6/3/03

Thomson scattering

$$\underset{\sim}{E} = \mathrm{Re}\, E\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t}$$

$$m \frac{d^2\underset{\sim}{r}}{dt^2} = e\left[\underset{\sim}{E} + \frac{\underset{\sim}{v}}{c} \times \underset{\sim}{B}\right]$$

$$\frac{|vB/c|}{|E|} = \frac{v}{c} << 1$$

↑ assume

$$m \frac{d^2\underset{\sim}{r}}{dt^2} \simeq e\,\underset{\sim}{E}(\underset{\sim}{r},t) \simeq \mathrm{Re}\, e\, E\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t}$$

$$\Delta r\, k \simeq \frac{\Delta r\, \omega}{c} \simeq \frac{v}{c} << 1$$

$$\Delta\underset{\sim}{r} \simeq \mathrm{Re}\, \frac{-eE}{m\omega^2}\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t}$$

$$\underset{\sim}{P}(t) = -e\,\Delta r(t) = + \mathrm{Re}\, \underbrace{\frac{e^2 E}{m\omega^2}}_{\underset{\sim}{P}} e^{i\underset{\sim}{k}\cdot r - i\omega t}$$

$$\frac{d\langle P\rangle}{d\Omega} = \frac{c}{8\pi}\left(\frac{\omega}{c}\right)^4 |P|^2 \sin^2\theta$$

$$\frac{d\sigma}{d\Omega} = \frac{\frac{d\langle P\rangle}{d\Omega}}{\frac{c}{8\pi}|E|^2} = \underbrace{\frac{\omega^4}{c^4}\frac{e^4}{m^2\omega^4}}_{\left(\frac{e^2}{mc^2}\right)^2 = r_{cl}^2} |\hat{E}\times\hat{r}|^2$$

question for class
force on electron in $\hat{k}$-direction?

Lienard-Wiechert Potentials

J. Chapter 14, L+L Vol 2, chapter 8

We want to calculate φ and $\underset{\sim}{A}$ for a charge moving along a given orbit $\underset{\sim}{r_0}(t)$.

$$\therefore \rho(\underset{\sim}{r},t) = e\,\delta[\underset{\sim}{r} - \underset{\sim}{r_0}(t)], \quad \underset{\sim}{J}(\underset{\sim}{r},t) = \dot{\underset{\sim}{r}}_0 e\,\delta[\ ]$$

$$\varphi(\underset{\sim}{r},t) = \int d^3\underset{\sim}{r}' \int dt' \, e \, \frac{\delta[\underset{\sim}{r}'-\underset{\sim}{r}_0(t')]}{|\underset{\sim}{r}-\underset{\sim}{r}'|} \delta\left[t-t'-\frac{|\underset{\sim}{r}-\underset{\sim}{r}'|}{c}\right]$$

$$\varphi(\underset{\sim}{r},t) = \int dt' \, \frac{e}{|\underset{\sim}{r}-\underset{\sim}{r}_0(t')|} \delta\left[t-t'-\frac{|\underset{\sim}{r}-\underset{\sim}{r}_0(t')|}{c}\right]$$

let $\underset{\sim}{R}(t') = \underset{\sim}{r} - \underset{\sim}{r}_0(t')$

for a given t define the retarded time t' such that

$$t' = t - \frac{R(t')}{c}$$

$$\varphi(\underset{\sim}{r},t) = \frac{e}{R(t')} \frac{1}{\left|\frac{d}{dt'}\left(t-t'-\frac{R(t')}{c}\right)\right|}$$

$$2\underset{\sim}{R}\cdot\underbrace{\frac{d\underset{\sim}{R}}{dt'}}_{-\underset{\sim}{v}_0(t')} = 2R\frac{dR}{dt'} \quad , \quad \frac{dR(t')}{dt'} = -\frac{\underset{\sim}{v}_0(t')\cdot\underset{\sim}{R}(t')}{R(t')}$$

$$\therefore \quad \phi(\underset{\sim}{r},t) = \frac{e}{\left[R - \underset{\sim}{\beta}\cdot\underset{\sim}{R}\right]_{t'}} \quad , \quad \underset{\sim}{\beta}(t) = \frac{\underset{\sim}{v}(t)}{c}$$

likewise

$$\underset{\sim}{A}(\underset{\sim}{r},t) = \left[\frac{e\underset{\sim}{\beta}}{R - \underset{\sim}{R}\cdot\underset{\sim}{\beta}}\right]_{t'}$$

