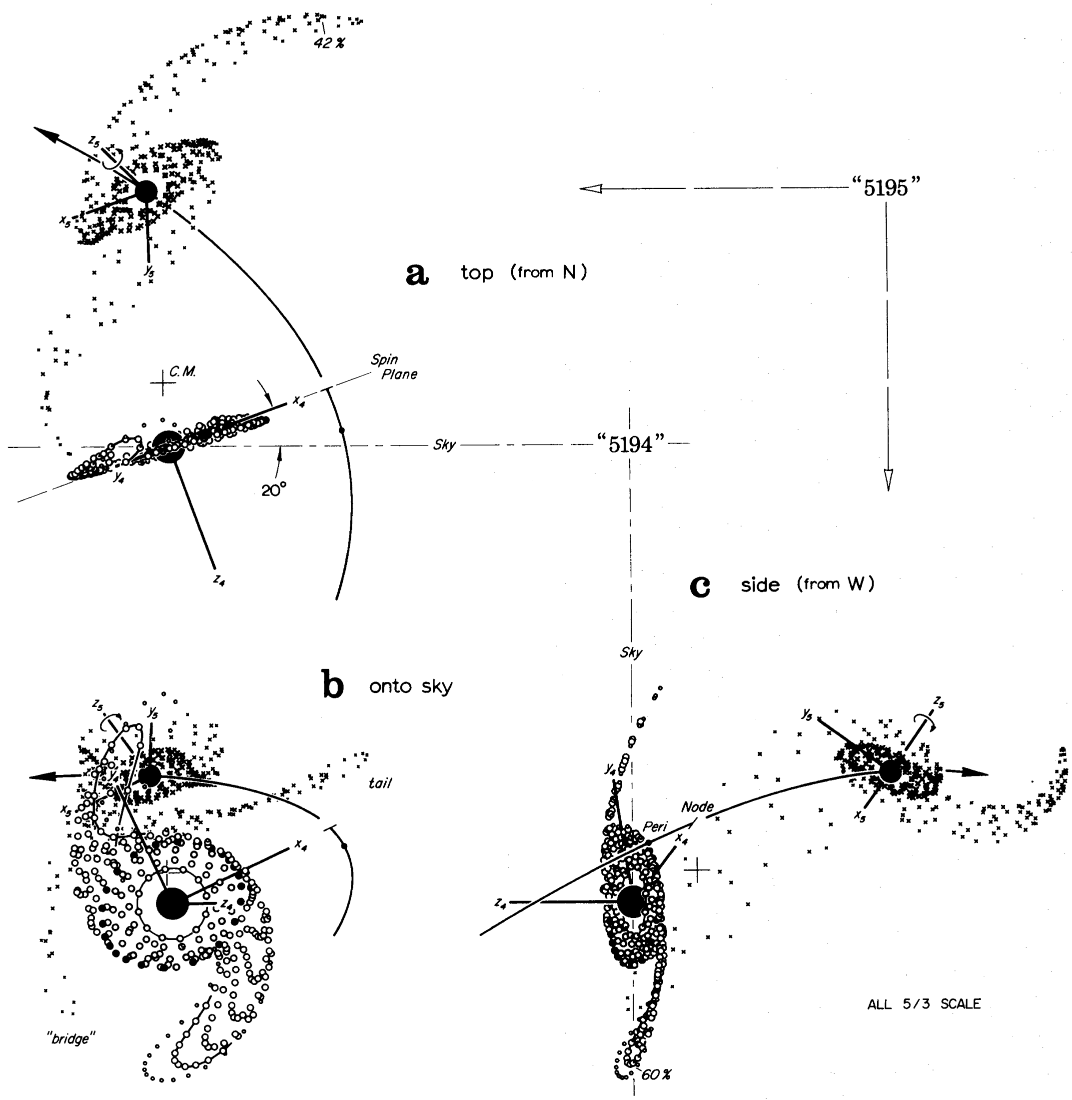

FIG. 21.—Model of the recent encounter between M51 and NGC 5195. Shown here at $t = 2.4$ are three mutually orthogonal views of the consequences of a highly elliptic $e = 0.8$ passage of a supposedly disklike "5195." This satellite was chosen to be one-third as massive, and of exactly 0.7 times the linear dimensions, of the "5194" primary—which itself contains particles from initial radii $0.2(0.05)0.4(0.033)0.633R_{\min}$. The orbit plane differs by an angle $i_4 = -70°$ from the initial spin plane of the larger disk and by $i_5 = -60°$ from that of the smaller; however, the arguments $\omega_4 = \omega_5 = -15°$ of the pericenters were here kept identical, to make the above nodal axes x_4 and x_5 exactly antiparallel. The three views show the combined system as it would appear not only (*b*) to us ($\lambda_4 = 65°$, $\beta_4 = -20°$), but also edge-on to our sky from (*a*) the "north" (−25°, 90°) and (*c*) the "west" (65°, 70°) directions.

In broad outline, the reader himself can retrace the logical steps needed to arrive at those diagrams:

1. On the tentative hypothesis that the relative mass of 5195 is roughly one-quarter, and the orbit roughly parabolic, figure 9 suggests at once that the orbital *inclination* i_4 relative to 5194 is *fairly high*, probably between $-60°$ and $-75°$. (The minus signs remind one that, to match the real thing, the orbits in fig. 9 should now be imagined going into the page.) At lesser inclinations, the "bridge" arm is clearly too long; at higher inclinations, assuming $\omega \simeq 0°$ so that the tidal damage remains interesting (cf. figs. 7 and 8), the satellite is too obscured by the victim in the above face-on projections.

2. Our actual view of M51 is, of course, not exactly face-on. The real viewing "inclination" (= our tilt β) is still known only poorly—with guesses that $|\beta| \simeq 35°$, based on apparent form, ranging back at least to Danver (1942). However, modern spectroscopic studies by the Burbidges (1964) and by Carranza, Crillon, and Monnet (1969) leave little doubt that it is the east side of M51 that is closest to us (on one assumption!), and that the corresponding "line of nodes" between the planes of that galaxy and our sky has position angle 0° plus or minus perhaps only 10°.

3. To be acceptable, therefore, any moderately tilted view of theoretical configurations such as those from figures 7–9 must seemingly place the companion onto, or preferably slightly forward of, the axis about which one did the visual tilting. This firmly excludes, for instance, the $i = 90°$, $\omega = 0°$ picture from figure 7, which one might otherwise have been tempted to press into service by a mere tilt about the x-axis. On the other hand, the $i = -60°$ or $-75°$, $\omega = -30°$ modifications to figure 9 become quite reasonable if viewed from slightly to the left—i.e., once tilted 30° or 40° around what we have christened the y-axis. What is needed, evidently, is not only that the inclination $-75° \lesssim i_4 \lesssim -60°$ but also that the pericenter *argument* ω_4 relative to 5194 be *slightly negative*.

