6-43. (a) For $x > 0$, $\hbar^2 k_2^2 / 2m + V_0 = E = \hbar^2 k_1^2 / 2m = 2V_0$

So, $k_2 = (2mV_0)^{1/2} / \hbar$. Because $k_1 = (4mV_0)^{1/2} / \hbar$, then $k_2 = k_1 / \sqrt{2}$

(b) $R = (k_1 - k_2)^2 / (k_1 + k_2)^2$ (Equation 6-68)

$= \left(1 - 1/\sqrt{2}\right)^2 / \left(1 + 1/\sqrt{2}\right)^2 = 0.0294$, or 2.94% of the incident particles

are

reflected.

(c) $T = 1 - R = 1 - 0.0294 = 0.971$

(d) 97.1% of the particles, or $0.971 \times 10^6 = 9.71 \times 10^5$, continue past the step in

the $+x$

direction. Classically, 100% would continue on.

6-44. (a) For $x > 0$, $\hbar^2 k_2^2 / 2m - V_0 = E = \hbar^2 k_1^2 / 2m = 2V_0$

So, $k_2 = (6mV_0)^{1/2} / \hbar$. Because $k_1 = (4mV_0)^{1/2} / \hbar$, then $k_2 = \sqrt{3/2}\, k_1$

(b) $R = (k_1 - k_2)^2 / (k_1 + k_2)^2$

$$R = (k_1 - k_2)^2 / (k_1 + k_2)^2 = \left(1 - \sqrt{3/2}\right)^2 / \left(1 + \sqrt{3/2}\right)^2 = 0.0102$$

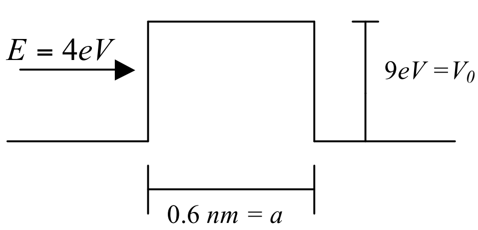
Or 1.02% are reflected at $x = 0$.

(c) $T = 1 - R = 1 - 0.0102 = 0.99$

(d) 99% of the particles, or $0.99 \times 10^6 = 9.9 \times 10^5$, continue in the $+x$ direction.

Classically, 100% would continue on.

6-45. (a)




$\alpha = \sqrt{2m(V_0 - E)}\hbar$

and $\alpha a = 0.6nm \times 11.46nm^{-1} = 6.87$

Since $\alpha a$ is not $\ll 1$, use Equation 6-75:

The transmitted fraction

$$T = \left[1 + \frac{\sinh^2 \alpha a}{4\left(E/V_0\right)\left(1 - E/V_0\right)}\right]^{-1} = \left[1 + \left(\frac{81}{80}\right)\sinh^2\left(6.87\right)\right]^{-1}$$

Recall that $\sinh x = \left(e^x - e^{-x}\right)/2$,

$$T = \left[1 + \frac{81}{80}\left(\frac{e^{6.87} - e^{-6.87}}{2}\right)^2\right]^{-1} = 4.3\times10^{-6}$$ is the transmitted fraction.

(b) Noting that the size of $T$ is controlled by $\alpha a$ through the $\sinh^2 \alpha a$ and increasing $T$ implies increasing $E$. Trying a few values, selecting $E = 4.5eV$ yields $T = 8.7\times10^{-6}$ or approximately twice the value in part (a).

6-48. Using Equation 6-76,

$$T \approx 16\frac{E}{V_0}\left(1 - \frac{E}{V_0}\right)e^{-2\alpha a}$$ where $E = 2.0eV$, $V_0 = 6.5eV$, and $a = 0.5nm$.

$$T \approx 16\left(\frac{2.0}{6.5}\right)\left(1 - \frac{2.0}{6.5}\right)e^{-2(10.87)(0.5)} \approx 6.5\times10^{-5}$$ (Equation 6-75 yields $T = 6.6\times10^{-5}$.)