$$\underset{\sim}{E} = -\frac{1}{c}\frac{\partial \underset{\sim}{A}}{\partial t} - \underset{\sim}{\nabla}\phi \quad , \quad \underset{\sim}{B} = \underset{\sim}{\nabla}\times\underset{\sim}{A}$$

to evaluate derivatives we need

$$R(t') = c(t - t')$$

$$c\left(1 - \frac{\partial t'}{\partial t}\right) = \frac{\partial R}{\partial t} = \frac{\partial R}{\partial t'}\frac{\partial t'}{\partial t} = -\frac{\underset{\sim}{R}\cdot\underset{\sim}{v}_0}{R}\frac{\partial t'}{\partial t}$$

$$\frac{\partial t'}{\partial t} = \frac{1}{\left(1 - \frac{\underset{\sim}{\beta}\cdot\underset{\sim}{R}}{R}\right)_{t'}}$$

$$\nabla t' = -\frac{1}{c}\nabla R(t') = -\frac{1}{c}\nabla\left|\underset{\sim}{r} - \underset{\sim}{r}_0(t')\right|$$

$$= -\frac{1}{c}\frac{\underset{\sim}{R}}{R} - \frac{1}{c}\frac{\partial R}{\partial t'}\nabla t'$$

$$\underset{\sim}{\nabla} t' = \frac{-\underset{\sim}{R}/Rc}{1 + \frac{1}{c}\frac{\partial R}{\partial t'}} = \left[\frac{-\underset{\sim}{R}/c}{R - \underset{\sim}{\beta}\cdot\underset{\sim}{R}}\right]_{t'}$$