4. For any given passage, another item to be determined is the *time* of viewing. Obviously this judgment is more subjective. However, as the skeletal time-sequence in the top row of figure 20*a* conveys for a near-optimal choice of the other parameters, even this freedom is limited: Recall again that it is the northern arm that must by now be well developed and narrow, whereas the southern arm should still be rather broad. On both grounds, the $t = 1.6$ and perhaps even $t = 2.0$ frames in the chosen sequence seem clearly too early, and especially for the latter reason $t = 2.8$ is already too late. This roughly 20 percent "window" of acceptable epochs of viewing is typical of all our M51 searches. Also typical is the finding that test particles in the "bridge" arm have distinctly overtaken each other by the time the counterarm has become sufficiently narrow. That is welcome, for if real interstellar material had tried to do likewise, its self-impacts should have created a narrow lane of debris. There is in fact an intense absorption lane in the northern arm of M51.

5. In logic if not probability, the real NGC 5195 might just have passed M51 for its first and only time. However, anyone contemplating an eccentricity $e \gtrsim 1$ should consider the $0.7R_{\min}$ particles superposed on the $t = 2.4$ frame of figure 20*a*. These remind us again how sensitive all tidal damage is to the exact proximity of approach. Rather than burden a chance passerby with this further demand of accuracy, we prefer to think that the outer arms in M51—and even the streamers from 5195—exist in their relatively modest forms only because previous encounter(s) rid both systems of material far enough out to splatter excessively.

6. Almost the only observational clue to the *mass ratio* of 5194/5 seems to be that their integrated luminosities differ by almost 2 magnitudes. We doubt for dynamical reasons that the actual ratio could be as large as one-half: As another part of figure 20*a* suggests, the counterarm then tends to stick out too straight from the surviving disk; also, as figure 21 hints already, the satellite hulk then remains too extensive. On the other hand, there is evidently no objection of *shape* to a mass ratio of one-fifth. Only then one needs to *wait* longer.

7. Any such delay, however, aggravates one surprising difficulty with *velocities*. At first sight, the well-known fact that 5195 has a radial velocity perhaps 110 km s^{-1} greater than the nucleus of M51 (cf. Roberts and Warren 1970) provides nothing but welcome corroboration that the orbit is indeed one of high inclination. It is only when we compare that speed with the observed maximum speeds of approach and recession of perhaps 85 km s^{-1} (cf. Carranza *et al.* 1969) along the north-south tilt axis of M51 that a serious discrepancy unfolds: Had we, for instance, taken the $i = -60°$, $\omega = -30°$ configuration implied by figure 9 and indeed viewed it from $\lambda = -90°$, $\beta = 35°$, we would have found the peak line-of-sight speeds of, say, the blackened $0.4R_{\min}$ particles to be more than 1.5 times the Doppler speed of the companion. No mere scaling turns that ratio into the desired 85/110.

8. The above difficulty actually has a simple explanation: Any tilt as large as the $|\beta| \simeq 35°$ favored by Danver "exposes" too much of the rotational motion of the disk, and it turns too nearly sideways to the observer the remaining velocity vector of the satellite. Indeed, that vector itself both shrinks and "turns the corner" more with (i) increasing time and (ii) decreasing eccentricity.

9. To avoid this dilemma, certain parameters which earlier seemed beyond our competence must now themselves be constrained. Even with a mass ratio as large as one-third—which may in part mimic some amplification by the self-gravity of the victim disk—we find that the correct speed ratio is achieved at times when the simulated M51 shape has finally become reasonably optimal *only if* the tilt $|\beta|$ = roughly 15°, 20°, and 25° for eccentricities $e = 0.6$, 0.8, and 1.0, respectively. Moreover, with a mass ratio of one-fifth (and consequent time-delay), those implied tilts lessen by nearly another 5°. Evidently M51 is more nearly *face-on* than it has become customary to suppose, and the orbit of 5195 simply has to be *highly eccentric* as well as highly inclined.

This concludes our main chain of deduction. Its end product in figures 20*b* and 21*b* is at best only a caricature of the real M51: in outer shape it certainly resembles its subject, but the simulated counterarm, for instance, does not curve quite enough. Also, as regards figure 20*b* in particular, we must caution that the neglected self-gravity probably renders untrustworthy all departures of the reported particle speeds from purely circular motion—except near the *tips* of the two arms: There, at least, our model predicts unequivocally that the actual line-of-sight speeds should be systematically lower than the 50 or 60 km s^{-1} extrapolated to such distances on the premise that all material lies in a common plane and travels simply in Keplerian circles.[5]

Incidentally, if M51 really is 4 Mpc from us and if we equate the actual 4′.4 projected separation of 5194/5 to the $0.73R_{\min}$ distance in the model and also scale our nominal 86 km s^{-1} speed difference to exactly 110 km s^{-1}, then the parameters adopted for figures 20*b* and 21 imply an actual elapsed time of about 50 million years since pericenter, a full orbit period of 2.5×10^9 years, and a mass $M_{5194} \simeq 4.5 \times 10^{10}\ M_{\odot}$. Also, the unperturbed period of revolution of the $0.4R_{\min}$ particles in our "5194" is then 65 million years. If the real distance is 8 Mpc, all these quantities simply double; and so on.

Of course, all the above is only one side of the story. That there is another side can scarcely be doubted by anyone familiar not only with the general luminous mess near NGC 5195 but especially with the two long, faint plumes or streamers which extend perhaps 7′ west-northwest and at least 4′ southeast from that companion. These two extraordinary features are best viewed again in van den Bergh's (1969) published photograph (though it is only a pale cousin of an original he kindly gave us); they are also evident in Arp 85 and in Zwicky's (1953, 1959) slightly too enthusiastic sketches.

[5] In this respect, we are pleased to note that already Tully (1972) reports some abnormally low outer-arm velocities from his extensive Fabry-Perot observations of M51. Also encouraging is Tully's entirely independent estimate that $|\beta| \simeq$ only $20° \pm 5°$, based largely on his geometric effort to rectify the inner spiral structure of M51 using the kinematically determined position angle of the tilt axis that had been unavailable to Danver.

As figure 21 testifies in three dimensions, we believe the two streamers to be a tidal tail and a loose sort of bridge arm torn from 5195 by 5194 during their recent encounter. Unlike i_4 and ω_4, the orbit parameters relative to 5195 are here to be regarded as only suggestive. It was obviously a rash step to choose the unperturbed satellite itself cold and disklike. Nevertheless, based on the study of several dozen such examples, we believe the following to be true: (i) The encounter must have seemed also to 5195 as one of fairly high inclination—or else the "bridge" would have accreted too rapidly onto the primary (cf. fig. 5). (ii) The mean spins of 5194/5 must be more nearly antiparallel than parallel—or else the tail of 5195, when viewed by us, would point much too far northward; and the bridgelike plume, if any, would be altogether masked by M51. (iii) The argument ω_5 must itself be closer to 0° than to $\pm 90°$. (iv) It is impossible to decrease to completely realistic proportions the size of the main hulk of our "5195" without resorting to mass ratios of the order of only one-tenth. (Possibly the latter is just another clue that self-gravity tends to aggravate tidal damage.)