6-49. $$R = \frac{\left(k_1 - k_2\right)^2}{\left(k_1 + k_2\right)^2}$$ and $T = 1 - R$ (Equations 6-68 and 6-70)

(a) For protons:

$$k_1 = \sqrt{2mc^2E}/\hbar c = \sqrt{2\left(938MeV\right)\left(40MeV\right)}/197.3MeV\cdot fm = 1.388$$

$$k_2 = \sqrt{2mc^2\left(E - V_0\right)}/\hbar c = \sqrt{2\left(938MeV\right)\left(10MeV\right)}/197.3MeV\cdot fm = 0.694$$

$$R = \left(\frac{1.388 - 0.694}{1.388 + 0.694}\right)^2 = \left(\frac{0.694}{2.082}\right)^2 = 0.111 \quad \text{And } T = 1 - R = 0.889$$

(b) For electrons:

$$k_1 = 1.388\left(\frac{0.511}{938}\right)^{1/2} = 0.0324 \qquad k_2 = 0.694\left(\frac{0.511}{938}\right)^{1/2} = 0.0162$$

$$R = \left(\frac{0.0324 - 0.0162}{0.0324 + 0.0162}\right)^2 = 0.111 \quad \text{And } T = 1 - R = 0.889$$

No, the mass of the particle is not a factor. (We might have noticed that $\sqrt{m}$ could

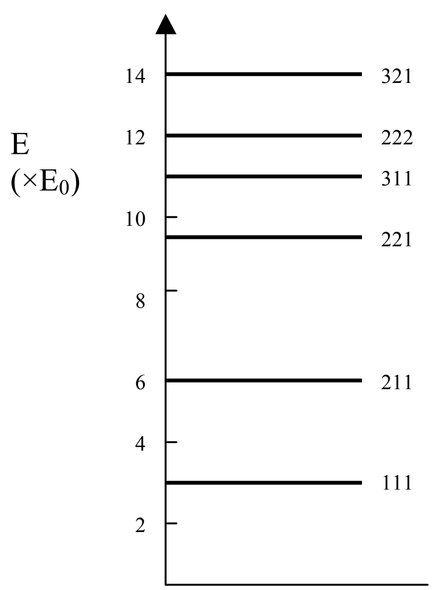
be canceled from each term.)

7-1. $E_{n_1 n_2 n_3} = \dfrac{\hbar^2\pi^2}{2mL^2}\left(n_1^2 + n_2^2 + n_3^2\right)$ (Equation 7-4)

$E_{311} = \dfrac{\hbar^2\pi^2}{2mL^2}\left(3^2 + 1^2 + 1^2\right) = 11E_0$ where $E_0 = \dfrac{\hbar^2\pi^2}{2mL^2}$

$E_{222} = E_0\left(2^2 + 2^2 + 2^2\right) = 12E_0$ and $E_{321} = E_0\left(3^2 + 2^2 + 1^2\right) = 14E_0$

The 1st, 2nd, 3rd, and 5th excited states are degenerate.



7-2. $$E_{n_1 n_2 n_3} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_1^2}{L_1^2}+\frac{n_2^2}{L_2^2}+\frac{n_3^2}{L_3^2}\right) = \frac{\hbar^2\pi^2}{2mL_1^2}\left(n_1^2+\frac{n_2^2}{4}+\frac{n_3^2}{9}\right) \qquad \text{(Equation 7-5)}$$

$n_1 = n_2 = n_3 = 1$ is the lowest energy level.

