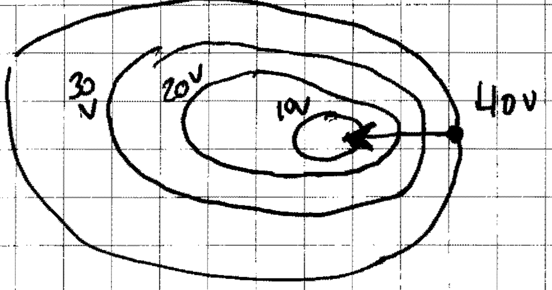
2.5 44



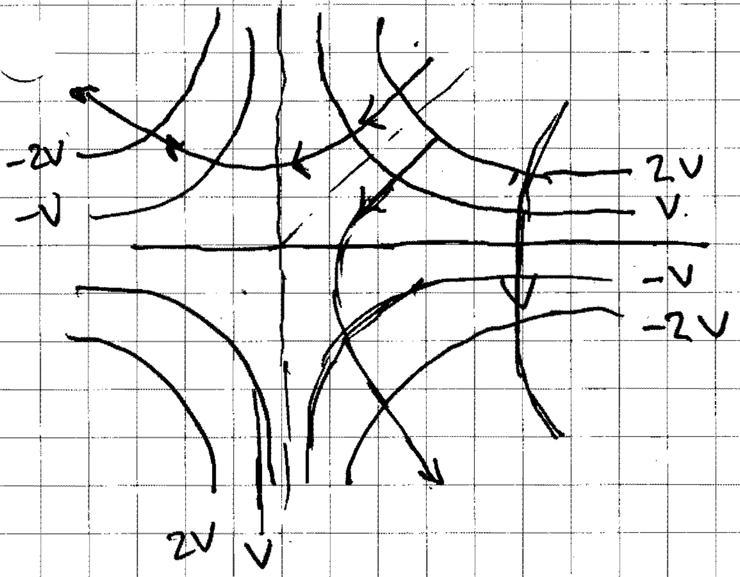
electric fields go from high potentials to low potentials and are perpindicular to equipotential lines

E α- rate of change of V over space

$$\left(E = -\nabla V \text{ where } \nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \ldots \right)$$

c) $\Delta X = +3$ $\Delta V = +30V$ so $E = -10\ N/C\ \hat{x}$
(and $\frac{dV}{dx}$ = constant)

45 $V = axy$ a) $E = -\nabla V = -ay\hat{x} - ax\hat{y}$



50)

$4q$ $(0,0)$ $\quad$ $-q$ $(a,0)$

$$V(x) = \frac{1}{4\pi\varepsilon_0}\frac{4q}{x} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{(x-a)}$$

$$V(x) = 0 \quad \frac{1}{4\pi\varepsilon_0}\left(\frac{4q}{x} - \frac{q}{(x-a)}\right) = 0$$

$$\frac{4}{x} - \frac{1}{x-a} = 0 \quad 4(x-a) = x$$

$$4x - 4a = x \quad 3x = 4a \quad x = \frac{4}{3}a$$

c) we could say $-\frac{\partial V}{\partial x} = E_x$

so $E_x = \frac{4q}{4\pi\varepsilon_0 x^2} - \frac{q}{4\pi\varepsilon_0 (x-a)^2}\,\hat{x}$

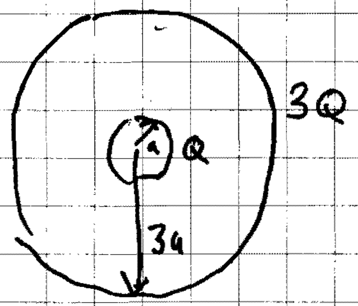
d) $E_x = 0 \Rightarrow \frac{4}{x^2} - \frac{1}{(x-a)^2} = 0$

$$4(x-a)^2 - x^2 = 0$$

$$3x^2 - 8xa + 4a^2 = 0$$

$$x = \frac{8a \pm \sqrt{64a^2 - 48a^2}}{6} = \frac{8a \pm 4a}{6} = 2a,\ \tfrac{2}{3}a$$