Like figure 21 itself, the first three of these items also imply that it should be roughly the east or southeast side of the main surviving body of NGC 5195 which most nearly approaches us. It will be interesting to compare this sense of rotation with observations when those become available.

Most important, all our satellite analyses indicate that if the plumes from 5195 are indeed tidal, then that galaxy must, *in whichever highly eccentric orbit*, have traveled since pericenter a distance at least comparable to its present tail length. We trust the reader will agree that this finding causes difficulty to any hypothesis that would currently place 5195 anywhere near the spin plane of M51.

Exactly the above construction was recently animated by us (Toomre and Toomre 1972) in yet a third movie.

c) The "Mice," NGC 4676

Nicknamed the "Mice" by Vorontsov-Velyaminov (1958), each of the two galaxies in NGC 4676 (= Ho 459*a*, *b* = VV 224 = Arp 242; see also Burbidge and Burbidge 1959, 1961, and Burbidge *et al.* 1963) possesses a long tail. The tail from the northern galaxy, 4676A, is intense, narrow, and almost straight; that from 4676B is fainter, more diffuse, and much more curved.

Figure 22 shows qualitatively how these remarkable tails could indeed have been produced by tides.

The logical cornerstone of this model is clearly tail A: Its straightness, together with its near collinearity both with the ragged dust lane and the major axis of its parent galaxy, indicates not only that (i) we must be viewing hulk A almost equatorially ($\beta_A \simeq 90°$) but also that (ii) the encounter must have seemed one of low inclination ($i_A \simeq 0°$) to that galaxy itself. Hence our view of the combined 4676 seems, fortuitously, to be from a direction that lies nearly in the orbit plane as well. This implies in turn that (iii) the orbital inclination i_B relative to the other galaxy must have been fairly high; otherwise also tail B would appear rather straight to us.

The next issue is: What is in front? Here we feel that the dust lane should *not* deceive one to think that the straight tail attaches to the *near* side of hulk A. That the opposite must be true is made clear by the Burbidges' (1961) observation (confirmed by Theys, Spiegel, and Toomre 1972) that it is the north end of 4676A that is receding, and the south end approaching, relative to the approximate 6500 km s^{-1} mean speed of that galaxy. Hence, as the top half of figure 22 suggests, (iv) had we instead been privileged to view the "Mice" from roughly our east, the sense of the orbit and of spin A would both have appeared counterclockwise.

The last key link in this deduction is figure 18. It shows that also (v) the orbital argument ω_B must have been very negative for tail B to have lifted itself archlike high above our nearly edge-on plane of the orbit. Otherwise, to make this model instructive

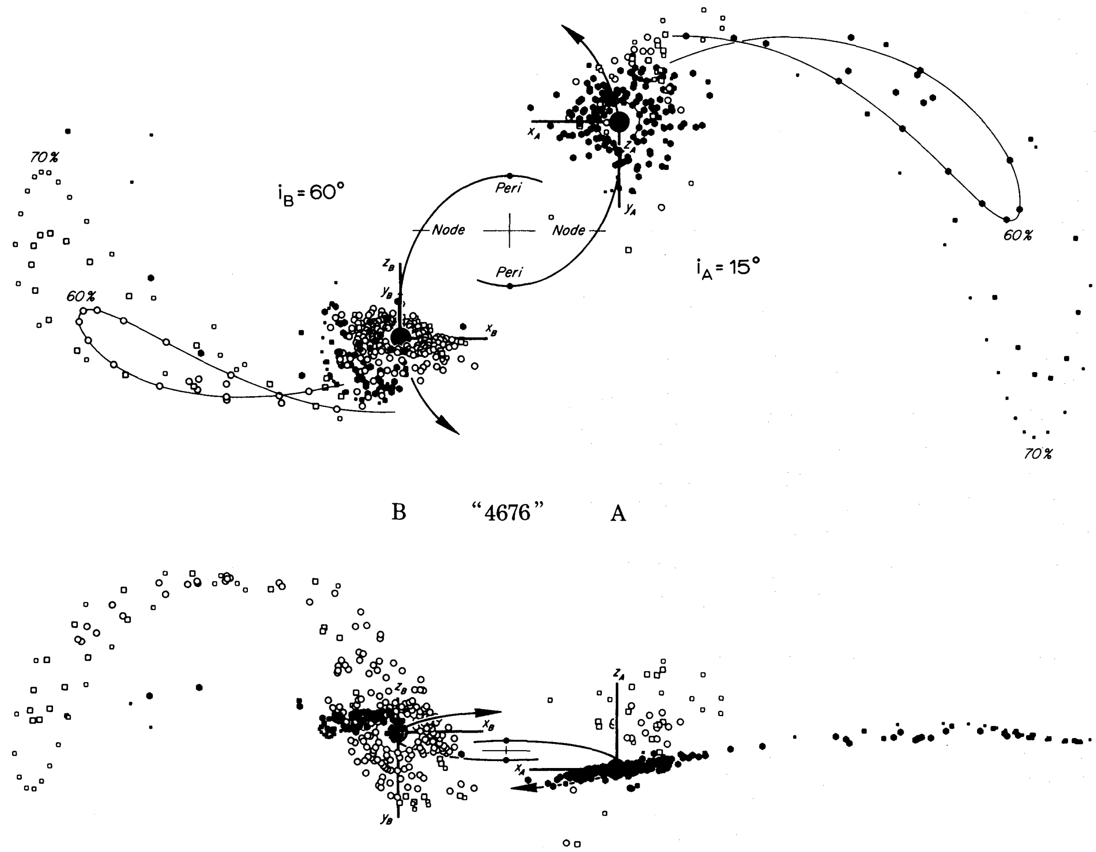

FIG. 22.—Model of NGC 4676. In this reconstruction, two equal disks of radius $0.7R_{\rm min}$ experienced an $e = 0.6$ elliptic encounter, having begun flat and circular at the time $t = -16.4$ of the last apocenter. As viewed from either disk, the adopted node-to-peri angles $\omega_A = \omega_B = -90°$ were identical, but the inclinations differed considerably: $i_A = 15°$, $i_B = 60°$. The resulting composite object at $t = 6.086$ (cf. fig. 18) is shown projected onto the orbit plane in the upper diagram. It is viewed nearly edge-on to the same—from $\lambda_A = 180°$, $\beta_A = 85°$ or $\lambda_B = 0°$, $\beta_B = 160°$—in the lower diagram meant to simulate our actual view of that pair of galaxies. The filled and open symbols distinguish particles originally from disks A and B, respectively.

rather than elaborate, we chose the masses and loadings to be identical, did the same with the simple $\omega_A = \omega_B = -90°$, picked the round values $i_A = 15°$, $i_B = 60°$, chose the viewing longitude to be simply along the line of pericenters, and retained both the eccentricity and the 135° viewing time already used in figure 18. Thus the B object in figure 22 is virtually an “off-the-shelf” item. It differs from its predecessor in figure 18 only in the coding of certain of its particles, the display of its not-too-offensive accretion cloud above mass A, and a 45° more advanced longitude and 10° different latitude of viewing.

