7-29. (a) Every increment of charge follows a circular path of radius R and encloses an area

πR^2, so the magnetic moment is the total current times this area. The entire charge

Q rotates with frequency $f = \omega / 2\pi$, so the current is

$$i = Qf = q\omega / 2\pi$$

$$\mu = iA = \left(Q\omega / 2\pi\right)\left(\pi R^2\right) = Q\omega R^2 / 2$$

$$L = I\omega = \frac{1}{2} MR^2\omega$$

$$g = \frac{2M\mu}{QL} = \frac{2MQ\omega R^2 / 2}{QMR^2\omega / 2} = 2$$

(b) The entire charge is on the equatorial ring, which rotates with frequency $f = \omega / 2\pi$.

$$i = Qf = Q\omega / 2\pi$$

$$\mu = iA = \left(Q\omega / 2\pi\right)\left(\pi R^2\right) = Q\omega R^2 / 2$$

$$g = \frac{2M\mu}{QL} = \frac{2MQ\omega R^2 / 2}{QMR^2\omega / 5} = 5/2 = 2.5$$

7-33. (a) There should be four lines corresponding to the four m_J values −3/2, −1/2, +1/2, +3/2.

(b) There should be three lines corresponding to the three m_ℓ values −1, 0, +1.

7-35. For

$\ell = 2, \quad L = \sqrt{\ell(\ell+1)}\hbar = \sqrt{6}\hbar = 2.45\hbar, \quad j = \ell \pm 1/2 = 3/2, 5/2$ and $J = \sqrt{j(j+1)}\hbar$

For $j = 3/2, \quad J = \sqrt{(3/2)(3/2+1)}\hbar = \sqrt{15/4}\hbar = 1.94\hbar$

For $j = 5/2, \quad J = \sqrt{(5/2)(5/2+1)}\hbar = \sqrt{35/4}\hbar = 2.96\hbar$

7-39. (a) $\boldsymbol{L} = \boldsymbol{L}_1 + \boldsymbol{L}_2$

$$\ell = (\ell_1 + \ell_2),\ (\ell_1 + \ell_2 - 1), \ldots, |\ell_1 - \ell_2| = (1+1),\ (1+1-1),\ (1-1) = 2, 1, 0$$

(b) $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$

$$s = (s_1 + s_2),\ (s_1 + s_2 - 1),\ldots,|s_1 - s_2| = (1/2 + 1/2),\ (1/2 - 1/2) = 1, 0$$

(c) $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$

$$j = (\ell + s),\ (\ell + s - 1),\ldots,|\ell - s|$$

For $\ell = 2$ and $s = 1$, $j = 3, 2, 1$

$\ell = 2$ and $s = 0$, $j = 2$

For $\ell = 1$ and $s = 1$, $j = 2, 1, 0$

$\ell = 1$ and $s = 0$, $j = 1$

For $\ell = 0$ and $s = 1$, $j = 1$

$\ell = 0$ and $s = 0$, $j = 0$

(d) $\boldsymbol{J}_1 = \boldsymbol{L}_1 + \boldsymbol{S}_1 \qquad j_1 = \ell_1 \pm 1/2 = 3/2,\ 1/2$

$\boldsymbol{J}_2 = \boldsymbol{L}_2 + \boldsymbol{S}_2 \qquad j2 = \ell_2 \pm 1/2 = 3/2,\ 1/2$

(e) $\boldsymbol{J} = \boldsymbol{J}_1 + \boldsymbol{J}_2 \qquad j = (j_1 + j_2),\ (j_1 + j_2 - 1),\ldots,|j_1 - j_2|$

For $j_1 = 3/2$ and $j_2 = 3/2$, $j = 3, 2, 1, 0$

$j_1 = 3/2$ and $j_2 = 1/2$, $j = 2, 1$

For $j_1 = 1/2$ and $j_2 = 3/2$, $j = 2, 1$

$j_1 = 1/2$ and $j_2 = 1/2$, $j = 1, 0$

These are the same values as found in (c).

