3-20. (a) $\lambda_m T = 2.898\times10^{-3}\,m\cdot K$ (Equation 3-5)

$$\lambda_m = \frac{2.898\times10^{-3}\,m\cdot K}{2\times10^{4}\,K} = 1.45\times10^{-7}\,m = 145nm$$

(b) λ_m is in the ultraviolet region of the electromagnetic spectrum.

3-21. Equation 3-4: $R = \sigma T^4$

$P_{abs} = \left(1.36\times10^{3}W/m^2\right)\left(\pi R_E^2 m^2\right)$ where R_E = radius of Earth

$$P_{emit} = \left(RW/m^2\right)\left(4\pi R_E^2\right) = \left(1.36\times10^{3}W/m^2\right)\left(\pi R_E^2 m^2\right)$$

$$R = \left(1.36\times10^{3}W/m^2\right)\left(\frac{\pi R_E^2}{4\pi R_E^2}\right) = \frac{1.36\times10^{3}}{4}\frac{W}{m^2} = \sigma T^4$$

$$T^4 = \frac{1.36\times10^{3}W/m^2}{4\left(5.67\times10^{-8}W/m^2\cdot K^4\right)} \quad \therefore \quad T = 278.3K = 5.3°C$$

3-22. (a) $\lambda_m T = 2.898\times10^{-3}\,m\cdot K \quad \therefore \quad \lambda_m = \dfrac{2.898\times10^{-3}\,m\cdot K}{3300K} = 8.78\times10^{-7}\,m = 878nm$

$$f_m = c/\lambda_m = \frac{3.00\times10^{8}\,m/s}{8.78\times10^{-7}\,m} = 3.42\times10^{14}\,Hz$$

(b) Each photon has average energy $E = hf$ and $NE = 40J/s$.

$$N = \frac{40J/s}{hf_m} = \frac{40J/s}{\left(6.63\times10^{-34}\,J\cdot s\right)\left(3.42\times10^{14}\,Hz\right)} = 1.77\times10^{20}\,photons/s$$

(c) At $5m$ from the lamp N photons are distributed uniformly over an area $A = 4\pi r^2 = 100\pi m^2$. The density of photons on that sphere is $\left(N/A\right)/s\cdot m^2$.

The area of the pupil of the eye is $\pi\left(2.5\times10^{-3}\,m\right)^2$, so the number of photons entering the eye per second is:

$$n=(N/A)(\pi)\left(2.5\times10^{-3}m\right)^2=\frac{\left(1.77\times10^{20}/s\right)(\pi)\left(2.5\times10^{-3}m\right)^2}{100\pi m^2}$$

$$=\left(1.77\times10^{20}/s\right)(\pi)\left(2.5\times10^{-3}m\right)^2=1.10\times10^{13}\ photons/s$$

3-50. (a) $\lambda_m T=2.898\times10^{-3}m\cdot K \quad \therefore \quad T=\dfrac{2.898\times10^{-3}m\cdot K}{82.8\times10^{-9}m}=3.50\times10^{4}K$

(b) Equation 3-18: $\dfrac{u(70nm)}{u(82.8nm)}=\dfrac{(70nm)^{-5}/\left(e^{hc/(70nm)kT}-1\right)}{(82.8nm)^{-5}/\left(e^{hc/(82.8nm)kT}-1\right)}$

where $\dfrac{hc}{(70nm)kT}=\dfrac{\left(6.63\times10^{-34}J\cdot s\right)\left(3.00\times10^{8}m/s\right)}{\left(70\times10^{-9}m\right)\left(1.38\times10^{-23}J/K\right)\left(3.5\times10^{4}K\right)}=5.88$ and

$\dfrac{hc}{(82.8nm)kT}=4.97 \qquad \dfrac{u(70nm)}{u(82.8nm)}=\dfrac{(70nm)^{-5}/\left(e^{5.88}-1\right)}{(82.8nm)^{-5}/\left(e^{4.97}-1\right)}=0.929$

Similarly, $\dfrac{u(100nm)}{u(82.8nm)}=\dfrac{(100nm)^{-5}/\left(e^{4.12}-1\right)}{(82.8nm)^{-5}/\left(e^{4.97}-1\right)}=0.924$

3-53. (a) Equation 3-18: $u(\lambda)=\dfrac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT}-1}$ Letting $C=8\pi hc$ and $a=hc/kT$

gives $u(\lambda)=\dfrac{C\lambda^{-5}}{e^{a/\lambda}-1}$

(b)

$$\frac{du}{d\lambda}=\frac{d}{d\lambda}\left[\frac{C\lambda^{-5}}{e^{a/\lambda}-1}\right]=C\left[\frac{\lambda^{-5}(-1)e^{a/\lambda}\left(-a\lambda^{-2}\right)}{\left(e^{a/\lambda}-1\right)^2}-\frac{5\lambda^{-6}}{e^{a/\lambda}-1}\right]$$

$$=\frac{C\lambda^{-6}}{\left(e^{a/\lambda}-1\right)^2}\left[\frac{a}{\lambda}e^{a/\lambda}-5\left(e^{a/\lambda}-1\right)\right]=\frac{C\lambda^{-6}e^{a/\lambda}}{\left(e^{a/\lambda}-1\right)^2}\left[\frac{a}{\lambda}-5\left(1-e^{a/\lambda}\right)\right]=0$$

The maximum corresponds to the vanishing of the quantity in brackets.