One almost incidental advantage of the present model is that, like the real tail A, ours looks slightly concave downward—or toward the west. More important is its agreement with the rough sense of the velocities measured in hulk B by the Burbidges: although our remnant B lacks the oval outline of the real object, its excess Doppler speeds are likewise positive and negative in the “north” and “south,” respectively. (Moreover, the fact that our remnant B happens to be viewed only 20° from face-on cautions that the actual rotation in 4676B could well be twice the observed ± 200 km s^{-1}, and hence also about twice the nominal speeds of our retained test particles. Thus if we equate the observed 4′ tip-to-tip length of 4676 with the $9R_{\min}$ total extent of this model, and if we also place 4676 at a reference distance of 100 Mpc, the model time since pericenter translates to roughly 1.2×10^8 years, and the full orbit period to only 6×10^8 years. This remarkably short timescale is a point we shall return to in § VII*b*.) Finally, even our "stolen" particles from A which now nestle beneath tail B in figure 22 may help explain the short *second* wisp or tail that accompanies the real 4676B.

The following difficulties remain:

1. It would be reassuring if the corresponding material from B predicted above A were itself plainly visible. (Of course, this material may simply be too splattered, or just fainter in proportion to the intensities of tails A and B themselves.)

2. In its simplicity, figure 22 contrives to view tail A too nearly *end*-on. Also, the two hulks seem too far apart. For these reasons alone, we would be tempted on a second round to diminish the viewing longitude λ_A by as much as 30°. (Face-on, our tail A may also seem too broad—but remember the likely "snowplowing.")

3. Though the detection by Theys *et al.* of [O II] λ3727 emission from almost the entire tail A indeed seems to corroborate that this appendage is not very exotic, their estimate (based on that *single* exposure and line doublet) of excess redshifts in the tail ranging up to at least $+400$ km s^{-1} relative to the remaining hulk is rather worrisome. The blunt truth is that even the *total* speeds relative to mass A of the farthest tail particles originally from the 0.55, 0.6, 0.65, and $0.7R_{\min}$ rings in figure 22 are only 50, 71, 89, and 92 percent of the unperturbed speed of rotation of the $0.4R_{\min}$ test particles. (We cite the latter because it seems representative of about the largest rotational motions we would reliably ascribe to our simulated remnant. In the real 4676A, such velocity extremes are perhaps ± 350 km s^{-1}, judging from a slight extrapolation and deconvolution of the Burbidges' ± 225 km s^{-1}.) Even worse, those relative velocity vectors are roughly *orthogonal* to the present line of sight!

At the very least, item 3 reinforces the impression from item 2 that our model should more correctly be viewed 30° or 45° further from the left (and perhaps slightly earlier as well, to keep the hulks clear of each other in projection). Certainly that way one obtains relative speeds that appear positive throughout tail A. However, even then—assuming that this tail is indeed tidal, and hence was not so much ejected as simply failed to turn the corner during the encounter—it seems very difficult for any of its line-of-sight velocities to exceed appreciably one-half the present maximum rotation in hulk A itself.

4. Most important, any tidal model of the "Mice" that we can imagine differs categorically with the receding sense of the $+100$ km s^{-1} relative motion of hulk B with respect to A that was reported by the Burbidges (1961)—that is to say, once one accepts, as we have done, their evidence on the *sense* of rotation of 4676A. Quite simply, the trouble is that the adoption of a counterclockwise spin A in the top view in figure 22 compels the sense of orbital motion to be likewise. It is then all but impossible geometrically for the center of mass of B not to be approaching us relative to that of A. Fortunately for us, the Burbidges' estimate of ΔV seems uncertain enough, since it stems from *separate* spectra. And of course also, it strictly refers only to the centers of light. However, unless convincing future observations manage to reverse it in sign—and preferably also to double it—our model will be fundamentally in error.

d) *The "Antennae," NGC 4038/9*

The reconstruction which gave us the most pleasure is the highly idealized one of NGC 4038/9 (= VV 245 = Arp 244; see also Duncan 1923; Shapley and Paraskevopoulos 1940; Zwicky 1956; Burbidge and Burbidge 1966; Rubin, Ford, and d'Odorico 1970) pictured in figure 23. Until we produced something like it, we had been concerned whether in fact it was possible to obtain seemingly *crossed* tails from tidal interactions. Figure 23 says we need not have worried.

By using figure 18, it is quite easy in retrospect to grasp the geometric essentials of this construction. Imagine that every bare companion in that ω survey carries an $i = +60°$ tail of its own, each such tail having been chosen from among the present four possibilities simply after a 180° visual rotation about the axis normal to the orbit