Homework

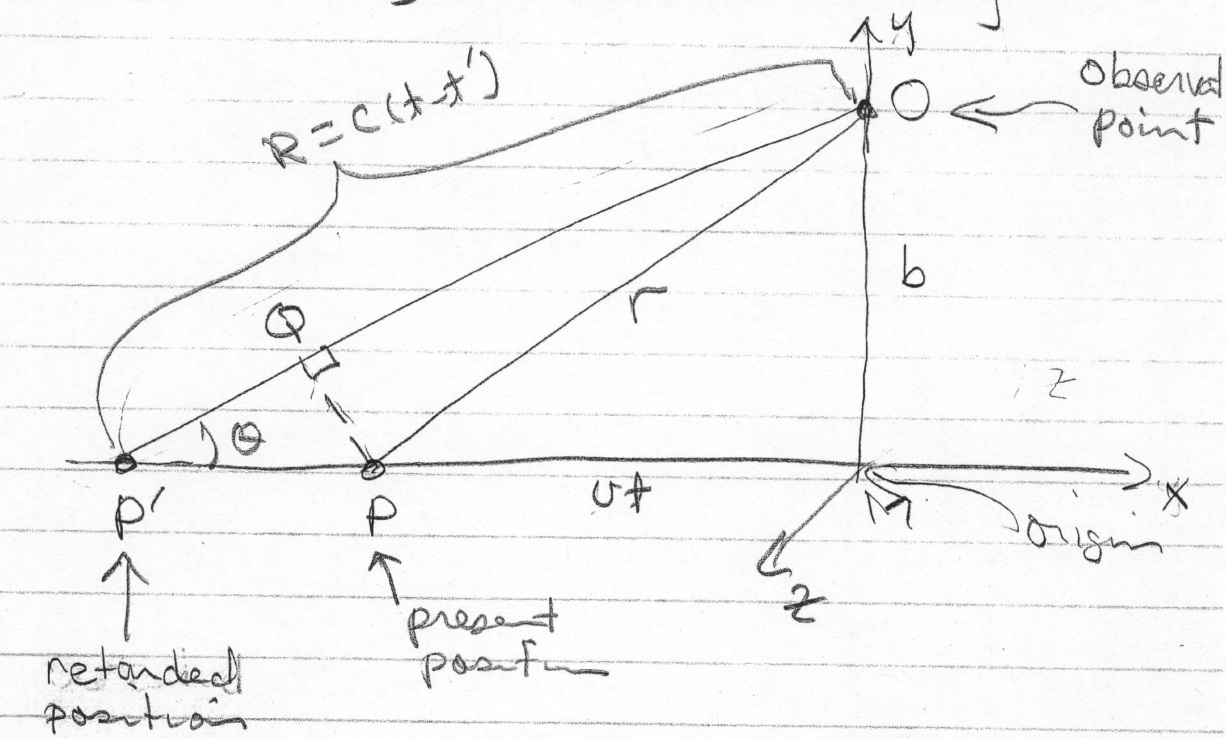
skip (30) Show that

radiation

$$\underset{\sim}{E} = \left[\frac{e(1-\beta^2)(\hat{n}-\underset{\sim}{\beta})}{(1-\underset{\sim}{\beta}\cdot\hat{n})^3 R^2} + \frac{e}{c}\frac{\hat{n}\times(\hat{n}-\underset{\sim}{\beta})\times\dot{\underset{\sim}{\beta}}}{(1-\underset{\sim}{\beta}\cdot\hat{n})^3 R}\right]_{t'}$$

$$\underset{\sim}{B} = \left[\hat{n}\times\underset{\sim}{E}\right]_{t'} \qquad \hat{n} = \frac{\underset{\sim}{R}}{R}$$

charge moving at constant velocity



$$P'P = v(t-t') = \beta R$$

$$P'Q = \beta R\cos\theta = \underset{\sim}{\beta}\cdot\underset{\sim}{R} \quad , \quad PQ = \beta R\sin\theta$$

$$QO = R - \underset{\sim}{\beta}\cdot\underset{\sim}{R}$$

$$(QO)^2 = r^2 - (PQ)^2 = r^2 - \beta^2\underbrace{R^2\sin^2\theta}_{b^2}$$

$$\therefore \quad (R - \underset{\sim}{\beta}\cdot\underset{\sim}{R})^2 = b^2 + v^2t^2 - \beta^2 b^2$$

$$= (1-\beta^2)\,b^2 + v^2t^2 = \frac{1}{\gamma^2}\left[b^2 + \gamma^2 v^2 t^2\right]$$

$$\underset{\sim}{E} = \left[\frac{e}{\gamma^2}\frac{(\underset{\sim}{R}-\underset{\sim}{\beta}R)}{(\underset{\sim}{R}-\underset{\sim}{\beta}\cdot\underset{\sim}{R})^3}\right]_{t'}$$

$$E_y = \left[\frac{e\,b}{\gamma^2(\underset{\sim}{R}-\underset{\sim}{\beta}\cdot\underset{\sim}{R})^3}\right]_{t'+} = \frac{e\,b\,\gamma}{[b^2+\gamma^2v^2t^2]^{3/2}}$$

note that $[\gamma]_{t'+} = \gamma$

Homework

(31) Show that

$$E_x = \frac{e\gamma v t}{[b^2+\gamma^2v^2t^2]^{3/2}}$$

$$B_z = \beta E_y$$

$$E_z = B_x = B_y = 0$$

start

Radiation

In far zone, we drop fields that fall off faster than $1/R$.

$$\underset{\sim}{E} = \left[\frac{e}{c} \frac{\hat{n} \times (\hat{n} - \underset{\sim}{\beta}) \times \dot{\underset{\sim}{\beta}}}{(1 - \underset{\sim}{\beta} \cdot \hat{n})^3 R} \right]_{t'}, \qquad \underset{\sim}{B} = [\hat{n} \times \underset{\sim}{E}]_{t'}$$

Consider accelerating charge in reference frame where $\beta \ll 1$

$$\underset{\sim}{E} \simeq \left[\frac{e}{c} \frac{\hat{n} \times \hat{n} \times \dot{\underset{\sim}{\beta}}}{R} \right]_{t'} \qquad \underset{\sim}{B} = [\hat{n} \times \underset{\sim}{E}]_{t'}$$

$$\frac{dP}{d\Omega} = \frac{c}{4\pi} \frac{e^2}{c^4} |\hat{n} \times \dot{\underset{\sim}{v}}|^2_{t'} = \frac{e^2 |\dot{\underset{\sim}{v}}|^2 \sin^2\Theta}{4\pi c^3}$$

not time average

at retarded time

$$P = \int d\Omega \frac{dP}{d\Omega} = \frac{2}{3} \frac{e^2 |\dot{\underset{\sim}{v}}|^2}{c^3} \quad \text{Larmor formula}$$

relation to electric dipole radiation

consider a localized collection of non-relativistic charges

$$\underset{\sim}{E} = \sum_j \left[\frac{e_j}{c^2} \frac{\hat{n}_j \times \hat{n}_j \times \dot{\underset{\sim}{v}}_j}{R_j} \right]_{t'_j} \underset{\uparrow}{\simeq} \left[\frac{\hat{n} \times \hat{n} \times \sum_j e_j \dot{\underset{\sim}{v}}_j}{c^2 R} \right]_{t'}$$