Thus, $5\lambda\left(1-e^{-a/\lambda}\right)=a$

(c) This equation is most efficiently solved by trial and error; i.e., guess at a value for a/λ in the expression $5\lambda\left(1-e^{-a/\lambda}\right)=a$, solve for a better value of a/λ; substitute the new value to get an even better value, and so on. Repeat the process until the calculated value no longer changes. One succession of values is 5, 4.966310, 4.965156, 4.965116, 4.965114, 4.965114. Further iterations repeat the same value (to seven digits), so we have $\frac{a}{\lambda_m} = 4.965114 = \frac{hc}{\lambda_m kT}$

(d) $\lambda_m T = \frac{hc}{(4.965114)k} = \frac{(6.63\times10^{-34}\,J\cdot s)(3.00\times10^{8}\,m/s)}{(4.965114)(1.38\times10^{-23}\,J/K)}$

Therefore, $\lambda_m T = 2.898\times10^{-3}\,m\cdot K$ Equation 3-5

3-34. Equation 3-24: $\lambda_m = \frac{1.24\times10^{3}}{V}nm = \frac{1.24\times10^{3}}{80\times10^{3}V} = 0.016nm$

3-36. $\lambda_2 - \lambda_1 = \frac{h}{mc}(1-\cos\theta) = \frac{(6.63\times10^{-34}\,J\cdot s)(1-\cos110°)}{(9.11\times10^{-31}kg)(3.00\times10^{8}\,m/s)} = 3.26\times10^{-12}m$

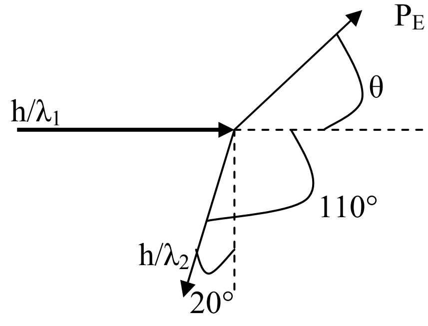
$$\lambda_1 = \frac{hc}{E_1} = \frac{(6.63\times10^{-34}\,J\cdot s)(3\times10^{8}\,m/s)}{(0.511\times10^{6}eV)(1.60\times10^{-19}\,J/eV)} = 2.43\times10^{-12}m$$

$$\lambda_2 = \lambda_1 + 3.26\times10^{-12}m = (2.43+3.26)\times10^{-12}m = 5.69\times10^{-12}m$$

$$E_2 = \frac{hc}{\lambda_2} = \frac{1240eV\cdot nm}{5.69\times10^{-3}nm} = 2.18\times10^{5}eV = 0.218MeV$$

Electron recoil energy $E_e = E_1 - E_2$ (Conservation of energy)

$E_e = 0.511MeV - 0.218MeV = 0.293MeV$. The recoil electron momentum makes an angle θ with the direction of the initial photon.



$$\frac{h}{\lambda_2}\cos 20° = p_e \sin\theta = (1/c)\sqrt{E^2 - (mc^2)^2}\,\sin\theta \qquad \text{(Conservation of momentum)}$$

$$\sin\theta = \frac{(3.00\times10^8 m/s)(6.63\times10^{-34} J\cdot s)\cos 20°}{(5.69\times10^{-12} m)\left[(0.804MeV)^2 - (0.511MeV)^2\right]^{1/2}(1.60\times10^{-13} J/MeV)}$$

$$= 0.330 \text{ or } \theta = 19.3°$$

3-37. $\Delta\lambda = \lambda_2 - \lambda_1 = \Delta\lambda = \frac{h}{mc}(1-\cos\theta) = 0.01\lambda_1$ Equation 3-25

$$\lambda_1 = (100)\frac{h}{mc}(1-\cos\theta) = (100)(0.00243nm)(1-\cos 90°) = 0.243nm$$

3-38. (a) $E_1 = \frac{hc}{\lambda_1} = \frac{1240eV\cdot nm}{0.0711nm} = 1.747\times10^4 eV$

(b) $\lambda_2 = \lambda_1 + \frac{h}{mc}(1-\cos\theta) = 0.0711 + (0.00243nm)(1-\cos 180°) = 0.0760nm$

(c) $E_2 = \frac{hc}{\lambda_2} = \frac{1240eV\cdot nm}{0.0760nm} = 1.634\times10^4 eV$