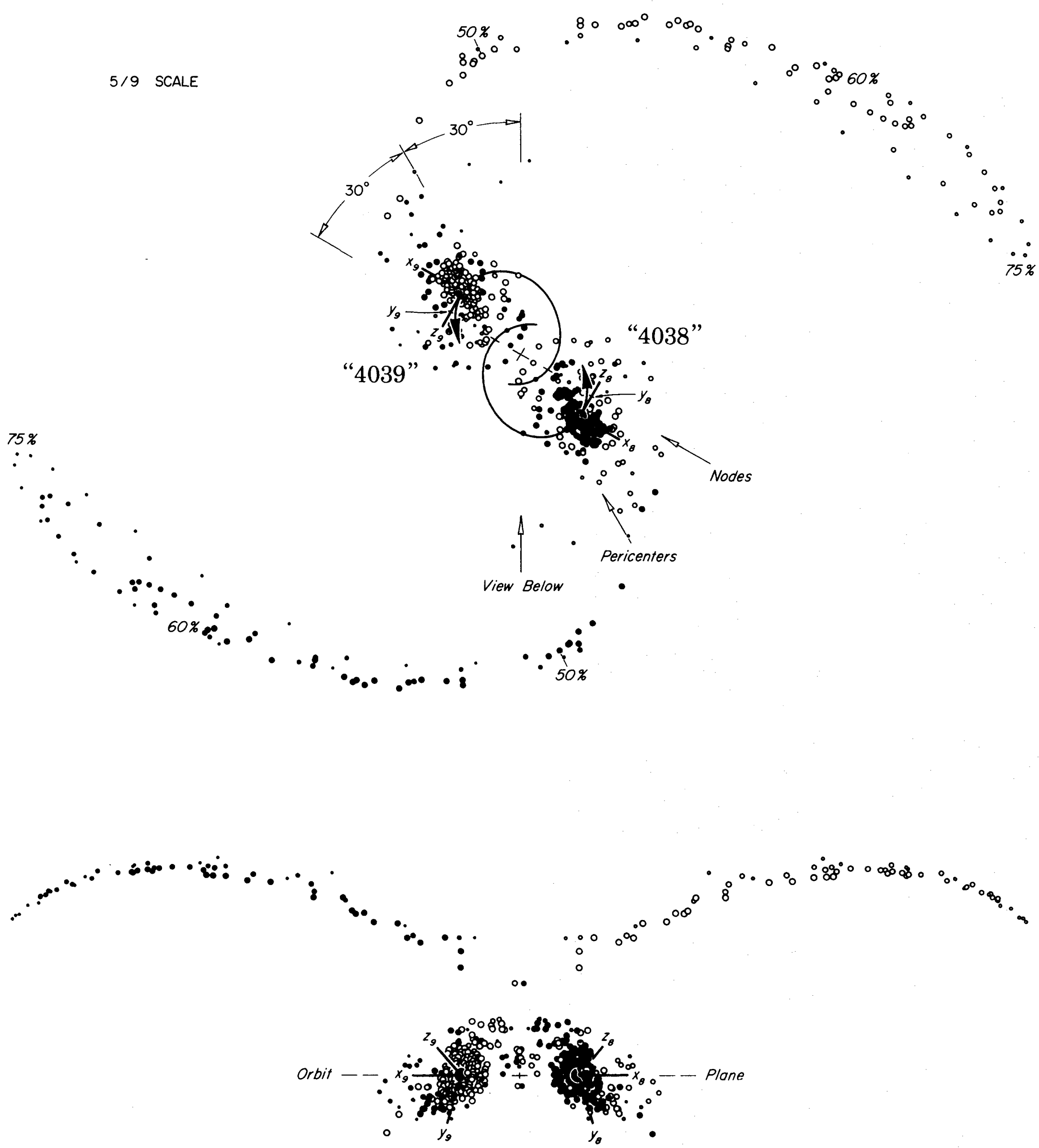

FIG. 23.—Symmetric model of NGC 4038/9. Here two identical disks of radius $0.75R_{\min}$ suffered an $e \approx 0.5$ encounter with orbit angles $i_8 = i_9 = 60°$ and $\omega_8 = \omega_9 = -30°$ that appeared the same to both. The above all-inclusive views of the debris and remnants of these disks have been drawn exactly normal and edge-on to the orbit plane; the latter viewing direction is itself 30° from the line connecting the two pericenters. The viewing time is $t = 15$, or slightly past apocenter. The filled and open symbols again disclose the original loyalties of the various test particles.

plane.[6] Clearly, then, no tail crossings are to be found in any view that is roughly perpendicular to the orbits. We can also exclude all other views from near the great circle normal to the line that presently connects the two masses; in such views, too, no part of one tail can be much closer to the other than the distance between the hulks themselves.

By contrast, views from more or less along that connecting line offer rich possibilities. Indeed exactly from that direction, as one of the lower views in figure 18 implies, the present $\omega = -45°$ tail together with its identical twin attached to the other mass looks very much like a pair of symmetric gull wings. The same can be said of the $\omega = -90°$ and $\omega = 0°$ tails when paired with their respective twins, and almost the same about either of them partnered with the $\omega = -45°$ ribbon.[7] Only the $\omega = +45°$ shape lacks obvious merit in this respect. To obtain crossed tails as in figure 23, it remains only to view something like the above $\omega = -45°$ pair from a slightly reduced orbital longitude.

The precise orbital parameters of figure 23 were chosen merely to conform with our 4038/9 movie (Toomre and Toomre 1971). Here the objects are viewed proportionately later than the "Mice" in figure 22, to match better the ratio of tail lengths to body diameters actually seen in the "Antennae." In this final example alone, the forces from each massive body were *softened* gradually at close range by pretending the potentials to vary with distance r as $-GM/(r^2 + a^2)^{1/2}$ (where $a = 0.2R_{\min}$), instead of as $-GM/r$. Without this gravity softening to mimic the force from the distributed mass of the real objects, too many near-side particles extracted from either disk would otherwise have splattered into distracting sheets (cf. fig. 16) beyond each aggressor. A side benefit of this lessened near-gravity was the further thinning of the tails.

Certainly this symmetric model is too simple: (1) The real tails are unequal in length. (2) The actual 4038 tail is more curved than ours. (3) The rotations of the real hulks seem to be such (Rubin *et al.* 1970) that their adjacent ends approach and recede alike, whereas in our model, given the symmetry, those senses are necessarily opposite. (Here we have assigned the correct rotation to the "4038" hulk because in reality it carries the longer tail. This also helps it to recede relative to "4039" as observed.)

The first item above is not serious. It can be remedied by slight inequalities of either the masses or the inclinations or the initial outer radii. The second item, together with the remarkably disturbed "ring-tailed" (Shapley *et al.* 1940) appearance and seeming proximity of the actual galaxy hulks, is perhaps an indication that those actors are about to do it again. The third item, however, seems to require a substantial change of at least the argument ω_9 from $-30°$ to almost $-90°$, besides a further reduction of the longitude of viewing. So far, all our test-particle efforts with such ω_9 have resulted in "4039" tails which appear rather too thick. But without the self-gravity, we cannot say whether that is a real handicap.

Happily, all these labors relate not only to NGC 4038/9: As Arp first told us, there is a small similar pair of crossed tails, discovered by Herzog, at $\alpha(1970) = 16^{h}11^{m}$, $\delta = +52°30'$.

VII. BROADER ISSUES

Taken together with the pioneering work of Pfleiderer and Siedentopf—plus the recent efforts of Tashpulatov, Yabushita, and Wright—the present demonstrations

[6] As a check, note that no such marriage, even with identical ω's, yet results in a combined object that is symmetric *through* the center of mass; in our terminology, such symmetry would require one of the two inclinations i to be reversed in sign as well.

[7] A pair of tails of *opposite* curvatures and looking rather like NGC 2623 (= Arp 243) would have been obtained in this viewing if the inclinations i of such partners had been chosen of opposite sign; in that case, one tail would have arched above and the other below the orbit plane.

should suffice as antidotes to such often-voiced sentiments as "a tidal perturbation of a galaxy can alter its shape, but cannot draw out a long narrow filament" (Gold and Hoyle 1959) or that "the bridges and tails of interacting galaxies . . . can hardly exist without a regular magnetic field" (Pikel'ner 1968).