$E_{111} = E_0\left(1+1/4+1/9\right) = 1.361E_0$ where $E_0 = \dfrac{\hbar^2\pi^2}{2mL_1^2}$

The next nine levels are, increasing order,

*(Problem 7-2 continued)*

| $n_1$ | $n_2$ | $n_3$ | $E\left(\times E_0\right)$ |
|---|---|---|---|
| 1 | 1 | 2 | 1.694 |
| 1 | 2 | 1 | 2.111 |
| 1 | 1 | 3 | 2.250 |
| 1 | 2 | 2 | 2.444 |
| 1 | 2 | 3 | 3.000 |
| 1 | 1 | 4 | 3.028 |
| 1 | 3 | 1 | 3.360 |
| 1 | 3 | 2 | 3.472 |
| 1 | 2 | 4 | 3.778 |

7-3. (a) $\psi_{n_1 n_2 n_3}\left(x,y,z\right) = A\cos\frac{n_1\pi x}{L}\sin\frac{n_2\pi y}{L}\sin\frac{n_3\pi z}{L}$

(b) They are identical. The location of the coordinate origin does not affect the energy

level structure.

7-4. $\psi_{111}\left(x,y,z\right) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{\pi z}{3L_1}$

$$\psi_{112}\left(x,y,z\right) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{2\pi z}{3L_1}$$

$$\psi_{121}\left(x,y,z\right) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{L_1}\sin\frac{\pi z}{3L_1}$$

$$\psi_{122}\left(x,y,z\right) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{L_1}\sin\frac{2\pi z}{3L_1}$$

$$\psi_{113}\left(x,y,z\right) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{\pi z}{L_1}$$

7-5.

$$E_{n_1 n_2 n_3} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{(2L_1)^2} + \frac{n_3^2}{(4L_1)^2}\right) = \frac{\hbar^2\pi^2}{2mL_1^2}\left(n_1^2 + \frac{n_2^2}{4} + \frac{n_3^2}{16}\right) \quad \text{(from Equation 7-5)}$$

$$E_0 = \left(n_1^2 + \frac{n_2^2}{4} + \frac{n_3^2}{16}\right) \quad \text{where } E_0 = \frac{\hbar^2\pi^2}{2mL_1^2}$$

*(Problem 7-5 continued)*

(a)

| $n_1$ | $n_2$ | $n_3$ | $E\left(\times E_0\right)$ |
|---|---|---|---|
| 1 | 1 | 1 | 1.313 |
| 1 | 1 | 2 | 1.500 |
| 1 | 1 | 3 | 1.813 |
| 1 | 2 | 1 | 2.063 |
| 1 | 1 | 4 | 2.250 |
| 1 | 2 | 2 | 2.250 |
| 1 | 2 | 3 | 2.563 |
| 1 | 1 | 5 | 2.813 |
| 1 | 2 | 4 | 3.000 |
| 1 | 1 | 6 | 3.500 |

(b) 1,1,4 and 1,2,2

7-7. $$E_0 = \frac{\hbar^2\pi^2}{2mL^2} = \frac{\left(1.055\times10^{-34}\,J\cdot s\right)^2\pi^2}{2\left(9.11\times10^{-31}kg\right)\left(0.10\times10^{-9}m\right)^2\left(1.609\times10^{-19}J/eV\right)} = 37.68eV$$

$$E_{311} - E_{111} = \Delta E = 11E_0 - 3E_0 = 8E_0 = 301eV$$

*(Problem 7-7 continued)*

$$E_{222} - E_{111} = \Delta E = 12E_0 - 3E_0 = 9E_0 = 339eV$$

$$E_{321} - E_{111} = \Delta E = 14E_0 - 3E_0 = 11E_0 = 415eV$$

7-8. (a) Adapting Equation 7-3 to two dimensions (i.e., setting $k_3 = 0$), we have

$$\psi_{n_1 n_2} = A\sin\frac{n_1\pi x}{L}\sin\frac{n_2\pi y}{L}$$

(b) From Equation 7-5, $E_{n_1 n_2} = \dfrac{\hbar^2\pi^2}{2mL^2}\left(n_1^2 + n_2^2\right)$

(c) The lowest energy degenerate states have quantum numbers $n_1 = 1$, $n_2 = 2$, and $n_1 = 2$,

$n_2 = 1$.