7-40. (a) $E_{3/2} = \dfrac{hc}{\lambda}$ Using values from Figure 7-22,

$$E_{3/2} = \frac{1239.852eV\cdot nm}{588.99nm} = 2.10505eV \qquad E_{1/2} = \frac{1239.852eV\cdot nm}{589.59nm} = 2.10291eV$$

(b) $\Delta E = E_{3/2} - E_{1/2} = 2.10505eV - 2.10291eV = 2.14 \times 10^{-3} eV$

(c) $\Delta E = 2\mu_B B \rightarrow B = \dfrac{\Delta E}{2\mu_B} = \dfrac{2.14 \times 10^{-3} eV}{2(5.79 \times 10^{-4} eV/T)} = 18.5T$

7-43. (a) For electrons: Including spin, two are in the $n = 1$ state, two are in the $n = 2$ state, and

one is in the $n = 3$ state. The total energy is then:

$E = 2E_1 + 2E_2 + E_3 \quad \text{where } E_n = \dfrac{n^2\hbar^2\pi^2}{2mL^2} \quad E = 2E_1 + 2\left(2^2 E_1\right) + \left(3^2 E_1\right) = 19E_1$

where

$$E_1 = \frac{(hc)^2 \pi^2}{2m_e c^2 L^2} = \frac{(197.3)^2 \pi^2}{2\left(0.511\times10^6\right)(1.0)^2} = 0.376eV \qquad E = 19E_1 = 7.14eV$$

(b) Pions are bosons and all five can be in the $n = 1$ state, so the total energy is:

$$E = 5E_1 \quad \text{where } E_1 = \frac{0.376eV}{264} = 0.00142eV \qquad E = 5E_1 = 0.00712eV$$

7-44. (a) Carbon: $Z = 6$; $1s^2 2s^2 2p^2$

(b) Oxygen: $Z = 8$; $1s^2 2s^2 2p^4$

(c) Argon: $Z = 18$; $1s^2 2s^2 2p^6 3s^2 3p^6$

7-45. Using Figure 7-34:

Sn ($Z = 50$)

$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6\ 4d^{10}\ 5s^2\ 5p^2$

Nd ($Z = 60$)

$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6\ 4d^{10}\ 5s^2\ 5p^6\ 4f^4\ 6s^2$

Yb ($Z = 70$)

$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6\ 4d^{10}\ 4f^{14}\ 5s^2\ 5p^6\ 6s^2$

Comparison with Appendix C.

Sn: agrees

Nd: $5p^6$ and $4f^4$ are in reverse order

Yb: agrees

7-46. Both *Ga* and *In* have electron configurations $(ns)^2(np)$ outside of closed shells $(n-1,s)^2(n-1,p)^6(n-1,d)^{10}$. The last *p* electron is loosely bound and is more easily removed than one of the *s* electrons of the immediately preceding elements *Zn* and *Cd*.

7-48. $E_n = -\frac{Z_{eff}^2 E_1}{n^2}$ (Equation 7-25)

$$Z_{eff} = n\sqrt{\frac{-E_n}{E_1}} = 3\sqrt{\frac{5.14eV}{13.6eV}} = 1.84$$

7-49. (a) Fourteen electrons, so $Z = 14$. Element is silicon.

(b) Twenty electrons, so $Z = 20$. Element is calcium.