We also suspect, however, that this work has somewhat wider implications than just that certain thin bridges or tails of a few pathological galaxies *can* indeed have arisen from violent tides. We conclude by discussing four such points, on the pretext that here, as in medicine, pathology seems instructive.

a) *Eccentric Bound Orbits*

Our first topic will probably seem much overdue to every thoughtful reader: Though one can certainly postulate some very close encounters on a computer, how did nearly enough of the real galaxies recently get into such a predicament?

For much of the tidal damage considered here, we believe emphatically that the answer lies *not* in accidental, pairwise hyperbolic passages of two galaxies, not even as otherwise bound members of a larger group or cluster. The objections are threefold. First, among the nearly 20 examples cited thus far, neither the apparent separations of the presumed participants, nor the relative tail lengths or quality of the bridges, nor even any differential redshifts exactly encourage the view that the relative orbits could have been very hyperbolic. (Judging from those shapes and separations—plus the few calculations we have performed for $e > 1$—a reasonable upper bound to the eccentricity is perhaps $e = 2$, and in some cases clearly lower.) Second, the probability that enough NGC galaxies in *loose* groups or clusters should have suffered close encounters during, say, the last 10^9 years seems marginal at best if one insists that none were bound pairwise. It becomes prohibitive if we require each encounter to have been just about grazing, and its speed not much greater than parabolic. Third, very striking tails and bridges seem, if anything, underabundant in *dense* clusters of galaxies where chance hyperbolic passages are obviously most numerous (cf. Vorontsov-Velyaminov 1958, 1961; Zasov 1968). On the other hand, it is just there that any initially formed long-period binaries would have stood in greatest danger of dissolution by the transit of a third galaxy.

Hence one must presume that the partners in most cases were already bound to each other prior to their latest encounter. By this we mean only that their approach orbits must have been subparabolic. We do not remotely suggest they orbited in circles.

Of course, such inferences that the incoming $0.5 \leqslant e < 1$ refer explicitly to only a handful of galaxies. Yet one imagines that those recent interactors are typical of at least a tenfold larger population, namely, all those similar pairs which today happen to be in considerably more distant—or near but approaching—phases of their own close-plunging orbits.

We wish to emphasize that the above are not the only hints that galactic binary orbits tend to be highly eccentric: (1) The Large Magellanic Cloud could not possibly have warped the Galaxy enough by its gravity from anything like its present distance (Hunter and Toomre 1969). (2) The like must be true of M66 (= NGC 3627 = Arp 16), which is probably the culprit that bent NGC 3628 and also caused its faint linear extension already known to Zwicky (1956, fig. 6). That the blame indeed belongs to M66 is corroborated not so much by its asymmetric spiral structure as by two hooklike faint extensions to the northwest of M66 which can be recognized in the group portrait Arp 317 and which are unmistakable in a deep IIIaJ photograph that Sandage showed us recently. (3) Rood (1965) and especially Faber (1973) have proposed that the envelope of the compact elliptical galaxy NGC 4486B has been severely truncated by tides during its orbital passage(s) much closer to M87. (4) Present tides seem even less adequate between the noticeably lopsided M101 and its strange companion NGC 5474 whose nucleus is especially off-center or "the front of which is disintegrated" (Vorontsov-Velyaminov 1958). Yet, as Beale and Davies (1969) remarked in a brief but eloquent discussion of that situation, "an approach to within one radius would produce additional mechanical effects which could severely distort the outer regions of both the parent and the companion." We can only agree.

b) *Orbit Decay*

Our second concern is with changes to the galactic orbits themselves as the result of an encounter. We claim almost no expertise here, since our idealized central masses obviously suffered no such changes. Yet the circumstantial evidence below seems serious enough to warrant the following questions:

When two roughly equal disk galaxies in a previously long-period orbit at last experience a close approach and raise really violent tides in each other, can it be that they also surrender a significant fraction of their orbital energy (and angular momentum) to the tail-making particles, many of which of course escape to infinity? And hence would not their remnants drop into orbits of progressively shorter periods, until at last they lose altogether their separate identities and simply blend or tumble into a single three-dimensional pile of stars?

If true, this seems a scenario for nothing less than the delayed formation of some elliptical galaxies—or at least of major stellar halos from otherwise gas-rich disks. But more to the point, it would also help solve two vexing observational puzzles, stated below.

1. It is curious that the main surviving galactic bodies not only in the well-known NGC 4038/9 and 4676 but also in others, such as NGC 7592 or the Herzog gull wings, appear separated only by small fractions of their tail lengths. Indeed, we can think of only one NGC system with a *pair* of major tails, 2992/3 (= Arp 245), of which this cannot be said. Yet in NGC 2623 (= Arp 243) it is already difficult to distinguish the surviving hulks from one another. Even allowing for accidents of projection, it seems scarcely credible, from figure 16 among others, that the eccentricities of most of those *present* orbits could exceed 0.5 or their periods exceed 10^9 years. Without orbital deceleration, then, it is a complete puzzle why such galaxies, if of a nominal age of 10^{10} years, had not destroyed each other long ago. And even if much younger, what will happen to them and their tails in the next 10^9 years? Should not one also observe such more advanced remnants?

2. Indeed there do exist several strange-looking NGC galaxies, notably 6621/2, 3509, 520, 7727, 3921, 7252, 455, and perhaps even 7603[8] (= Arp 81, 335, 157, 222, 224, 226, 164, and 92), which seem plausible examples of progressively later stages of the kind of coalescence envisaged above. Each of the first two, discussed in rather different terms by Burbidge *et al.* (1963), exhibits one broad crescent of a tail which to us seems clearly tidal. Yet the culprit especially in 3509, lacking a major tail of its own, seems sufficiently intermingled with the victim that it "cannot be located with certainty at the present time"—to borrow words which Zwicky (1956, plate IX; = Arp 188) once applied to his similarly puzzling "spiral galaxy with one long arm." Our next candidate, NGC 520, was indeed classified by Sandage in the *Hubble Atlas* as but a *single* Irr II galaxy! In the last five cases, as in IC 883 (= Arp 193), there is only a single luminous ball, from which protrude several tentacles or filaments, much as one would imagine at a fairly advanced stage of a merger.

Even these thoughts are not novel in principle. The literature abounds with warnings, dating back at least to the ingenious and still astonishingly relevant optical-numerical

[8] NGC 7603 appears linked by a faint double "bridge" to a small companion which Arp (1971*a*) recently found to be redshifted 8000 km s^{-1} more than the main galaxy. Even if one imagines that those two galaxies superpose merely by chance, there remains the problem of why 7603 itself exhibits those and other wisps. Our remarks here are directed only toward this second question.

N-body computations of Holmberg (1941), that low-velocity galactic encounters are apt to be inelastic. The only shortage is one of reliable estimates of the amount of that "stickiness"—inasmuch as the calculations of Holmberg must be deemed at least slightly suspect because they did not exhibit any of the by-now-notorious intrinsic instabilities of the separate disks, and since such more recent work as that of Alladin (1965) merely treats the tidal action as impulsive. Perhaps the best evidence so far consists of the incidental findings of both Hohl and Prendergast (1971, private communications) from their respective $\sim 10^5$-body numerical experiments that various condensations or subclusters which form during the violent Jeans instability of a single cold disk tend eventually to congeal back into one.