(d) $E_e = E_1 - E_2 = 1.128\times10^3 eV$

3-41. (a) Compton wavelength = $\frac{h}{mc}$

electron: $\frac{h}{mc} = \frac{6.63\times10^{-34} J\cdot s}{(9.11\times10^{-31} kg)(3.00\times10^8 m/s)} = 2.43\times10^{-12} m = 0.00243nm$

proton: $\frac{h}{mc} = \frac{6.63\times10^{-34} J\cdot s}{(1.67\times10^{-27} kg)(3.00\times10^8 m/s)} = 1.32\times10^{-15} m = 1.32\,fm$

(b) $E = \frac{hc}{\lambda}$

(i) electron: $E = \dfrac{1240 eV\cdot nm}{0.00243 nm} = 5.10\times10^{5} eV = 0.510 MeV$

(ii) proton: $E = \dfrac{1240 eV\cdot nm}{1.32\times10^{-6} nm} = 9.39\times10^{8} eV = 939 MeV$

4-1. $\dfrac{1}{\lambda_{mn}} = R\left(\dfrac{1}{m^2} - \dfrac{1}{n^2}\right)$ where $R = 1.097\times10^{7} m^{-1}$ (Equation 4-2)

The Lyman series ends on m =1, the Balmer series on m =2, and the Paschen series on

m =3. The series limits all have $n = \infty$, so $\dfrac{1}{n} = 0$.

$$\frac{1}{\lambda_L} = R\left(\frac{1}{1^2}\right) = 1.097\times10^{7} m^{-1}$$

$$\lambda_L(\text{limit}) = 1.097\times10^{7} m^{-1} = 91.16\times10^{-9} m = 91.16 nm$$

$$\frac{1}{\lambda_B} = R\left(\frac{1}{2^2}\right) = 1.097\times10^{7} m^{-1}/4$$

$$\lambda_B(\text{limit}) = 4/1.097\times10^{7} m^{-1} = 3.646\times10^{-7} m = 364.6 nm$$

$$\frac{1}{\lambda_P} = R\left(\frac{1}{3^2}\right) = 1.097\times10^{7} m^{-1}/9$$

$$\lambda_P(\text{limit}) = 9/1.097\times10^{7} m^{-1} = 8.204\times10^{-7} m = 820.4 nm$$

4-2. $\dfrac{1}{\lambda_{mn}} = R\left(\dfrac{1}{m^2} - \dfrac{1}{n^2}\right)$ where $m = 2$ for Balmer series (Equation 4-2)

$$\frac{1}{379.1 nm} = \frac{1.097\times10^{7} m^{-1}}{10^{9} nm/m}\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{4} - \frac{1}{n^2} = \frac{10^{9} nm/m}{379.1 nm\left(1.097\times10^{7} m^{-1}\right)} = 0.2405$$

$$\frac{1}{n^2} = 0.2500 - 0.2405 = 0.0095$$

$$n^2 = \frac{1}{0.0095} \quad \rightarrow \quad n = (1/0.0095)^{1/2} = 10.3 \quad \rightarrow \quad n = 10$$

$$n = 10 \rightarrow n = 2$$

4-3. $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$ where $m = 1$ for Lyman series (Equation 4-2)

$$\frac{1}{164.1nm} = \frac{1.097 \times 10^7 m^{-1}}{10^9 nm/m}\left(1 - \frac{1}{n^2}\right)$$

$$\frac{1}{n^2} = 1 - \frac{10^9 nm/m}{164.1nm\left(1.097 \times 10^7 m^{-1}\right)} = 1 - 0.5555 = 0.4445$$

$$n = (1/0.4445)^{1/2} = 1.5$$

No, this is not a hydrogen Lyman series transition because n is not an integer.

4-4. $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$ (Equation 4-2)

For the Brackett series $m = 4$ and the first four (i.e., longest wavelength lines have $n = 5$,

6, 7, and 8.

$$\frac{1}{\lambda_{45}} = 1.097 \times 10^7 m^{-1}\left(\frac{1}{4^2} - \frac{1}{5^2}\right) = 2.468 \times 10^5 m^{-1}$$

$$\lambda_{45} = \frac{1}{2.68 \times 10^5 m^{-1}} = 4.052 \times 10^{-6} m = 4052nm.$$ Similarly,

$$\lambda_{46} = \frac{1}{3.809 \times 10^5 m^{-1}} = 2.625 \times 10^{-6} m = 2625nm$$

$$\lambda_{47} = \frac{1}{4.617 \times 10^5 m^{-1}} = 2.166 \times 10^{-6} m = 2166nm$$

$$\lambda_{48} = \frac{1}{5.142 \times 10^5 m^{-1}} = 1.945 \times 10^{-6} m = 1945nm$$

These lines are all in the infrared.