54)



$$\sigma_{small} = \frac{Q}{4\pi a^2}$$

$$\sigma_{large} = \frac{3Q}{4\pi(3a)^2} = \frac{Q}{3(4\pi a^2)} = \frac{\sigma_{small}}{3}$$

of course all of that is irrelevant

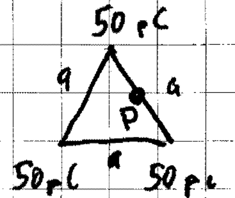
$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$ so $E_{small} = \frac{Q}{4\pi\varepsilon_0 a^2}$

and $E_{large} = \frac{3Q}{4\pi\varepsilon_0 (3a)^2}$

V can be found by replacing a with r (or 3a) then integrating

$V_{small} = \frac{Q}{4\pi\varepsilon_0 a}$ $V_{large} = \frac{Q}{4\pi\varepsilon_0 (3a)}$

59)



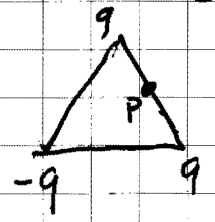
$a = 1.5\,mm$

$q = 50\,pC$

$V_\infty = 0 \qquad V_p = \frac{2q}{4\pi\varepsilon_0 a/2} + \frac{q}{4\pi\varepsilon_0 a\frac{\sqrt{3}}{2}}$

so $W = q_p \Delta V = \underbrace{q_p}_{\text{charge of proton}}(V_p - V_\infty)$

60) if we were to change it around



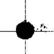
$V_p = \frac{2q}{4\pi\varepsilon_0 a/2} - \frac{q}{4\pi\varepsilon_0 a\frac{\sqrt{3}}{2}}$

$W = q_{P_{\text{proton}}} V_P$

26.7) W needed to assemble = total energy stored in fields

first note which is easier to find (likely the former)

first charge is free to move in



second charge goes from $V=0$ to $V(r)$ where r is a away from the first charge

$W_1 = q\,\Delta V = \frac{q^2}{4\pi\varepsilon_0 a}$

the next charge sees effects from both charges there but, both are the same distance from where its going so their potentials are the same

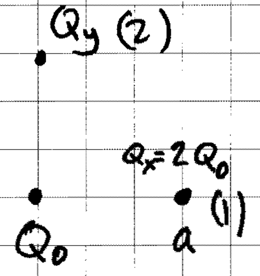
$W_2 = q\,\Delta V = \frac{2q^2}{4\pi\varepsilon_0 a}$

the last charge sees three charges there

$$W_3 = q\,\Delta V = \frac{3q^2}{4\pi\varepsilon_0 a}$$

$$W_T = W_1 + W_2 + W_3 = \frac{6q^2}{4\pi\varepsilon_0 a}$$ (note that a tetrahedron has 6 identical edges) ~~this is~~

8)



Work to bring Q_x in is $\frac{2Q_0^2}{4\pi\varepsilon_0 a} = W_1$

work to bring Q_y in is $Q_y\left(\frac{Q_0}{4\pi\varepsilon_0 a} + \frac{2Q_0}{4\pi\varepsilon_0\sqrt{2}a}\right) = W_2$

$W_2 = 2W_1$ so $\quad 2\,\frac{2Q_0^{\cancel{2}}}{\cancel{4\pi\varepsilon_0 a}} = Q_y\left(\frac{\cancel{Q_0}}{\cancel{4\pi\varepsilon_0 a}} + \frac{2\cancel{Q_0}}{\cancel{4\pi\varepsilon_0}\sqrt{2}\cancel{a}}\right)$

$$4Q_0 = Q_y(1+\sqrt{2})$$

$$Q_y = Q_0\frac{4}{1+\sqrt{2}}$$

12) $W = q\,\Delta V$ so $\Delta V = \frac{W}{q}$ $\qquad q = C\Delta V$ so $q^2 = CW$