lowest order $(t'_j \simeq t'_i = t')$

$$\sum_j e_j \dot{\underset{\sim}{v}}_j = \frac{d^2}{dt^2} \sum_j e_j \underset{\sim}{r}_j = \ddot{\underset{\sim}{p}}$$

$$\underset{\sim}{E} \simeq \left[\frac{\hat{n} \times \hat{n} \times \ddot{\underset{\sim}{p}}}{c^2 R} \right]_{t'}$$

$$\frac{dP}{d\Omega} = \frac{1}{4\pi c^3} |\hat{n} \times \ddot{\underset{\sim}{p}}|^2 = \frac{1}{4\pi c^3} |\ddot{\underset{\sim}{p}}|^2 \sin^2\theta$$

at retarded time

not time average

$$P = \int d\Omega \frac{dP}{d\Omega} = \frac{2}{3} \frac{|\ddot{p}|^2}{c^3}$$

for $\underset{\sim}{P}(t) = R_0 P_0 e^{-i\omega t}$

$$\langle P \rangle = \frac{1}{\cancel{2}} \frac{\cancel{2}}{3} \frac{|P_0|^2 \omega^4}{c^3}$$

L+L extend this treatment to magnetic dipole and electric quadrupole (note where these terms enter $\gamma_j' \mp \gamma_2'$)

review

$$P = \frac{2}{3} \frac{|\ddot{\underset{\sim}{p}}|^2}{c^3} + \frac{2}{3} \frac{|\ddot{\underset{\sim}{m}}|^2}{c^3} + \frac{1}{180 c^5} \sum_{\alpha\beta} |\dddot{Q}_{\alpha\beta}|^2$$

start

<u>radiation during collision of non-relativistic particles</u>

1. electron-ion (bremsstrahlung)
(electric-dipole radiation)

dipole moment

$$\underset{\sim}{p} = e(\underset{\sim}{r}_1 - \underset{\sim}{r}_2) = e\underset{\sim}{r}$$

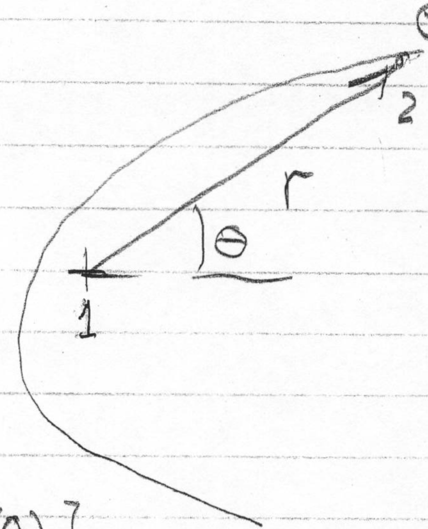
dipole

$$\ddot{\underset{\sim}{p}} = e\ddot{\underset{\sim}{r}} = \frac{e}{\mu}(\mu \ddot{\underset{\sim}{r}}) = -\frac{e^3}{\mu} \frac{\underset{\sim}{r}}{r^3}$$

$$dt = dt' \qquad t - t' = \frac{R}{c}$$

$$W \simeq \frac{2}{3} \frac{1}{c^3} \frac{e^6}{\mu^2} \int_{-\infty}^{+\infty} \frac{dt}{r^4(t)}$$

along orbit neglecting radiation



$$\mu r^2 \dot{\theta} = l = \text{const.}$$

$$dt = \frac{\mu r^2 d\theta}{l}$$

$$\frac{1}{r} = \frac{\mu e^2}{l^2}\left[1 - \sqrt{1 + \frac{2El^2}{\mu e^4}}\cos(\theta)\right]$$

$$W \simeq \frac{2}{3}\frac{e^6}{c^3 \mu^2}\frac{\mu}{l}\int \frac{d\theta}{r^2(\theta)}$$

etc.