As a small step in the same direction, we ourselves have performed a few computations which took the former test particles to be not quite massless. In the spirit of classical perturbation theory, and to first order in their masses, we merely integrated their back-reactions upon the initially parabolic motions of the heavy central masses. This preliminary work indicates that a net effect of all the tidal "splash" that occurs during a moderately *high-inclination* passage of equal disks is indeed to *decrease* the orbital eccentricity, and hence also the period since the pericenter distance remains relatively unaltered. Moreover, an obviously shaky extrapolation of those perturbation results to a substantial disk mass suggests that a change $\Delta e = -0.2$ or even -0.3 might be achieved during a single encounter that is about as close as the one in our NGC 4038/9 simulation. This change may not seem large. But notice that a decrease from $e = 0.8$ to $e = 0.5$ would already reduce the period by almost a factor of four.[9]

It is also instructive to estimate the number of very old tumbled remains that one should see today. For this let us suppose that each of the above dozen or so examples would visibly retain the incriminating tail(s) and similar evidence for at most 10^9 years. Then if galaxy pairs in the past merged at only the current rate, one would expect at least 10 times as many remnants altogether from the past 10^{10} years. But it is also clear that the "storage" of any pair of potential interactors in a very eccentric orbit with period of order 10^{10} years is *a priori* quite unlikely: For instance, if a population of otherwise identical binaries had been created with a flat statistical distribution of relative *speeds* down at the same pericenter distance, then the number dN of long periods in the range $(P, P + dP)$ would vary about as $dN \propto P^{-5/3}dP$. Hence if indeed practically all binary galaxies formed long ago, our recent violent mergers have probably been occurring at a rate at least three times less frequent than, say, at an epoch when the universe was only one-half as old. Conversely, the total number of all "elliptical" remnants of the sort envisaged here may well exceed 500 of the 4000-odd existing NGC galaxies.

c) *Stoking the Furnace?*

We have deliberately not touched earlier on the well-known tendency (e.g., Burbidge *et al.* 1963; Zwicky 1967; Arp 1969*b*, 1971*b*; Stockton 1972) of the various tails, plumes, and intergalactic bridges to involve at least one galaxy whose own color or spectrum is often unusual, or which has a high surface brightness, or which contains oddly placed absorbing material and/or emitting regions.

That such intrinsic evidence of "strangeness" has itself contributed to the reluctance to regard the external features as tidal is both clear and understandable. Nevertheless —well short of such really exotic cases as the "jets" of M87 and 3C 273—we cannot help feeling that even this share of reluctance has been somewhat excessive: Would not the violent mechanical agitation of a close tidal encounter—let alone an actual

[9] Quite understandably, partners of very unequal mass seem to suffer much less deceleration. Also, we have noticed for several mass ratios that the orbital eccentricity and angular momentum in *low-inclination* passages can actually increase slightly at the expense of spin.

merger—already tend to bring *deep* into a galaxy a fairly *sudden* supply of fresh fuel in the form of interstellar material, either from its own outlying disk or by accretion from its partner? And in a previously gas-poor system or nucleus, would not the relatively mundane process of prolific star formation thereupon mimic much of the "activity" that is observed?

d) *Certain Exceptional Spirals?*

Lastly, we must return to those unusually fine spirals M51 and NGC 7753. As regards the former, we saw in § VI*b* that both its outer arms are plausibly tidal. Yet the same test particles showed *no* appreciable spiral structure in those deeper parts of the disk where the optical arms are most prominent, and where the recent Westerbork 21-cm continuum observations of Mathewson, van der Kruit, and Brouw (1972) and the Fabry-Perot work of Tully (1972) provide impressive support for *some* kind of a density wave—if not just the "quasi-stationary" and locally "self-sustained" spiral wave pattern first envisaged by Lin and Shu (1964, 1966) and claimed (rather vaguely) to have been corroborated for M51 by Shu, Stachnik, and Yost (1971).

In NGC 7753, where the outer structure seems again tidal, and where again one arm appears to lead to a small and *intrinsically interesting* (Arp 1969*b*) companion, the inner structure seems even better developed and more regular than in M51. Naturally one wonders in both cases if there might not be some logical connection between the outside tidal damage and the presumed waves deeper inside.

Of course, it is conceivable that the tidal arms in both M51 and NGC 7753 were merely superposed onto preexisting fine spirals. However, there exist two serious reasons for suspecting that even the density waves in these exceptionally striking galaxies may be *transient* rather than long-lived, and that they were caused *indirectly* by the recent external influences. These reasons follow.

1. Once we accept any tidally provoked "material arms" near the periphery of a disk, those arms—so long as they persist—obviously represent substantial and relatively slowly rotating mass asymmetries whose own gravity must in reality be felt even by the interior. In fact, each tidal arm should have about the same gravitational effect upon the inside disk as would a very close-orbiting direct satellite. And in that connection, recall that already Julian and Toomre (1966, esp. fig. 11) showed that an orbiting concentrated mass within a galactic disk tends to evoke a disproportionately strong *forced* spiral *wave* in its immediate vicinity.

2. We also know from Toomre's (1969) study of group transport that any slow spiral wave which locally obeys the Lin-Shu dispersion relation must propagate *inward*—once tightly enough wrapped, as in principle by the kinematic shear pictured in figure 5 of that paper.[10] It eventually disappears unless somehow replenished.

At the very least, these remarks caution against taking it for granted that either M51 or NGC 7753 is basically a "self-made" spiral. Conceivably this warning should now be extended even to another well-known galaxy: Especially on *Sky Survey* prints, its outer structure again seems suspiciously wide-open—indeed, it is slightly reminiscent of the $i = 60°$ picture in figure 13. And it again has an intriguing Irr II neighbor. *Et tu*, M81?

[10] Otherwise, that particular diagram contains two errors of emphasis which we would now like to rectify: (i) The very vehemence of the outer spiral-making studied in the present paper (see also Toomre 1970, 1972) seems to rule out the imagined *direct* close passage of the LMC. (ii) The old figure also overplayed the *immediate* role of tidal forces in exciting noticeable oval distortions, and hence spiral waves, in the interior. The present indirect sequence of the peripheral tidal arms in turn influencing the interior seems, on further thought, much more realistic.

Among all those who have generously advised, educated, and/or encouraged us in this undertaking, we thank especially Drs. H. C. Arp, S. van den Bergh, C. C. Lin, D. Lynden-Bell, J. Pfleiderer, K. H. Prendergast, A. R. Sandage, E. A. Spiegel, and R. B. Tully.