7-50. (a) For a *d* electron, $\ell = 2$, so $L_z = -2\hbar, -1\hbar, 0, 1\hbar, 2\hbar$

(b) For an *f* electron, $\ell = 3$, so $L_z = -3\hbar, -2\hbar, -1\hbar, 0, 1\hbar, 2\hbar, 3\hbar$

7-56. (a) $E_1 = -13.6eV(Z-1)^2 = -13.6eV(74-1)^2 = -7.25\times10^4 eV = -72.5keV$

(b) $E_1(\text{exp}) = -69.5keV = -13.6eV(Z-\sigma)^2 = -13.6eV(74-1)^2$

$$74 - \sigma = \left(69.5\times10^3 eV/13.6eV\right)^{1/2} = 71.49$$

$$\sigma = 74 - 71.49 = 2.51$$

7-58. (a) $\Delta E = hc/\lambda$

$$E(3P_{1/2}) - E(3S_{1/2}) = \frac{1240eV\cdot nm}{589.59nm} = 2.10eV$$

$$E(3P_{1/2}) = E(3S_{1/2}) + 2.10eV = -5.14eV + 2.10eV = -3.04eV$$

$$E(3D) - E(3P_{1/2}) = \frac{1240eV\cdot nm}{818.33nm} = 1.52eV$$

$$E(3D) = E(3P_{1/2}) + 1.52eV = -3.04eV + 1.52eV = -1.52eV$$

(b) For $3P$: $Z_{eff} = 3\sqrt{\frac{3.04eV}{13.6eV}} = 1.42$

For $3D$: $Z_{eff} = 3\sqrt{\frac{1.52eV}{13.6eV}} = 1.003$

(c) The Bohr formula gives the energy of the $3D$ level quite well, but not the $3P$ level.

7-59. (a) $\Delta E = gm_j\mu_B B$ (Equation 7-72) where $s = 1/2, \ell = 0$ gives $j = 1/2$ and

(from Equation 7-73) $g = 2$. $m_j = \pm 1/2$.

$$\Delta E = (2)(\pm 1/2)(5.79\times10^{-5}eV/T)(0.55T) = \pm 3.18\times10^{-5}eV$$

The total splitting between the $m_j = \pm 1/2$ states is $6.37\times10^{-5}eV$.

(b) The $m_j = 1/2$ (spin up) state has the higher energy.

(c) $\Delta E = hf \rightarrow f = \Delta E/h = 6.37\times10^{-5}eV/4.14\times10^{-15}eV\cdot s = 1.54\times10^{10}Hz$

This is in the microwave region of the spectrum.

7-61. (a) $\Delta E = \dfrac{e\hbar}{2m}B = (5.79\times10^{-5}eV/T)(0.05T) = 2.90\times10^{-6}eV$

(b) $|\Delta\lambda| = \dfrac{\lambda^2}{hc}\Delta E = \dfrac{(579.07nm)^2(2.90\times10^{-6}eV)}{1240eV\cdot nm} = 7.83\times10^{-4}nm$

(c) The smallest measurable wavelength change is larger than this by the ratio

$0.01nm/7.83\times10^{-4}nm = 12.8$. The magnetic field would need to be

increased by this

same factor because $B \propto \Delta E \propto \Delta\lambda$. The necessary field would be 0.638T.

7-66. $\theta_{\min} = \cos^{-1}\left[m_\ell\hbar/\sqrt{\ell(\ell+1)}\hbar\right]$ with $m_\ell = \ell$.

$\cos\theta_{\min} = \ell\sqrt{\ell(\ell+1)}$. Thus, $\cos^2\theta_{\min} = \ell^2/\left[\ell(\ell+1)\right] = 1-\sin^2\theta_{\min}$

or, $\sin^2\theta_{\min} = 1 - \dfrac{\ell^2}{\ell(\ell+1)} = \dfrac{\ell(\ell+1)-\ell^2}{\ell(\ell+1)} = \dfrac{\ell^2+\ell-\ell^2}{\ell(\ell+1)}$

And, $\sin\theta_{\min} = \left(\dfrac{1}{\ell+1}\right)^{1/2}$ For large ℓ, $\theta_{\min}$ is small.

Then $\sin\theta_{\min} \approx \theta_{\min} = \left(\dfrac{1}{\ell+1}\right)^{1/2} \approx \dfrac{1}{(\ell)^{1/2}}$