2. electron-electron collision

$$\underset{\sim}{\dot{p}} = \frac{e}{m}(m\underset{\sim}{v_1} + m\underset{\sim}{v_2})$$

$$\therefore \quad \underset{\sim}{\ddot{p}} = 0$$

$$\underset{\sim}{m} = \frac{e}{2c}\underset{\sim}{r_1} \times \underset{\sim}{v_1} + \frac{e}{2c}\underset{\sim}{r_2} \times \underset{\sim}{v_2} = \frac{e}{2cm}(\underset{\sim}{l_1} + \underset{\sim}{l_2})$$

$$\therefore \quad \ddot{\underset{\sim}{m}} = 0$$

∴ electric quadrupole radiation

$$W = \frac{1}{180c^5}\int dt \sum_{\alpha\beta} |\dddot{Q}_{\alpha\beta}|^2$$

see L&L sec 69 and 71 for several interesting problems

quals problem

<u>relativistic generalization of Larmor formula</u>

$dW = P\,dt$ is 0th component of 4-vector

∴ P is scalar

in rest frame of accelerating particle

momentum

$$P = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{d\vec{p}}{dt}\right)^2$$

covariant generalization

$$P = -\frac{2}{3}\frac{e^2}{m^2c^3}\frac{dP_\mu}{d\tau}\frac{dP^\mu}{d\tau}$$

$$-\frac{dP_\mu}{d\tau}\frac{dP^\mu}{d\tau} = \left(\frac{d\mathbf{p}}{d\tau}\right)^2 - \underbrace{\frac{1}{c^2}\left(\frac{d\mathcal{E}}{d\tau}\right)^2}_{\beta^2\left(\frac{d\mathbf{p}}{d\tau}\right)^2} \xrightarrow[\beta \to 0]{} \left(\frac{d\mathbf{p}}{dt}\right)^2$$

$$\mathcal{E}^2 = p^2c^2 + m^2c^4$$

$$2\mathcal{E}\frac{d\mathcal{E}}{d\tau} = 2p\frac{dp}{d\tau}c^2 \quad \therefore \quad \frac{d\mathcal{E}}{d\tau} = \underbrace{\frac{pc^2}{\mathcal{E}}}_{v}\frac{dp}{d\tau}$$

This is only generalization that involves only β and $\dot{\boldsymbol{\beta}}$. We know from Liénard-Wiechert fields that generalization can only involve $\boldsymbol{\beta}$ and $\dot{\boldsymbol{\beta}}$.

start

Using $\mathcal{E} = \gamma mc^2$, $\mathbf{p} = \gamma m\mathbf{v}$, $\frac{dt}{d\tau} = \gamma$ we find that

$$P = \frac{2}{3}\frac{e^2}{m^2c^3}\gamma^2\left[\left(\frac{d\mathbf{p}}{dt}\right)^2 - \underbrace{\frac{1}{c^2}\left(\frac{d\mathcal{E}}{dt}\right)^2}_{\beta^2\left(\frac{d\mathbf{p}}{dt}\right)^2}\right]$$

$\xrightarrow{\text{algebra}}$

$$P = \frac{2}{3}\frac{e^2\gamma^6}{c}\left[(\dot{\boldsymbol{\beta}})^2 - (\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}})^2\right]$$

linear versus transverse acceleration

linear $\quad \left|\frac{d\mathbf{p}}{dt}\right| = \left|\frac{dp}{dt}\right|$

$$P_L = \frac{2}{3}\frac{e^2}{m^2c^3}\underbrace{\gamma^2(1-\beta^2)}_{=1}\left|\frac{d\mathbf{p}}{dt}\right|^2$$

transverse $\quad \frac{d\mathbf{p}}{dt}\cdot\mathbf{p} = 0 \Rightarrow \frac{d\mathcal{E}}{dt} = 0$

$$P_T = \frac{2}{3}\frac{e^2}{m^2c^3}\gamma^2\left(\frac{d\mathbf{p}}{dt}\right)^2$$

For $\left|\frac{d\mathbf{p}}{dt}\right| = |\mathbf{F}| = \left|\frac{dp}{dt}\right|$, $\quad P_\perp = \gamma^2 P_L$.

Advantage of linear accelerators versus circular accelerators.

using velocity vectors

$$P = \frac{2}{3}\frac{e^2\gamma^6}{c}\left[(\dot{\underset{\sim}{\beta}})^2 - (\underset{\sim}{\beta}\times\dot{\underset{\sim}{\beta}})^2\right]$$

$$P_L = \frac{2}{3}\frac{e^2\gamma^6}{c}(\dot{\underset{\sim}{\beta}})^2 \ , \quad \text{since } \underset{\sim}{\beta}\times\dot{\underset{\sim}{\beta}} = 0$$

