

Formulas and constants:

$hc = 12,400 \text{ eV}\cdot\text{Å}$; $k_B = 1/11,600 \text{ eV/K}$; $ke^2 = 14.4 \text{ eV}\cdot\text{Å}$; $m_e c^2 = 0.511 \times 10^6 \text{ eV}$; $m_p / m_e = 1836$

Relativistic energy - momentum relation $E = \sqrt{m^2 c^4 + p^2 c^2}$; $c = 3 \times 10^8 \text{ m/s}$

Photons: $E = hf$; $p = E/c$; $f = c/\lambda$ Lorentz force: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Photoelectric effect: $eV_0 = \left(\frac{1}{2}mv^2\right)_{\max} = hf - \phi$, $\phi \equiv$ work function

Integrals: $I_n \equiv \int_0^\infty x^n e^{-\lambda x^2} dx$; $\frac{dI_n}{d\lambda} = -I_{n+2}$; $I_0 = \frac{1}{2}\sqrt{\frac{\pi}{\lambda}}$; $I_1 = \frac{1}{2\lambda}$; $\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$

Planck's law : $u(\lambda) = n(\lambda)\bar{E}(\lambda)$; $n(\lambda) = \frac{8\pi}{\lambda^4}$; $\bar{E}(\lambda) = \frac{hc}{\lambda} \frac{1}{e^{hc/\lambda k_B T} - 1}$

Energy in a mode/oscillator: $E_f = nhf$; probability $P(E) \propto e^{-E/k_B T}$

Stefan's law : $R = \sigma T^4$; $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$; $R = cU/4$, $U = \int_0^\infty u(\lambda)d\lambda$

Wien's displacement law : $\lambda_m T = hc/4.96k_B$

Compton scattering: $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos\theta)$; $\lambda_c \equiv \frac{h}{m_e c} = 0.0243 \text{ Å}$

Rutherford scattering: $b = \frac{kq_1 q_2}{m_\alpha v^2} \cot(\theta/2)$; $\Delta N \propto \frac{1}{\sin^4(\theta/2)}$

Electrostatics: $F = \frac{kq_1 q_2}{r^2}$ (force) ; $U = q_0 V$ (potential energy) ; $V = \frac{kq}{r}$ (potential)

Hydrogen spectrum: $\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$; $R = 1.097 \times 10^7 \text{ m}^{-1} = \frac{1}{911.3 \text{ Å}}$

Bohr atom: $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $E_n = -E_0 \frac{Z^2}{n^2}$; $a_0 = \frac{\hbar^2}{mke^2} = 0.529 \text{ Å}$; $E_0 = \frac{ke^2}{2a_0} = 13.6 \text{ eV}$; $L = mvr = n\hbar$

$E_k = \frac{1}{2}mv^2$; $E_p = -\frac{ke^2 Z}{r}$; $E = E_k + E_p$; $F = \frac{ke^2 Z}{r^2} = m\frac{v^2}{r}$; $hf = hc/\lambda = E_n - E_m$

Reduced mass: $\mu = \frac{mM}{m+M}$; X-ray spectra: $f^{1/2} = A_n(Z-b)$; K: $b=1$, L: $b=7.4$

de Broglie: $\lambda = \frac{h}{p}$; $f = \frac{E}{h}$; $\omega = 2\pi f$; $k = \frac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \frac{p^2}{2m}$; $\hbar c = 1973 \text{ eV}\cdot\text{Å}$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg : $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$

Wave function $\Psi(x,t) = |\Psi(x,t)| e^{i\theta(x,t)}$; $P(x,t) dx = |\Psi(x,t)|^2 dx =$ probability

Schrodinger equation : $-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi(x,t) = i\hbar \frac{\partial \Psi}{\partial t}$; $\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$

Time-independent Schrodinger equation: $-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx \psi^* \psi = 1$

∞ square well: $\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$; $E_n = \frac{\pi^2 \hbar^2 n^2}{2mL^2}$; $x_{op} = x$, $p_{op} = \frac{\hbar}{i} \frac{\partial}{\partial x}$; $\langle A \rangle = \int_{-\infty}^{\infty} dx \psi^* A_{op} \psi$

Eigenvalues and eigenfunctions: $A_{op} \Psi = a \Psi$ (a is a constant) ; uncertainty: $\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$

Harmonic oscillator: $\Psi_n(x) = C_n H_n(x) e^{-\frac{m\omega}{2\hbar} x^2}$; $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$; $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$; $\Delta n = \pm 1$

Step potential: $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\frac{2m}{\hbar^2}(E - V)}$

Tunneling: $\psi(x) \sim e^{-\alpha x}$; $T \sim e^{-2\alpha \Delta x}$; $T \sim e^{-2 \int_a^b \alpha(x) dx}$; $\alpha(x) = \sqrt{\frac{2m[V(x) - E]}{\hbar^2}}$

3D square well: $\Psi(x,y,z) = \Psi_1(x)\Psi_2(y)\Psi_3(z)$; $E = \frac{\pi^2 \hbar^2}{2m} \left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2} \right)$

Spherically symmetric potential: $\Psi_{n,\ell,m}(r,\theta,\phi) = R_{n\ell}(r)Y_{\ell m}(\theta,\phi)$; $Y_{\ell m}(\theta,\phi) = f_{\ell m}(\theta)e^{im\phi}$

Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$; $L_z = \frac{\hbar}{i} \frac{\partial}{\partial \phi}$; $L^2 Y_{\ell m} = \ell(\ell+1)\hbar^2 Y_{\ell m}$; $L_z = m\hbar$

Radial probability density: $P(r) = r^2 |R_{n,\ell}(r)|^2$; Energy: $E_n = -13.6eV \frac{Z^2}{n^2}$

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Spin 1/2: $s = \frac{1}{2}$, $|S| = \sqrt{s(s+1)}\hbar$; $S_z = m_s \hbar$; $m_s = \pm 1/2$; $\vec{\mu}_s = \frac{-e}{2m_e} g \vec{S}$

Total angular momentum: $\vec{J} = \vec{L} + \vec{S}$; $|J| = \sqrt{j(j+1)}\hbar$; $|l-s| \leq j \leq l+s$; $-j \leq m_j \leq j$

Orbital + spin mag moment: $\vec{\mu} = \frac{-e}{2m} (\vec{L} + g\vec{S})$; Energy in mag. field: $U = -\vec{\mu} \cdot \vec{B}$

Two particles : $\Psi(x_1, x_2) = +/ - \Psi(x_2, x_1)$; symmetric/antisymmetric

Screening in multielectron atoms: $Z \rightarrow Z_{\text{eff}}$, $1 < Z_{\text{eff}} < Z$

Orbital ordering:

$1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p < 6s < 4f < 5d < 6p < 7s < 6d \sim 5f$

Justify all your answers to all problems. Write clearly.