

Formulas and constants:

$hc = 12,400 \text{ eV}\cdot\text{Å}$; $k_B = 1/11,600 \text{ eV/K}$; $ke^2 = 14.4 \text{ eV}\cdot\text{Å}$; $m_e c^2 = 0.511 \times 10^6 \text{ eV}$; $m_p / m_e = 1836$

Relativistic energy - momentum relation $E = \sqrt{m^2 c^4 + p^2 c^2}$; $c = 3 \times 10^8 \text{ m/s}$

Photons: $E = hf$; $p = E/c$; $f = c/\lambda$ Lorentz force: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Photoelectric effect: $eV_0 = (\frac{1}{2}mv^2)_{\text{max}} = hf - \phi$, $\phi \equiv$ work function

Integrals: $I_n \equiv \int_0^\infty x^n e^{-\lambda x} dx$; $\frac{dI_n}{d\lambda} = -I_{n+1}$; $I_0 = \frac{1}{\lambda}$; $I_1 = \frac{1}{\lambda^2}$; $\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$

Planck's law : $u(\lambda) = n(\lambda)\bar{E}(\lambda)$; $n(\lambda) = \frac{8\pi}{\lambda^4}$; $\bar{E}(\lambda) = \frac{hc}{\lambda} \frac{1}{e^{hc/\lambda k_B T} - 1}$

Energy in a mode/oscillator: $E_f = nhf$; probability $P(E) \propto e^{-E/k_B T}$

Stefan's law : $R = \sigma T^4$; $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$; $R = cU/4$, $U = \int_0^\infty u(\lambda)d\lambda$

Wien's displacement law : $\lambda_m T = hc/4.96k_B$

Compton scattering: $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos\theta)$; $\lambda_c \equiv \frac{h}{m_e c} = 0.0243 \text{ Å}$

Rutherford scattering: $b = \frac{kq_\alpha Q}{m_\alpha v^2} \cot(\theta/2)$; $\Delta N \propto \frac{1}{\sin^4(\theta/2)}$

Electrostatics: $F = \frac{kq_1 q_2}{r^2}$ (force) ; $U = q_0 V$ (potential energy) ; $V = \frac{kq}{r}$ (potential)

Hydrogen spectrum: $\frac{1}{\lambda} = R(\frac{1}{m^2} - \frac{1}{n^2})$; $R = 1.097 \times 10^7 \text{ m}^{-1} = \frac{1}{911.3 \text{ Å}}$

Bohr atom: $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $E_n = -E_0 \frac{Z^2}{n^2}$; $a_0 = \frac{\hbar^2}{mke^2} = 0.529 \text{ Å}$; $E_0 = \frac{ke^2}{2a_0} = 13.6 \text{ eV}$; $L = mvr = n\hbar$

$E_k = \frac{1}{2}mv^2$; $E_p = -\frac{ke^2 Z}{r}$; $E = E_k + E_p$; $F = \frac{ke^2 Z}{r^2} = m \frac{v^2}{r}$; $hf = hc/\lambda = E_n - E_m$

Reduced mass: $\mu = \frac{mM}{m+M}$; X-ray spectra: $f^{1/2} = A_n(Z-b)$; K: $b=1$, L: $b=7.4$

de Broglie: $\lambda = \frac{h}{p}$; $f = \frac{E}{h}$; $\omega = 2\pi f$; $k = \frac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \frac{p^2}{2m}$; $\hbar c = 1973 \text{ eV}\cdot\text{Å}$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg : $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$

Justify all your answers to all problems

Problem 1 (10 pts)

(a) Compare the following properties of an electron in the lowest energy state in the hydrogen atom with an electron in the first excited state of the He^+ ion, ignoring the reduced mass correction. Denoting with subindex 1 the property of the electron in the H atom and with subindex 2 the property of the electron in the He^+ ion, with property (P) being energy, radius, etc, find the ratio P_2/P_1 for the following properties:

(i) total energy; (ii) speed ; (iii) angular momentum ; (iv) radius ; (v) kinetic energy (vi) potential energy ; (vii) ionization energy=energy required to remove the electron from the atom/ion.

Example: (i) find E_2/E_1 , where E_1 =total energy of electron in $n=1$ level of H, E_2 =total energy of electron in $n=2$ level of He^+ . etc.

(b) Repeat (a) taking into account the reduced mass correction. Use that the ratio of the nucleon mass (proton or neutron) to the electron mass is approximately 1836. Helium has four nucleons.

Problem 2 (10 pts)

Electrons of de Broglie wavelength $\lambda=0.08\text{\AA}$ hit a gold foil ($Z=79$) and cause X-rays to be emitted. A plot of the intensity of emitted X-rays versus their wavelength yields a smooth curve starting at a minimum wavelength and some sharp peaks.

(a) Find the kinetic energy of the incident electrons, in eV.

(b) Find the minimum wavelength of emitted X-rays, in \AA .

(c) With increasing wavelengths above this minimum wavelength, a first group of sharp peaks is seen in the X-ray intensity versus wavelength plot. Does it correspond to the K series or the L series? Find the value of the maximum wavelength in the group and give the quantum numbers associated with the transition it corresponds to.

Problem 3 (10 pts)

An electron located around the origin ($x=0$) is described by a wavepacket with wavenumbers in the range $k=10\text{\AA}^{-1}$ to $k=14\text{\AA}^{-1}$ at time $t=0$.

(a) Estimate the uncertainty in the position of the electron, in \AA .

(b) Estimate the uncertainty in the momentum of the electron, in eV/c .

(c) Find an expression for the wavefunction of this electron at $t=0$, assuming the amplitude of the waves in the range 10\AA^{-1} to 14\AA^{-1} is constant. Using this expression estimate the ratio of the magnitude of the wavefunction in the neighborhood of position $x\sim 20\text{\AA}$ to the magnitude of the wavefunction at $x=0$.

Justify all your answers to all problems