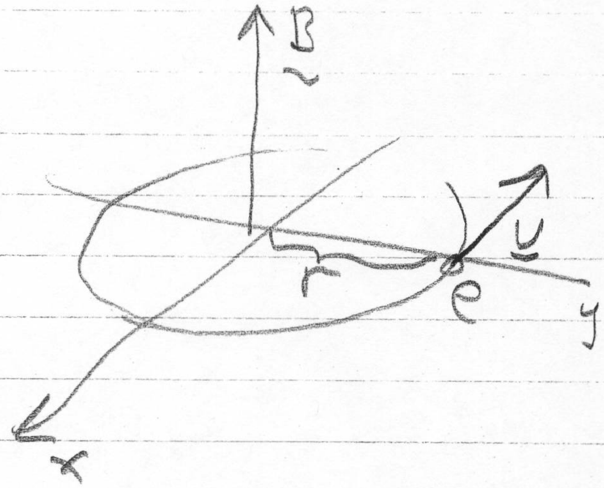
$$P_T = \frac{2}{3}\frac{e^2\gamma^6}{c}(1-\beta^2)(\dot{\underset{\sim}{\beta}})^2$$

$$= \frac{2}{3}\frac{e^2\gamma^4}{c}(\dot{\underset{\sim}{\beta}})^2$$

example J. (14.9)

An electron moves in a plane under the influence of the magnetic field $\underset{\sim}{B}$

Neglect radiation while calculating orbit



$$\frac{d\underset{\sim}{p}}{dt} = \frac{e\underset{\sim}{v} \times \underset{\sim}{B}}{c}$$

$$= -\frac{evB}{c}\hat{r}$$

(a) Calculate power radiated

$$P_r = \frac{2}{3}\frac{e^2}{m^2c^3}\gamma^2\frac{e^2v^2B^2}{c^2}$$

Power radiated

$$\frac{d}{dt}mc^2\gamma = -P_r = -\frac{2}{3}\frac{e^4B^2}{m^2c^3}\frac{v^2}{c^2}\gamma^2$$

We have assumed that

$$\frac{1}{\Omega_c}\left|\frac{d}{dt}\gamma mc^2\right| \ll \gamma mc^2$$

$$\frac{1}{\frac{eB}{\gamma mc}} \frac{e^4 B^2}{m^2 c^3}\frac{v^2}{c^2}\gamma^2 \ll \gamma mc^2$$

$$\frac{e^3 B}{m^2 c^4}\frac{v^2}{c^2}\gamma^2 \ll 1$$

$$\underbrace{\left(\frac{e^2/mc^2}{c}\right)}_{\frac{r_{cl}}{c}}\frac{eB}{mc}\frac{v^2}{c^2}\gamma^2 \ll 1$$

$$\left(10^{-23}\,\cancel{\text{sec}}\right)\left(\underbrace{10^7 \frac{B}{\text{Gauss}}}_{10^{12}}\frac{1}{\cancel{\text{sec}}}\right)\frac{v^2}{c^2}\gamma^2 \ll 1$$

for $B = 100$ kG $= 10$ tesla

(b) find $\gamma(t)$ for $\gamma_0 \gg 1$

$$\frac{v}{c} \simeq 1$$

$$\frac{d\gamma}{dt} = -\frac{2}{3}\frac{e^4 B^2}{m^3 c^5}\gamma^2$$

$$\frac{d}{dt}\frac{1}{\gamma} = +\frac{2}{3}\frac{e^4 B^2}{m^3 c^5}$$

$$\frac{1}{\gamma} - \frac{1}{\gamma_0} = \frac{2}{3}\frac{e^4 B^2}{m^3 c^5}t$$

(c) find $\frac{mv^2(t)}{2}$ for $\frac{v}{c} \ll 1$

$$\frac{d}{dt}\frac{mv^2}{2} = -\frac{2}{3}\frac{e^4 B^2 v^2}{m^2 c^5} = -\frac{4}{3}\frac{e^4 B^2}{m^3 c^5}\frac{mv^2}{2}$$

$$\frac{mv^2(t)}{2} = \frac{mv^2(0)}{2}e^{-\frac{4 e^4 B^2}{3 m^3 c^5}t}$$

Angular distribution of radiation

from Lienard - Wiechert.