$C = \frac{\varepsilon_0 A}{d}$ $\qquad q^2 = \frac{\varepsilon_0 A W}{d}$ $\qquad d = \frac{\varepsilon_0 A W}{q^2}$

$q = 45\,\mu C \quad A = \pi(.15\,m)^2 \quad W = 6.3\,J$

15)

3 2 -1

a) at each surface

$V_{a+} = \frac{q}{4\pi\varepsilon_0 a}$ $V_{a-} = \frac{-q}{4\pi\varepsilon_0 a}$

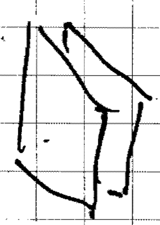
(note $V_\infty = 0$)

so $\Delta V = V_{a+} - V_{a-} = \frac{2Kq}{a}$

b) $dW = dq \frac{2Kq}{a}$

c) $W = \int_0^Q \frac{2Kq}{a} dq = \frac{KQ^2}{a}$

20)



$u_E = \frac{1}{2}\varepsilon_0 E^2 = 4.5 \text{ kJ/m}^2$

$E = \sqrt{2u_E/\varepsilon_0}$

$Q = CV = CEd = \frac{\varepsilon_0 A}{\cancel{d}} E\cancel{d} = \varepsilon_0 A\sqrt{2u_E\varepsilon_0}$

where $A = .10^2$

23 $U = \frac{1}{2}\varepsilon_0 \int E^2 dV$

$E = \frac{Q}{4\pi\varepsilon_0 r^2}$ for $r \geq R$ 0 for $r < R$

$U = \frac{1}{2}\varepsilon_0 K^2 Q^2 \int_R^\infty \frac{1}{r^4} r^2 \int_{\text{angles}} \sin\theta \, dr \, d\theta \, d\phi$ note $\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \sin\theta \, d\theta \, d\phi = 4\pi$

$= \frac{4\pi\varepsilon_0}{2} K^2 Q^2 \int_R^\infty \frac{1}{r^2} dr = -\frac{KQ^2}{2r}\Big|_R^\infty = \frac{KQ^2}{2R}$

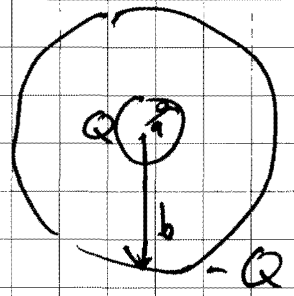
34) $Q = CV$ $\quad C = \frac{\varepsilon_0 A}{d}$ $\quad d = 1.1\,mm$

$Q = 2.3\,\mu C$

$V = 150$

$Q = \frac{\varepsilon_0 A}{d} V$ $\quad A = \frac{Q d}{V \varepsilon_0}$

36)

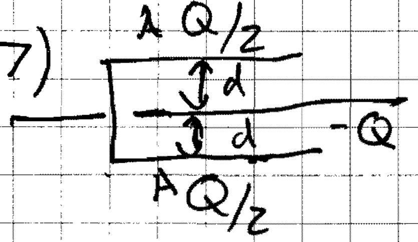


$V_{inside} = \frac{KQ}{r}$ $\quad \Delta V = \frac{KQ}{a} - \frac{KQ}{b}$

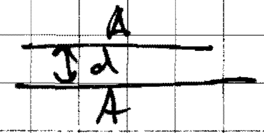
$Q = CV$

$\frac{K}{a} - \frac{K}{b} = 1/C$ $\quad C = \frac{ab}{K(b-a)}$

37)



compare w/



$\cancel{E_{inside\,(top)} = \frac{Q}{2A\varepsilon_0} + \frac{Q}{A\varepsilon_0} + \frac{Q}{2A\varepsilon_0} = +\frac{Q}{A\varepsilon_0}\ \text{so}\ \Delta V = \frac{dQ}{A\varepsilon_0}}$

