Homework 7: *Due March 11*

1. Consider two samples of low-density gases A and B, such that $P_B = \frac{1}{6}P_A$ and $T_B = 2T_A$. If the mean free path λ_B is 3 times the mean free path λ_A, what is the ratio of the molecular diameter d_B to the molecular diameter d_A?

2. Consider a sample of carbon dioxide (molecular weight 44, molecular diameter $4.6 \times 10^{-10} m$) at 1 atmosphere and 300 K. Determine the collision frequency, the mean free path and the ratio of the mean free path to the molecular diameter.

3. The mean free path of air molecules (O_2 or N_2) in a sample at 1 atmosphere and 300 K is almost $1.02 \times 10^{-7} m$. Using the same value of the molecular diameter d as used in the class, determine the mean free path of air molecules at a height of 10 kilometers above the surface of the Earth and at a temperature 300 K.

   [For simplicity, assume the atmosphere to be isothermal and use the fact that under this assumption,

   $$n(z) = n(0)exp\left[-\frac{mgz}{kT}\right],$$

   where the various symbols have their usual meanings. For m, the mass of an air molecule, take 29 amu].

4. In a room containing "still" air at 1 atmosphere and 25 °C, focus your attention on one molecule (of O_2 or N_2). At t = 0, this molecule is at a particular location which may be designated as $\vec{r} = 0$. Estimate the amount of time it will take for the molecule to reach a point 10 centimeters away from its initial location.

5. (Optional) Consider a large group of Brownian particles diffusing in and dispersing through a background fluid. The number density $n(\vec{r}, t)$ of these particles is governed by the diffusion equation

   $$\frac{\partial n(\vec{r}, t)}{\partial t} = D\,\nabla^2 n(\vec{r}, t),$$

   where D is the coefficient of diffusion and ∇^2 the laplacian operator.

   (a) Show, by substitution, that the function

   $$n(\vec{r}, t) = \frac{N_0}{(4\pi D\, t)^{3/2}} \exp\left(-\frac{r^2}{4\, D\, t}\right)$$

   satisfies the diffusion equation.
   [Note that the dependence of $n(\vec{r}, t)$ on $\vec{r}$ is through the magnitude r only, so ∇^2 is only radial.]

   (b) Using this distribution function, evaluate the mean-square value of the variable r at time t.