$$\underset{\sim}{E} = \left[\frac{e}{c} \frac{\hat{n} \times [(\hat{n} - \underset{\sim}{\beta}) \times \dot{\beta}]}{R(1 - \hat{n} \cdot \beta)^3} \right]_{t'} \qquad \underset{\sim}{B} = [\hat{n} \times \underset{\sim}{E}]_{t'}$$

$$\hat{n} = \underline{R}(t') / R(t')$$

$$[\underset{\sim}{S} \cdot \hat{n}] = \frac{c}{4\pi} \frac{e^2}{c^2} \left| \frac{\hat{n} \times [(\hat{n} - \underset{\sim}{\beta}) \times \dot{\beta}]}{R(1 - \hat{n} \cdot \underset{\sim}{\beta})^3} \right|^2_{t'}$$

The useful quantity is the radiated power in terms of the charge's own time.

$$\frac{dP}{d\Omega}(t') = [R^2 \underset{\sim}{S} \cdot \hat{n}] \frac{dt}{dt'}$$

where

$$\frac{dW}{d\Omega} = \int_{t_1'}^{t_2'} \frac{dP(t')}{d\Omega} dt'$$

Short acceleration period so $\hat{n}$, $R(t')$, $d\Omega$ are const. during acceleration page 130 notes

using $\frac{dt}{dt'} = (1-\beta\cdot\hat{n})_{t'}$

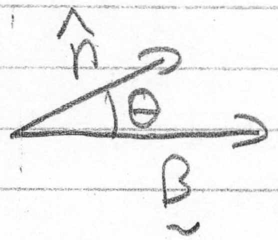
$$\frac{dP(t')}{d\Omega} = \frac{e^2}{4\pi c}\frac{\left|\hat{n}\times[(\hat{n}-\beta)\times\dot{\beta}]\right|^2}{(1-\hat{n}\cdot\beta)^5}\Bigg|_{t'}$$

(1) linear acceleration $\rightarrow \beta\times\dot{\beta} = 0$

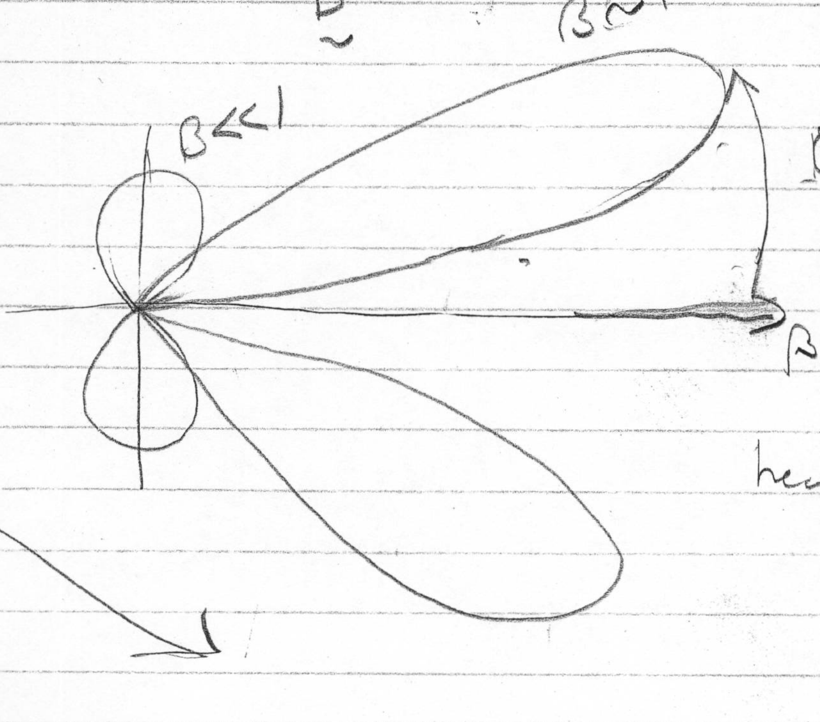
$$\frac{dP(t')}{d\Omega} = \frac{e^2\dot{v}^2}{4\pi c^3}\frac{\sin^2\theta}{(1-\beta\cos\theta)^5}$$

for $\beta\to 1$, $\theta \ll 1$

$$\frac{\theta^2}{[\underbrace{1-\beta}_{\frac{1}{2\gamma^2}}+\frac{\theta^2}{2}]^5}$$



$\theta_{max} \sim \frac{1}{\gamma}$

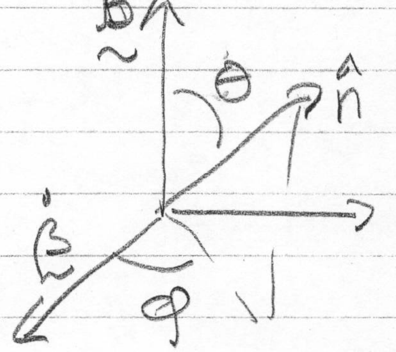


head lights

$$\int \frac{dP(t)}{d\Omega}d\Omega = \frac{2}{3}\frac{e^2\dot{v}^2}{c^3}\gamma^6$$