~~bottom~~ $\cancel{= -\frac{Q}{A\varepsilon_0}}$ ~~but our dl is in opposite direction~~

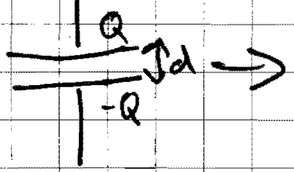
~~so ΔV~~

w/out changing charge we have doubled surface area

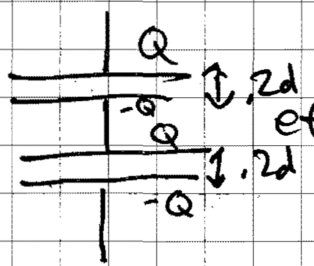
so $C = \frac{2A\varepsilon_0}{d}$

(you could also find the total energy stored in fields and compare with that of a capacitor

47)



$\rightarrow$

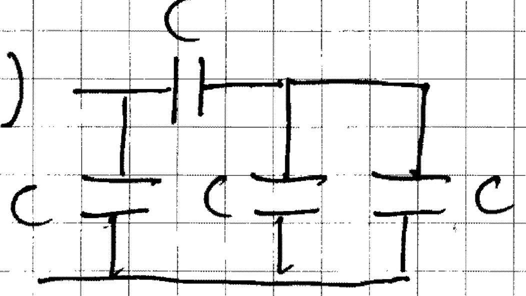


effectively

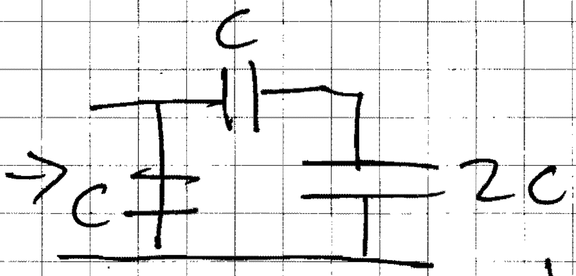
$C = \frac{A\varepsilon_0}{d}$ $\quad C_{top} = C_{bottom} = \frac{A\varepsilon_0}{.2d}$

$\frac{1}{C_{eff}} = \frac{1}{C_{top}} + \frac{1}{C_{bottom}} = \frac{.4d}{A\varepsilon_0} \Rightarrow C_{eff} = \frac{A\varepsilon_0}{.4d} = \frac{1}{.4} C$

54)

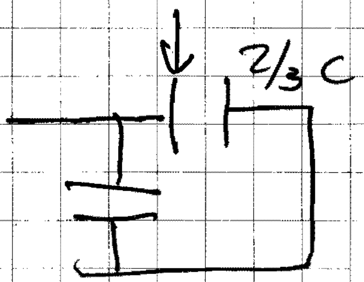


→

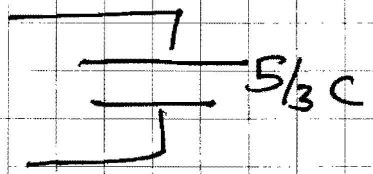


$\frac{1}{\frac{1}{2}+1} = \frac{1}{3/2} = 2/3$

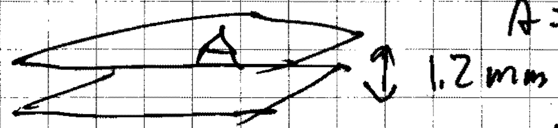
↓



←



68)



$A = 76\,cm^2$

$V = 900\,V$

$C_0 = \frac{\varepsilon_0 A}{d}$ $\qquad Q_0 = C_0 V_0$

same, charge is conserved

$C_1 = \frac{\varepsilon A}{d} = \frac{\varepsilon}{\varepsilon_0} C_0$ $\qquad Q_0 = C_1 V_1$ $\qquad V_1 = \frac{Q_0}{C_1} = \frac{Q_0}{\frac{\varepsilon}{\varepsilon_0} C_0} = \frac{\varepsilon_0}{\varepsilon} V_0$