This work has been supported in part by the National Science Foundation. Yet it would scarcely have been possible without also (i) a Guggenheim Fellowship and the warm hospitality of Drs. P. Goldreich and J. L. Greenstein and various other Pasadena astronomers to A. T. in 1969–1970, and (ii) the computing and especially the graphic display facilities of the Goddard Institute for Space Studies that were made available to J. T. by Dr. R. Jastrow.

REFERENCES

Alladin, S. M. 1965, *Ap. J.*, **141**, 768.
Arp, H. 1962, *Ap. J.*, **136**, 1148.
———. 1966, *Atlas of Peculiar Galaxies* (Pasadena: California Institute of Technology); also *Ap. J. Suppl.*, **14**, No. 123.
———. 1969*a*, *Sky and Tel.*, **38**, 385.
———. 1969*b*, *Astr. and Ap.*, **3**, 418.
———. 1971*a*, *Ap. Letters*, **7**, 221.
———. 1971*b*, *Science*, **174**, 1189.
Beale, J. S., and Davies, R. D. 1969, *Nature*, **221**, 531.
Bergh, S. van den. 1969, *Ap. Letters*, **4**, 117.
Burbidge, E. M., and Burbidge, G. R. 1959, *Ap. J.*, **130**, 23.
———. 1961, *Ap. J.*, **133**, 726.
———. 1964, *ibid.*, **140**, 1445.
———. 1966, *ibid.*, **145**, 661.
Burbidge, E. M., Burbidge, G. R., and Hoyle, F. 1963, *Ap. J.*, **138**, 873.
Carranza, G., Crillon, R., and Monnet, G. 1969, *Astr. and Ap.*, **1**, 479.
Contopoulos, G., and Bozis, G. 1964, *Ap. J.*, **139**, 1239.
Danver, C. G. 1942, *Ann. Obs. Lund*, No. 10.
Duncan, J. C. 1923, *Ap. J.*, **57**, 137.
Faber, S. M. 1973, *Ap. J.*, **179**, in press.
Gold, T., and Hoyle, F. 1959, in *Paris Symposium on Radio Astronomy* (I.A.U. Symp. No. 9), ed. R. N. Bracewell (Palo Alto: Stanford University Press), p. 583.
Holmberg, E. 1941, *Ap. J.*, **94**, 385.
Hoyle, F., and Narlikar, J. V. 1971, *Nature*, **233**, 41.
Hunter, C., and Toomre, A. 1969, *Ap. J.*, **155**, 747.
Julian, W. H., and Toomre, A. 1966, *Ap. J.*, **146**, 810.
Lin, C. C., and Shu, F. H. 1964, *Ap. J.*, **140**, 646.
———. 1966, *Proc. Nat. Acad. Sci.*, **55**, 229.
Lindblad, P. O. 1960, *Stockholm Obs. Ann.*, **21**, No. 3.
Mathewson, D. S., Kruit, P. C. van der, and Brouw, W. N. 1972, *Astr. and Ap.*, **17**, 468.
Pfleiderer, J. 1963, *Zs. f. Ap.*, **58**, 12.
Pfleiderer, J., and Siedentopf, H. 1961, *Zs. f. Ap.*, **51**, 201.
Pikel'ner, S. B. 1965, *Astr. Zh.*, **42**, 515 (English transl. in *Soviet Astr.—AJ*, **9**, 408).
———. 1968, *Ann. Rev. Astr. and Ap.*, **6**, 165.
Roberts, M. S., and Warren, J. L. 1970, *Astr. and Ap.*, **6**, 165.
Rood, H. J. 1965, *A.J.*, **70**, 689.
Rubin, V. C., Ford, W. K., Jr., and d'Odorico, S. 1970, *Ap. J.*, **160**, 801.
Sandage, A. 1961, *The Hubble Atlas of Galaxies* (Washington: Carnegie Institution of Washington).
Shapley, H., and Paraskevopoulos, J. S. 1940, *Proc. Nat. Acad. Sci.*, **26**, 31.
Shu, F. H., Stachnik, R. V., and Yost, J. C. 1971, *Ap. J.*, **166**, 465.
Stockton, A. 1972, *Ap. J.*, **173**, 247.
Tashpulatov, N. 1969, *Astr. Zh.*, **46**, 1236 (English transl. in *Soviet Astr.—AJ*, **13**, 968).
———. 1970, *Astr. Zh.*, **47**, 277 (English transl. in *Soviet Astr.—AJ*, **14**, 227).
Theys, J. C., Spiegel, E. A., and Toomre, J. 1972, *Pub. A.S.P.*, **84**, in press.
Toomre, A. 1969, *Ap. J.*, **158**, 899.
———. 1970, in *The Spiral Structure of Our Galaxy* (I.A.U. Symp. No. 38), ed. W. Becker and G. Contopoulos (Dordrecht: Reidel Publishing Co.), p. 334.
———. 1972, *Quart. J.R.A.S.*, **13**, 266.
Toomre, A., and Toomre, J. 1970, *Bull. A.A.S.*, **2**, 350.
———. 1971, *ibid.*, **3**, 390.
———. 1972, *ibid.*, **4**, 214.

Tully, R. B. 1972, unpublished Ph.D. thesis, University of Maryland.
Vorontsov-Velyaminov, B. A. 1958, *Astr. Zh.*, **35**, 858 (English transl. in *Soviet Astr.—AJ*, **2**, 805).
———. 1959, *Atlas and Catalog of Interacting Galaxies* (Moscow: Sternberg State Astronomical Institute).
———. 1961, in *Problems of Extra-Galactic Research* (I.A.U. Symp. No. 15), ed. G. C. McVittie (New York: Macmillan), p. 194.
———. 1964, *Astr. Zh.*, **41**, 814 (English transl. in *Soviet Astr.—AJ*, **8**, 649).
Wild, P. 1953, *Pub. A.S.P.*, **65**, 202.
Wright, A. E. 1972, *M.N.R.A.S.*, **157**, 309.
Yabushita, S. 1971, *M.N.R.A.S.*, **153**, 97.
Zasov, A. V. 1967, *Astr. Zh.*, **44**, 975 (English transl. in *Soviet Astr.—AJ*, **11**, 785).
———. 1968, *Astrofizika*, **4**, 427 (English transl. in *Astrophysics*, **4**, 173).
Zwicky, F. 1952, *Pub. A.S.P.*, **64**, 242.
———. 1953, *Phys. Today*, **6**, No. 4, p. 7.
———. 1956, *Ergebnisse der Exakten Naturwissenschaften*, **29**, 344.
———. 1959, *Hdb. d. Phys.*, **53**, 373.
———. 1963, *A.S.P. Leaflet*, No. 403.
———. 1967, *Adv. Astr. and Ap.*, **5**, 267.
Zwicky, F., and Humason, M. L. 1961, *Ap. J.*, **133**, 794.