(2) transverse acceleration

$\underset{\sim}{\beta} \cdot \dot{\underset{\sim}{\beta}} = 0$

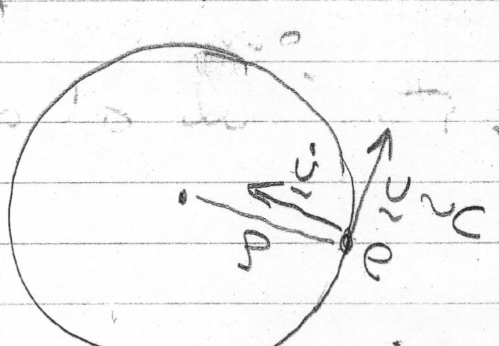


$$\frac{dP}{d\Omega}(t') = \frac{e^2}{2\pi c^3} \frac{|\dot{\underset{\sim}{v}}|^2}{(1-\beta\cos\theta)^3}\left[1 - \frac{\sin^2\theta\cos^2\varphi}{\gamma^2(1-\beta\cos\theta)^2}\right]$$

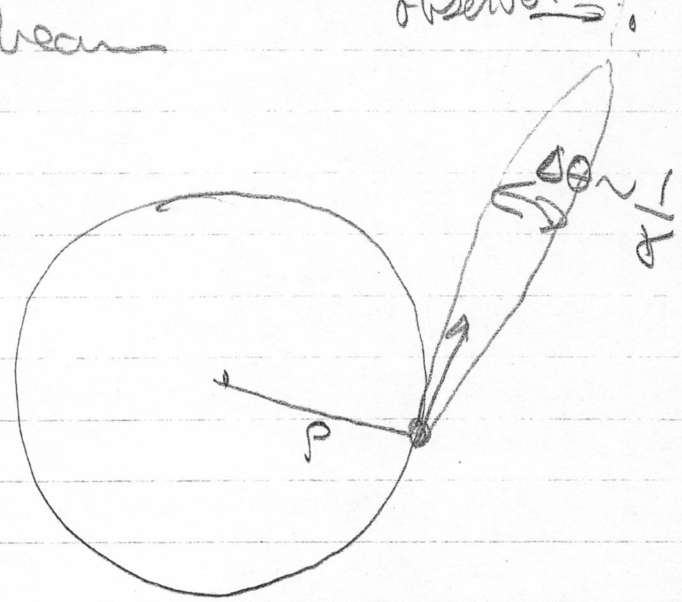
forward peaked $\theta_{max} \sim 1/\gamma$

Consider a charge that moves relativistically in circular motion

$\dot{\underset{\sim}{v}} = \frac{v^2}{\rho} \simeq \frac{c^2}{\rho}$



radiation is directed forward in narrow beam



$d \equiv$ distance traveled by particle during time particle moves through $\Delta\theta \sim 1/\gamma$

$$\therefore \quad d = \rho\Delta\theta \simeq \rho/\gamma$$

particle's time $\rightarrow$ $$\Delta t' = \frac{d}{c} \simeq \frac{\rho}{\gamma c}$$

observers time $\rightarrow$ $$\Delta t = \frac{dt}{dt'}\Delta t' \simeq \underbrace{(1-\beta)}_{\frac{1}{\gamma^2}}\Delta t' \simeq \frac{\rho}{\gamma^3 c}$$

The short pulse length implies high Fourier components

$$\omega \simeq \frac{1}{\Delta t} \simeq \underbrace{\frac{c}{\rho}}_{\omega_0} \gamma^3$$

$\omega_0 \leftarrow$ freq of circular motion

$$\therefore\ n = \frac{\omega}{\omega_0} \sim \gamma^3$$ can be very large

<u>Example</u> 10 GeV synchrotron at Cornell

$$\gamma_{max} \simeq \frac{10\ \text{GeV}}{.5\ \text{MeV}} = 2.\times 10^4$$

$$\omega_0 \simeq 3\times 10^6\ \text{sec}^{-1}$$

$$\omega_{max} \sim 3\times 10^6\ \text{sec}^{-1}\ (2\times 10^4)^3 \simeq 24\times 10^{18}\ \text{sec}^{-1}$$

16 keV x-rays